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DEVELOPING A NEW METHOD OF TEACHING,

(From Principles, in preference to Dogmatic Rules,)

THE SCIENCE AND ART OF COMPUTATION.

WITH UPWARDS OF

EIGHT THOUSAND EXAMPLES FOR PRACTICE.

BY

JAMES A. CHRISTIE, C.S., M.C.P.,
HEAD MASTER OF THE DUKE OF BEDFORD'S SCHOOL, MILTON ABBOT, TAVISTOCK.
Author of the Constructive Etymological Spelling-Book.

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PREFACE.

IN proposing the exposition of a system by which Numerical Computation may be effectually taught from Principles, addressed to, and serving to cultivate, the pupil's judgment, instead of by dogmatic Rules, demanding from him no more than passive obedience and the exercise of mere mechanical memory, THE CONSTRUCTIVE ARITHMETIC does but follow, however remotely, in the path marked out by many a treatise of established and extended reputation. From all these, however, its noble predecessors, it differs as essentially in plan and mechanism as it accords with them in aim and purpose: its peculiar features being so many and so marked, as to confer on it, in the absence of whatever other recommendation, the distinction, at least, of possessing a positive and clearly-defined individuality. Among its principal peculiarities are these:

I. *Nothing is taught which requires to be afterwards contradicted.* It has been the practice of even eminent arithmetical writers to commence with two statements: *a.* That, in any line of figures, the right-hand place is that of units. *b.* That ciphers to the left of a figure do not affect its value. Both which propositions, being only true of figures representing Integers, must be abandoned and contradicted when the Notation of Decimals is encountered. But, by the very simple and natural device of restoring the Decimal Point (see page 2·) to its legitimate position as the origin of local value in figures, we at once escape the necessity of this false start in Notation of Integers, and provide for that of Decimals, by the easiest of gradations, as soon as the nature of a Fraction shall have been explained. (See PRINCIPLES I. and II., page 3·)

II. *Vulgar and Decimal Fractions are introduced at the earliest possible stage.* Considering that Fractions cannot be avoided in Simple Division, few proceedings can be less logical than postponing the exposition of them to a comparatively remote stage of arithmetical study, or treating them as constituting a distinct department of the science of numbers. The leading modern Arithmeticians have, therefore, unanimously renounced this vicious arrangement, and are constantly bringing Fractions nearer and nearer to the threshold of their methods. Going to the utmost point in this direction, we here exhibit the Formation of Fractions, Vulgar and Decimal, immediately after Notation and Numeration of Integers.

III. *Decimals are taught with Integers in the elementary Rules.* By this early exposition of Decimals, we are enabled to restore them to their true connection with Integers, and to teach the treatment of both at once, in the rudimentary processes of Addition, Multiplication, Subtraction, and Division. This is, in itself, an important advantage. And it leads to others; among which are the prominence necessarily given to the truth, that such Multipliers (and Divisors) as 6000· and ·006 are strictly analogous in their effect on the Product (or Quotient), and the consequent exclusion of every pretext for separating, under different and widely-sundered rules, as is invariably done, two operations which are essentially identical in principle and in effect.

IV. *Tracing the Process.* A fundamental idea pervading the entire of this work, and more distinctly conspicuous in the second part of it, is that: not only should the pupil's judgment be satisfied that the process which he is taught to adopt in the solution of any question is the true one,—that the prescribed route must, of necessity, conduct him to the desired goal,—but, also, that he should, during and throughout his journey, everywhere find himself in a position to demonstrate that each particular step which he is, at any moment, taking, is accurately directed thither. So that, like a skilful navigator, after taking a true departure, on the true course, and, pausing at any point in his voyage, he may perceive precisely where he is, how and why he arrived there, and whether he is still steering towards his destined port.

The advantages of this practice are too obvious for advocacy. In order to secure them, the pupil is called on, throughout the exercises of The Constructive Arithmetic, to explain in writing every line of the work of every solution which he effects. Than this minute recognition and progressive record of the part which each successive line and each individual figure performs in building up the ultimate result of the operation, there is no more valuable feature in the method here developed. It frequently unveils important truths which might otherwise elude observation. (See pages 335· to 337·)

v. *Improved Elementary Rules.* The practice of thus Tracing the Process through every line and every figure, leads, also, to the detection of such obscurity or irreconcileable contradiction as imposes on us the necessity of modifying certain of the ordinary elementary rules. As an example of the former defect, take 649 × 358 in the ordinary way, where, by the injudicious omission of ciphers, a young student is, so to speak, *tempted* to misapprehend the second and third lines of the work, which (having learned to call the right-hand place units) he would be almost certain to read, and would, in fact, be justified in reading, as 3245 and 1947, instead of 32,450· and 194,700·, as they really are.

$$\begin{array}{r} 649 \\ 358 \\ \hline 5192 \\ 3245 \\ 1947 \\ \hline 232342 \\ \hline\hline \end{array}$$

$$\begin{array}{rl} 649\cdot & \\ 358\cdot & \\ \hline 5192\cdot = & 8\cdot \text{ times } 649\cdot \\ 32450\cdot = & 50\cdot \text{ times } 649\cdot \\ 194700\cdot = & 300\cdot \text{ times } 649\cdot \\ \hline 232342\cdot = & 358\cdot \text{ times } 649\cdot \\ \hline\hline \end{array}$$

PRINCIPLES VI. and XXV.

We effectually seal this source of error by the constant use of the Decimal Point, and by the restoration of the improperly-omitted ciphers. With this slight modification, the operation becomes clear and logical, and Tracing the Process, easy.

$$\begin{array}{r} 389 \\ \cdot 25 \\ \hline 1945 \\ 778 \\ \hline 97\cdot 25 \\ \hline\hline \end{array}$$

As an instance of palpable contradiction, Multiply 389 by ·25 according to the established method. Here, in attempting to trace the process, we are confronted by three astounding statements:

a. That (389 × ·05) that is ($\frac{1}{20}$ of 389) = 1945 !
b. That (389 × ·2) that is ($\frac{1}{5}$ of 389) = 778(0) !
c. That 1945 + 778(0) = 97$\frac{1}{4}$!

the last line being only made true by flatly contradicting the two preceding ones, from which it is derived;

for, to interpret those two lines so as to make them consistently conducive to the result which is thus illogically extorted from them, is manifestly impossible.

$$\begin{array}{l} 389\cdot \\ \underline{\quad \cdot 25} \\ 19{\cdot}45 = {\cdot}05 \text{ of } 389\cdot \\ \underline{77{\cdot}8\ \ } = {\cdot}2 \text{ of } 389\cdot \\ \underline{\underline{97{\cdot}25}} = {\cdot}25 \text{ of } 389\cdot \end{array}$$

By our method, on the contrary, the example is so worked that every line is kept to its true meaning, and then the derivation of the result from the foregoing lines becomes obvious and natural. See PRINCIPLES XII. and IX. Page 171·

VI. *Elementary Processes based on Experiment.* In teaching Arithmetic, nothing can be more proper than to found the pupil's primary notions of numbers and their relations upon actual, experimental, manipulation of real objects. Hence, the collection, diminution, repetition, and distribution of manageable units, as well as the partition of single units into Fractions, are now universally adduced, by arithmetical writers, as the groundwork for their expositions of the essential elementary operations in numbers. Following so sound an example, The Constructive Arithmetic is constantly appealing, in its rudimentary department, to actual experiment with peas, pebbles, strips of paper, the strings of balls composing the Arithmeticon, or Abacus table, &c. It does more than this. It takes effectual care that, however variously an elementary process may be afterwards applied, it shall never be so distorted or obscured as to be no longer demonstrable by the humble experiment by which it was at first elucidated. This is by no means a vain precaution. It is especially necessary in treating of the applications of Division; for in these varied applications there is a constantly-recurring danger of misapprehending the true nature of that operation. This danger originates in the fact, that: "Dividing by any number produces the same figures as Multiplying by its Reciprocal;" and, therefore, Division *seems* (only *seems*) to yield the same result as Multiplication. That this is only an apparent, not a real, identity—a coincidence, and not an equality, of results—we are enabled to shew conclusively, by keeping incessantly in view the true nature of pure experimental Division. See pages 101· to 103·

If Dividing by any Number really produced the same result as Multiplying by its Reciprocal, then Division and Multiplication must be essentially the same process, which no one pretends them to be.

Neither is Division the reverse of Multiplication, as is commonly said; for Division is nothing more or less than *Distribution*, and, therefore, neither increases nor diminishes the Dividend, but merely re-arranges it; whereas, Multiplication, if by an Integer, *increases*, and, if by a Fraction, *decreases* the Multiplicand. See page 98· and Principle xiv., page 99·

The importance of maintaining, conspicuously, the distinction between coincident and equal numerical expressions, has been held to justify the introduction of a new arithmetical symbol ($\doteq$), to be employed as the sign of numerical coincidence. Page 103·

viii. *The Implicit Change of Unit.* The true idea of actual, mechanical Division being consistently adhered to throughout this work, we are, under its guidance, led to the discovery of the hidden, or, as we say, implicit change of unit, producing, in Compound Division, a succession of Concrete Divisors, so that each denomination in a Compound Dividend shall have, as it necessarily must have, a Divisor of its own kind: Pounds being only divisible by Pounds, Shillings by Shillings, and so on. Page 218· and Principle xiv., page 99· It is only by the light of this discovery that Compound Division can be intelligibly analysed, or shewn to be identical with that pure and simple distribution of actual units which it professedly represents. Thus elucidated, however, it becomes an interesting study. Page 224·

The Implicit Change of Unit is, also, the only key to Concrete Reduction. For, Multiplication by an Integer being only repetition, how can Multiplying Pounds by 20·, that is repeating them 20· times, turn them into Shillings? Page 84·

Practice, too, is founded on the Change of Unit. Page 253· Barter cannot be rationally analysed without encountering it.

In deriving numbers representing Superficial or Solid units from factors expressing Lineal units, whether by Decimal or by Duodecimal Multiplication, successive implicit changes of unit constitute the very essence of the process. See pages 269· to 276·, and 280· to 292· In short, it would be difficult to discover any application of Arithmetic to concrete computations into which "Change of Unit" does not enter as an essential and pervading element.

IX. *Principles enunciated as Substantive Propositions.* The Constructive Arithmetic employs the word "Principle" with more than ordinary precision. The term does not here mean, as in some excellent treatises, a more or less terse and lucid explanation of the "Reason of the Rule;" nor, as in others, the evasion of a dogmatic rule by substituting a common-sense application of elementary processes, in the same way as we supersede the Rule of Three by working "without reference to ratio" (see pages 312· *et seq.*), which some writers would describe as "Solving by First Principles;" nor does it signify the expression of a truth, or description of a process, by means of an algebraic formula.

A "Principle" here, is a fundamental arithmetical truth, originating either in the nature of numbers or in the properties of the Decimal Notation, and on which one or more arithmetical operations is wholly or in part founded. Twenty-seven propositions, with four corollaries, embody all the arithmetical truths necessary to establish whatever is taught in this volume. These are successively evolved in the course of the work, demonstrated, (by experiment or analogy, and without algebraic aid,) propounded in brief form, and numbered for subsequent reference. The first twenty-six are collected on pages 170· to 172· The twenty-seventh, illustrating Duodecimal Multiplication, will be found on page 275·

X. *Minute Discussion of advanced Rules.* Although by the possession of these Principles the student is, to a great extent, rendered independent of special rules, and although by an unusually ample development of the Four Elementary Processes, Simple and Compound, (Vulgar and Decimal Fractions being everywhere made available,) we have largely encroached on the ground ordinarily occupied by such special rules; having, in fact, gone far towards entirely superseding very many of them; yet they are here not only given, but demonstrated and illustrated as elaborately as if they constituted the staple of the volume.

Lord Bacon recommends that "Books should be tasted." To facilitate this mode of testing the present work, the following may be mentioned as some of the *morceaux* which are most thoroughly saturated with its peculiar and characteristic flavour.

Subtraction, Simple and Compound, which are here effected, in figures, precisely as they would be with the actual units. The common circuitous method of increasing both quantities being abandoned, carrying is no longer necessary. See pages 82· and 207·

Greatest Common Measure, demonstration simplified and condensed. Page 132·

Least Common Multiple, rule improved, and established by a new demonstration. Page 135·

Recurring Decimals, simplified, and rules combined. Page 142·

The Tables are arranged on a new and natural system, and developed with peculiar amplitude and precision of detail and illustration.

The Proposed Decimal Coinage is fully discussed, and the comparative advantages of £ *s*. *d*. and of the "Pound and Mil scheme," are minutely and impartially displayed. Pages 234· to 246·

Square and Cubic Measures, by Decimal and by Duodecimal Multiplication, dissected and illustrated with minute precision, and with the exposition of some new methods. See pages 269·, 271·, &c.

Practical Applications of Rudimentary Mensuration to Bricklayers', Masons', Carpenters', Painters', and Paperhangers' work, as well as to the computation of simple Land-areas, Weight of Hay-ricks, Weight of live Cattle, Bulk and Weight of Timber, Content of Cisterns, &c., &c., have received careful attention, and are all presented under aspects more or less novel and attractive. Pages 277· to 294·

Specific Gravity is explained in a simple manner; and a number of interesting and useful calculations founded upon it. Pages 294· to 307·

Proportion, or Rule of Three, into which, it is hoped, new light and interest have been thrown, by the minute discussion of its principle and applications; and, especially, by the curious considerations detected in tracing the process in Inverse Proportion examples, and by the exhibition of an interesting device for mutual transmutation of Inverse and Direct Proportions. See pages 145·, 330·, 335·, *et seq.*

Barter: ordinary errors of reasoning exposed and corrected. Pages 338· to 342·

Compound Proportion, or Double Rule of Three, where the construction of the question, by the combination of two or more Simple Proportions, is exhibited in a new and easily-intelligible manner, and the process simplified and clearly traced. Pages 345· and 347·

Equation of Payments: the usual rule extricated from its chronic uncertainty, by referring the balance of advantages to the end of the Term, instead of to the day of Equated Payment. Page 382·

Other points of peculiarity will be readily discovered upon inspection of the Table of Contents, which is in sufficient detail to serve as an analytical index.

The efficacy of any instrument, however well adapted to its intended purpose, must ever depend on the fidelity and skill with which it is employed. The effect of a drama calmly perused in the closet does not more feebly suggest the vivid picture which it may present when informed with light, life, and reality, by the actor's genius and zeal, than a system of teaching, however elaborately propounded on paper, fails to represent its own power as applied in actual practice by skilful instructors. On such of these as may so appreciate and absorb its spirit as to transfuse that spirit with quickened energy into their own teachings, this humble work must rely for its production of those valuable practical results, which the Author has spared neither toil nor time to render it competent, in their hands, to achieve.

THE DUKE OF BEDFORD'S SCHOOL,
MILTON ABBOT, TAVISTOCK,
October, 1865·

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ERRATA.

Page 45· Ex. 16· For "numbers" read "number."
" 123· Line 9· from foot. Read $\frac{6}{8}$ instead of $\frac{4}{8}$.
" 142· Line 14· from foot. Omit "or by nines."
" 166· Ex. 33· For "apples" read "pears."
" 269· Line 10· from foot. For "twelve" read "eight."
" 275· Line 19· Read "Duodecimal Multiplication."
" 276· Line 19· and following. For "lg." read "wide."
" " Line 24· For "14· ft. 11¼ in. long," read "9· ft. 6· in. 9· li. wide."
" 297· Line 5· from foot. For "many" read "may."
" 298· Line 18· For "40· feet" read "30· feet."
" 323· Line 14· from foot. For "And 9· times" read "And 19· times."
" " Line 13· from foot. For "4357· times" read "4347· times."
" 333· Ex. 12· For $\frac{1}{16}$ read $\frac{15}{16}$

⁂ The notes, throughout the work, are addressed to Pupil-Teachers.

THE CONSTRUCTIVE ARITHMETIC.

◆

PART I.

PRINCIPLES, AND ELEMENTARY PROCESSES.

Arithmetic is the science of number, and teaches the art of computation.

UNITS AND INTEGERS.

Any single undivided thing is called, in arithmetic, a unit.

Thus, a unit may be *an apple, a ball, an animal;* a certain weight, as *a pound, a hundred-weight, a ton;* or the unit may be a certain length, as *a foot, a mile;* or a space of time, as *a minute, a month, a year;* or it may be a surface of some certain size, or any other single entire thing.

Any number of units of the same kind make an integer or whole number.

If we take for our unit a square of card-board of a certain size, then, two of those squares will be the integer two, three of the squares will be the integer three, and so on.

One unit is called **unity**.

NOTATION.

Numbers are expressed by ten characters; nine digits or figures, and a cipher or nought.

The art of expressing numbers by digits or figures is called Notation.

One unit is	expressed	by the digit	1·	which is	called	"one."
Two units are	,,	by the digit	2·	,,	,,	"two."
Three units	,,	by the digit	3·	,,	,,	"three."
Four units	,,	by the digit	4·	,,	,,	"four."
Five units	,,	by the digit	5·	,,	,,	"five."
Six units	,,	by the digit	6·	,,	,,	"six."
Seven units	,,	by the digit	7·	,,	,,	"seven."
Eight units	,,	by the digit	8·	,,	,,	"eight."
Nine units	,,	by the digit	9·	,,	,,	"nine."
Nothing is	,,	by the cipher	0	,,	,,	"nought."

Significant Figures. The digits 1, 2, 3, 4, 5, 6, 7, 8, 9, are called significant figures, because each of them signifies a number. The cipher is not a significant figure, for it signifies no number. Its use will be hereafter shewn.

EXERCISE I.

*Write answers to the following questions.** What is arithmetic? What is taught by it? What is meant by the word "unit"? Name some units. Explain the word "integer." If the unit were an apple, what would the integer eight be? What would five apples be? How are numbers expressed? What is the art of expressing them in this way called? How many significant figures are there? Why are they called significant figures? Which of the ten arithmetical characters is not a significant figure? What is meant by "notation" in arithmetic?

NOTATION OF INTEGERS.

DECIMAL POINT, PLACES OF FIGURES.

Eight units are expressed thus, 8· The dot on the right of the digit is called the **Decimal Point.** Places of digits are counted from the decimal point. Digits representing integers, stand on the left of the decimal point.

The first digit to the left of the decimal point is in *the first place of integers.* The second digit to the left of the decimal point is in *the second place of integers.* The third digit to the left of the decimal point is in *the third place of integers*, and so on.

Hence, in the following line of figures, 873695041· digit 8 is in ninth place of integers; 6 is in sixth place of integers; 5 is in fourth place of integers; cipher is in third place of integers.

TENS, HUNDREDS, THOUSANDS, MILLIONS.

Ten units make .... *one ten.*
Ten tens make .... *one hundred.*
Ten hundreds make .... *one thousand.*
A thousand thousand make.. *one million.*

One unit and *one ten* make *eleven.*
Two units and *one ten* ,, *twelve.*
Three units and *one ten* ,, *thirteen.*
Four units and *one ten* ,, *fourteen.*
Five units and *one ten* ,, *fifteen.*
Six units and *one ten* ,, *sixteen.*
Seven units and *one ten* ,, *seventeen.*
Eight units and *one ten* ,, *eighteen.*
Nine units and *one ten* ,, *nineteen.*

Two tens make *twenty.*
Three tens ,, *thirty.*
Four tens ,, *forty.*
Five tens ,, *fifty.*
Six tens ,, *sixty.*
Seven tens ,, *seventy.*
Eight tens ,, *eighty.*
Nine tens ,, *ninety.*
Ten tens ,, *one hundred.*

---

* NOTE TO TEACHER. Every answer should include the terms of the question, so as to form in itself a complete proposition. Thus, the answer to the question, "What is arithmetic?" should be, "Arithmetic is the science of number," and not simply, "The science of number," words which convey no complete idea, when detached from the question. This method of replying ought to be enforced in every examination on whatever subject.

Since *four and ten* are *fourteen; six and ten, sixteen;* and so on, it appears that the syllable "*teen*" stands for the words "*and ten*." In the same way, the syllable "*ty*" stands for the word "*tens*," *sixty* meaning "*six tens*."

To express four units, put digit 4 in first place of integers, that is, next on the left to decimal point; thus, 4·

One ten is expressed by digit 1 *in second place of integers*, the first place being filled by a cipher; thus, 10·

If the cipher be removed, the 1 will then stand in the first place of integers, and will therefore represent but *one unit* instead of *one ten*, as it does now. The cipher, therefore, standing between the decimal point and a significant figure, alters the value of that figure. But, if a cipher, or any number of ciphers, stand on the left of a digit in any place of integers, no change will be made in the value of that digit. 01·, 001·, 0001·, are all alike in value, because, in all three, the 1 is in *the first place of integers.* Remember, then, the following important principles:

PRINCIPLE I. *The same digit is made to represent different numbers by changing its place, counting from the decimal point.*

PRINCIPLE II. *The cipher is employed to fill places where significant figures are not required; and the cipher is only of service when it stands between a significant figure and the decimal point.*

To express a number consisting of tens and units.

Separate the number into tens and units.

Then express the units by the proper digit in the first place of integers, and the tens by the proper digit in the second place of integers.

Eleven, being *one ten* and *one unit*,	is expressed	thus,	11·
Twelve, or *one ten* and *two units*	„	„	12·
Thirteen, or *one ten* and *three units*	„	„	13·
Twenty, or *two tens*	„	„	20·
Fifty, or *five tens*	„	„	50·
Ninety-four, or *nine tens* and *four units*	„	„	94·

EXERCISE II.

How are units expressed? What is the dot placed on the right of the digit called? What is the use of the decimal point? On which side of the decimal point do digits stand to represent integers? When is a digit said to be in the first place of integers? Name the place of each figure in the following line: 314625· What is a hundred? What is a million? How many tens make a thousand? How many hundreds make a million? What is meant by "fourteen," "eleven," "twelve"? What is meant by "sixty," "seventy," "eighty," "ninety"? What words does the syllable "teen" stand for? What does the syllable "ty" represent? How is ten expressed? What is the use of the cipher in this expression? Where would the significant figure be, and what would it represent, if the cipher were removed? How is the cipher employed? Where only is the cipher of use? Give an instance in which four ciphers would make no alteration in the value of a digit. Why is this? How is the same digit made to represent different numbers? In proceeding to express, by digits, a number consisting of tens and units, two steps are necessary; which is the first, and which the second step? How do you express eleven; and why do you so express it? How thirteen, ninety-six, fifty-five; and why? Write, in figures, the following numbers,* placing them in a column, with all the decimal points in one vertical line: nineteen, eleven, thirty-one, twenty-four, forty-two, sixteen, sixty-one, seventeen, seventy-one, fifteen, fifty-one, thirteen, thirty-one, fourteen, forty-one, twelve, twenty-one. Write, in figures, all the numbers from one to ninety-nine.

Hundreds are expressed by digits in third place of integers.

For one hundred, write digit 1 in third place of integers, filling first and second places by ciphers; thus, 100·

To express a number consisting of hundreds, tens, and units.

Separate the number into hundreds, tens, and units. Then express the units by the proper digit in first place of integers; the tens in second place; and the hundreds in third. Six hundred and eighty-three, being 6 hundreds, 8 tens, and 3 units, will be written thus, 683·

EXERCISE III.

Write in figures the following numbers. One hundred and sixty-four; four hundred and sixty-one; six hund. and fourteen; six hund. and forty-one; four hund. and sixteen; one hund. and forty-six; one hund.; two hund.; and so on, up to nine hund.; four hund. and three; three hund. and forty; four hund. and thirty; three hund. and four; one hund. and twelve; two hund. and eleven; one hund. and twenty-one.

Thousands are expressed by digits in fourth place of integers.

For one thousand, write digit 1 in fourth place of integers, filling first, second, and third places by ciphers; thus, 1000·

---

* Note to Teacher. The decimal point must on no account be omitted in these exercises. Its suppression is one of the most mischievous features of the illogical system on which arithmetic has been too long, and is, even now, too generally taught. The decimal point must not be mistaken for the full stop.

EXERCISE IV.

*Write in figures the following numbers.** One thousand four hundred and eighty-six; one thousand four hund. and sixty-eight; one thous. six hund. and forty-eight; one thous. six hund. and eighty-four; one thous. eight hund. and forty-six; one thous. eight hund. and sixty-four; four thous. one hund. and sixty-eight; four thous. one hund. and eighty-six; four thous. eight hund. and sixteen; four thous. eight hund. and sixty-one; four thous. six hund. and eighteen; four thous. six hund. and eighty-one; six thous. four hund. and eighteen; six thous. four hund. and eighty-one; six thous. eight hund. and fourteen; six thous. eight hund. and forty-one; six thous. one hund. and eighty-four; six thous. one hund. and forty-eight; eight thous. six hund. and fourteen; eight thous. six hund. and forty-one; eight thous. four hund. and sixteen; eight thous. four hund. and sixty-one; eight thous. one hund. and sixty-four; eight thous. one hund. and forty-six.

Observe that the foregoing twenty-four numbers are expressed by the same four digits differently placed. Principle I. page 3.

Ten thousand are expressed by digit 1 *in fifth place of integers, the other four places being filled by ciphers; thus,* 10000·

EXERCISE V.

Write in figures the following numbers. Twenty thousand; thirty thous.; forty thous.; fifty thous.; sixty thous.; seventy thous.; eighty thous.; ninety thous.

Ninety-two thous.; seventy-three thous. and one; eighteen thous. and forty; twenty-one thous. and twelve; eleven thous. and nineteen; fifty-four thous. one hund. and ten; seventeen thous. four hund. and fourteen; ten thous. ten hund. and ten;† fifteen thous. fifteen hund. and fifty; seventeen hund. and twelve; twenty-nine tens.

Eighty thous. three hund. and twelve; eighty-one thous. two hund. and three; eighty-three thous. and twenty-one; eighty-two thous. and thirteen; eighty-three thous. one hund. and two; eighty-one thous. two hund. and thirty; eighty thous. two hund. and thirty-one; eighty-three thous. and twelve; eighty-two thous. one hund. and thirty; eighty-one thous. and thirty-two; eighty-three thous. two hund. and one; eighty-one thous. and twenty-three; eighty-three thous. one hund. and twenty; eighty thous. one hund. and thirty-two; eighty-three thous. two hund. and ten; eighty-two thous. three hund. and ten; eighty thous. three hund. and twenty-one; eighty-two thous. three hund. and one; eighty thous. two hund. and thirteen; eighty-one thous. three hund. and twenty; eighty thous.

---

* NOTE TO TEACHER. It is considered unnecessary to repeat the direction to separate the numbers into their component parts. See pages 3 and 4.

† See Note, page 6.

one hund. and twenty-three; eighty-two thous. and thirty-one; eighty-one thous. three hund. and two; eighty-two thous. one hund. and three.

Observe that the foregoing twenty-four numbers are expressed by the same four digits and a cipher. In all these twenty-four numbers, the digit 8 occupies the fifth place of integers. If the 8 be moved from place to place, like the others, then the four digits and cipher will suffice to express *ninety-six* different numbers. And if a significant figure be employed instead of the cipher, then the *five* digits will be capable of expressing *one hundred and twenty* different numbers. See Principle I. page 3.

For one hundred thousand, we write digit 1 *in sixth place of integers; thus,* 100000·

EXERCISE VI.

Write in figures the following numbers. Six hundred thousand; seven hund. thous.; eight hund. thous.; nine hund. thous.; two hund. thous.; three hund. thous.; four hund. thous.; five hund. thous.

Eight hund. and ninety-five thous. four hund. and twelve; three hund. and eleven thous. and thirteen; five hund. thous. and twenty-nine.

Nine hund. and forty-three thous. six hund.; six hund. and nine thous. and seventy; four hundred and eighty thous. one hund. and six; seventy thous. three hund. and sixty-one; six hund. and fifty-four thous. three hund. and twenty-one; one hund. and sixty-four thous. three hund. and fifty-six; four hund. and twenty-one thous. five hund. and thirty-six; three hund. and fifty-four thous. six hund. and twenty-one; six hund. and fourteen thous. five hund. and thirty-two; six hund. and fifty-three thous. two hund. and forty-one.

Any six significant figures will express seven hundred and twenty different numbers. Examples of this will be given under the head "Numeration."

For one million, or one thousand thousand, we write digit 1 *in seventh place of integers;* thus, 1000000·

EXERCISE VII.

Write in figures the following numbers. One million one hund. thous. and sixty-four; two millions; three millions; four millions; five millions; six millions; seven millions; eight millions; nine millions; two millions four hund. and six thous. five hund. and three; seven millions and fifty-four hundreds;*

---

* NOTE TO TEACHER. This, and subsequent similar examples, are purposely put in this form, that the pupil may remember that a thousand is ten hundreds; two thousand, twenty hundreds, and so on.

As the places required for the notation of a number become so numerous eight millions ten hund. and twelve; twelve hund. and fifty thous. six hund. and seventy; five million one hund. and forty-six thous. three hund. and sixteen; eighteen hund. thous. eighteen hund. and five; three million four hund. and fifteen thous. five hund. and fourteen; eleven hund. and ten thous. and eleven.

For ten millions, or ten thousand thousand, we write digit 1 *in eighth place of integers; thus,* 10000000·

EXERCISE VIII.

Write in figures the following numbers. Eleven millions; twelve millions; thirteen millions; fourteen millions; fifteen millions; sixteen millions; seventeen millions; eighteen millions; nineteen millions; twenty millions; thirty millions; forty millions; fifty millions; sixty millions; seventy millions; eighty millions; ninety millions; eighty-seven million six hund. and fifty-four thous. three hund. and twenty-one; thirty million five hund. and eight thous. and forty; twelve million twelve thous. and twelve; forty million sixty thous. nine hund. and sixty-four; fourteen million one hund. thous. one hund. and forty-one; ninety million sixty-one hund. and four; thirty-six million one hund. and eighty thous. and twenty-nine.

For one hundred millions, we write digit 1 *in ninth place of integers; thus,* 100000000·

EXERCISE IX.

Write in figures the following numbers. Two hund. millions; three hund. mill.; four hund. mill.; five hund. mill.; six hund. mill.; seven hund. mill.; eight hund. mill.; nine hund. mill.

Seven hund. and one mill. eight hund. and forty-three thous. two hund. and six; five hund. mill. four hund. and fifteen thous. and eight; three hund. mill. fifty thous. and seven; four hund. and ten mill. and sixteen; five hund. and forty mill. three hund. and ten; seven hund. and two mill. seven thous. forty tens; six hund. mill. three hund. and forty-two thous.; nine hund. and three mill. five hund. and sixty thous. nine hund. and fifty-three; eight hund. and ten mill. five hund. and one thous. and thirty-seven; one hund. and ten mill. one hund. and ten thous. one hund. and ten; three hund. and five mill. sixty-one thous. four hund. and eight.

The explanations and examples which have been given shew that integers are represented by digits arranged according to the following table.

---

as to embarrass the young pupil, he may be assisted either by permitting him to fill all the necessary places with ciphers at first, and then to substitute for them the significant figures, one by one, or by allowing him to rule vertical lines to separate spaces for the places of the digits.

FIRST NOTATION TABLE:

FOR INTEGERS ONLY.

To be read from decimal point upward.

&c.	*&c.*	*&c.*
Hundreds of millions	are expressed by	digits in ninth place of integers.
Tens of millions	,,	digits in eighth place of integers.
Millions	,,	digits in seventh place of integers.
Hundreds of thousands	,,	digits in sixth place of integers.
Tens of thousands	,,	digits in fifth place of integers.
Thousands	,,	digits in fourth place of integers.
Hundreds of units	,,	digits in third place of integers.
Tens of units	,,	digits in second place of integers.
Units, or single ones	,,	digits in first place of integers.

· DECIMAL POINT. Places of integers are counted from decimal point towards the left.

NUMERATION.

Numeration is the art of reading in words numbers expressed by digits. And, since notation is the art of expressing numbers by digits, numeration is the opposite process to notation. By reversing the above notation table we obtain the

FIRST NUMERATION TABLE:

FOR INTEGERS ONLY.

To be read from decimal point upward.

&c.	*&c.*	*&c.*
Digits in ninth place of integers	represent	Hundreds of millions.
Digits in eighth place of integers	,,	Tens of millions.
Digits in seventh place of integers	,,	Millions.
Digits in sixth place of integers	,,	Hundreds of thousands.
Digits in fifth place of integers	,,	Tens of thousands.
Digits in fourth place of integers	,,	Thousands.
Digits in third place of integers	,,	Hundreds of units.
Digits in second place of integers	,,	Tens of units.
Digits in first place of integers	,,	Units, or single ones.

· DECIMAL POINT. Digits to the left of decimal point, represent integers.

In reading off a number, two steps are necessary. First, notice the place of each digit, and thus ascertain the value for which it stands.

Second, read off those values in succession, beginning at the left of the line.

In the following line of figures, 342080697·

3	in	ninth place of integers	stands for	Three hund. mill.
4	,,	eighth ,,	,,	Forty mill.
2	,,	seventh ,,	,,	Two mill.
8	,,	fifth ,,	,,	Eighty thous.

6 in third place of integers stands for Six hund.
9 „ second „ „ Nine tens (ninety).
7 „ first „ „ Seven (units).

Therefore the whole line of digits represents the number, three hund. and forty-two mill. eighty thous. six hund. and ninety-seven.

EXERCISE X.

Read, or write in words, the following numbers. 314· 406·
500· 512· 9001· 8010· 30106· 5400· 16016· 3216·
3126· 3162· 3261· 3612· 3621· 2316· 2361· 2136·
2163· 2631· 2613· 1263· 1236· 1362· 1326· 1632·
1623· 6321· 6312· 6213· 6231· 6132· 6123·

Observe that the last twenty-four numbers are expressed by the same four digits differently placed. This shews the advantage of giving to each digit a local value, that is, a value depending on its place. See Principle I. p. 3.

Arithmetical Symbols. To save time and space in arithmetic, many symbols are employed instead of words. For example, instead of writing the words "*is equal to*," we employ the *sign of equality*, which is formed by two horizontal and equal lines, one under the other, in this way =; so that "100· = ten tens," means that "100· is equal to ten tens."

Seeing, then, that 100· = ten tens; 200· = twenty tens; 300· = thirty tens; we may read the number 316· in two ways; either as "three hundred and sixteen," or as "thirty-one tens and six."

And, because 1000· = ten hund. or = one hundred tens;
And 2000· = twenty hund. or = two hundred tens;
And 3000· = thirty hund. or = three hundred tens;
We may therefore read the number 3562· in four different ways.*
For 3562· = three thous. five hund. and sixty-two;
And 3562· = thirty-five hund. and sixty-two;
And 3562· = three thous. fifty-six tens and two;
And 3562· = three hund. and fifty-six tens and two.

EXERCISE XI.

Read, or write in words, the following numbers, in as many ways as possible. 914· 50019· 406915· 90108604·
610842306· 840· 603· 15116· 390004·
891406574· 16410050· 104040810· 714653065· 30425·
500011· 71040682· 965008423· 131· 26104·
614817· 6184211· 870264245· 800405613· 412·

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* NOTE TO TEACHER. The pupil should be well grounded in this method of reading the same number in a variety of denominations; for not only will the practice ensure an intelligent dexterity in notation and numeration, but it will also be of essential service hereafter in simplifying the processes of Long Division, one of the most perplexing operations to the learner. In every example of numeration, the pupil should be required to declare the *separate local value of each individual digit.*

181416· 780000· 84200015· 452197164· 341542361·
1645· 370500· 210121· 76000104· 94634341·
724361800· 8037· 841615· 39· 140142·
80004060· 111111111· 900014106· 5104· 720450·
794631· 91001004· 100101101· 7045· 804000537·
181316· 821976· 53846219· 11014· 908416350·
718432615·

Before reading a number of many places of figures, it should be divided into *periods* and *half periods*. A period is six places of figures, and a half period three; and the division spoken of is effected by inserting a comma between the third and fourth places, a semicolon between the sixth and seventh, a comma between the ninth and tenth, a semicolon between the twelfth and thirteenth, and so on. The last of the foregoing lines of figures, when divided into periods and half periods, stands thus, 718;432,615· Then, in reading, it is only necessary to numerate from stop to stop, so that never more than three figures are read off at once; observing to say "*millions*," on arriving at the semicolon nearest to the decimal point, and "*thousands*" at *every comma*. For a digit in tenth place of integers represents "thousands" (of millions). The following line of twelve places, 801900064005·, is divided thus, 801,900;064,005·; and read thus, "eight hund. and one thous. nine hund. mill.; sixty-four thous. and five."

The "eight hund. and one" of the left hand half period are thousands of millions.*

EXERCISE XII.

Write answers to the following questions. How is the same digit made to represent different numbers? Where will you put digit 6 to represent sixty? Where will you find the second place of integers? What does 9 stand for in the first, second, third, fourth, fifth, sixth, seventh, eighth, and ninth places of integers respectively? What are the two steps to be taken to write a number in figures? If the number consist of millions, tens of thous., hund., and units, how many significant figures would be employed, and how many ciphers? What places would the ciphers occupy? If the decimal point be removed from the following line of figures, 83142· in what place would each digit be then? Why is it absolutely necessary to true notation that the decimal point should always be expressed? In how many ways may 91642· be read? What is the greatest number that can be written in three figures? What are the greatest that can be written in four, in five, and in six places?

What is meant by the *local* value of a digit? What advantage arises from giving digits a *local value?*

---

* NOTE TO TEACHER. This example is carried three places beyond the notation and numeration tables on pp. 8 and 9; but the pupil who has thoroughly mastered what precedes, will have no difficulty in thus far extending the application of the principle of notation, of which he is supposed by this time to have acquired a clear idea from the many examples he has had opportunities of examining. The principle of decimal notation will be categorically enunciated further on. It has not been considered advisable to carry the examples generally beyond nine places, which are more than sufficient for every practical purpose in ordinary arithmetic. Billions, trillions, and so forth, may be hereafter discussed.

FRACTIONS

A fraction is any part or parts of a unit.

THE FORMATION OF SIMPLE FRACTIONS.

Take any convenient unit, as a square of paper or cardboard. Cut this unit into two exactly equal parts. Each of these parts is one half of the unit. Therefore

One unit = two halves.

Cut another unit into three equal parts. Each of these is one third part, or one third of the unit. Therefore

One unit = three thirds.

Cut a unit into four equal parts. These are fourth parts or fourths or quarters of the unit. Hence

One unit = four fourths, or quarters.

Similarly, by cutting the unit into five equal parts, you obtain fifths of the unit. By cutting it into six equal parts, you obtain sixths. Proceeding in this way, you find that

One unit = five fifths, or *six sixths*, or *seven sevenths*, or *ten tenths*, or *nineteen nineteenths*, &c.

And you remark, also, that the number of equal parts into which the unit is cut, if more than two, decides the name of the part.

If the unit be cut into nine equal parts, they will be ninths; if into fifty equal parts, they will be fiftieths; if into three hundred equal parts, they will be three-hundredths; and so on.

To obtain nine tenths of a unit, two steps are necessary.

First. Divide the unit into ten tenths.

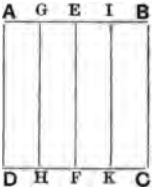
Second. Take nine of those tenths.

Fractions formed by thus dividing the unit into equal parts, and then taking a certain number of those parts, are **Simple Fractions.**

EXERCISE XIII.

Draw for your unit a line of any convenient length, and divide it into fifths. Divide another line of the same length into eighths. Divide another into tenths. Draw twenty horizontal lines, of equal length in a vertical column. Considering each of these lines as a unit, mark off the upper one into halves, the next into thirds, the next into fourths, the next into fifths; proceed thus until the lowest of the twenty lines is divided into *twenty twenty-first* parts.

Diagram 1.



I describe the square A B C D (*Diagram* 1), and consider it as a unit. I put E exactly in the middle of A B, and F exactly in the middle of D C. How much of A B is A E? How much of D C is D F or F C? I draw the line E F, dividing the square into two equal parts. How much of the unit is the part A F? I put G exactly midway between A and E; I midway between E and B; H midway between D and F; and K midway between F and C. Then I draw the lines G H and I K. What parts of A B are A G, G E, E I, I B, A I, G I, G B? What parts of D C are D K, H C, H K? Into what parts do the three vertical lines G H, E F, and I K divide the unit? How much of the unit is A K? How much of the unit is G K? The fractions A F, G K, E C have each two names, what are they?

Draw for your unit a square of any convenient size. Divide it into fifths by four vertical lines. Place letters at the corners of the fifths, as in *Diagram* 1.

Point out four pieces each containing two fifths, three pieces each containing three fifths, two pieces each containing four fifths.

Divide another unit-square into sixths, and letter them. Point out five pieces each containing two sixths, four pieces each containing three sixths, three pieces each containing four sixths, two pieces each containing five sixths. Draw a vertical line down the middle of each of the sixths, the unit will then be divided into twelfths. Letter these, and then point out eleven pieces each consisting of two twelfths, ten of three twelfths, nine of four twelfths, eight of five twelfths, seven of six twelfths, six of seven twelfths, five of eight twelfths, four of nine twelfths, three of ten twelfths, and two of eleven twelfths.

I am to divide an apple among three boys, so that Charles and George shall have equal portions, and Joseph as much as both the other two. How shall I do this, and what fraction of the apple will each boy have? Answer. Divide the apple into four fourths, give one fourth to Charles, one fourth to George, and two fourths to Joseph.

Shew how to apportion a cake among the same three boys, giving George double as much as Charles, and Charles double as much as Joseph.

Prove the correctness of your answer, by dividing a square in the same way as you would the cake.

NOTATION OF FRACTIONS.

NUMERATOR AND DENOMINATOR.

In writing a fraction we have two things to shew: first, the number of parts into which unity is divided, (which determines also the name of those parts;) and second, the number of those parts in the fraction spoken of.

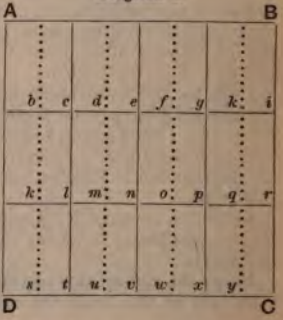
Therefore, every fraction has two numbers to express it; the *denominator* or namer, placed below a horizontal line, and the *numerator* or numberer, placed above it. Six elevenths are written thus, $\frac{8}{11}$ where the denominator 11· shews that the unit is supposed to be divided into *eleven* equal parts, which are therefore named *elevenths;* and the numerator 6· shews the number of those elevenths in this fraction.* One half is written thus, $\frac{1}{2}$, because the unit is divided into *two* equal parts (halves), and *one* of those halves is spoken of.

EXERCISE XIV.

Write in figures the following fractions. Four fifths, three sevenths, six elevenths, seven eighths, eleven thirteenths, forty-two sixty-sevenths, thirty-nine one-hundredths, fifteen one-thousandths, one hund. and eleven ninety-fifths, three hund. and fifty-four eleven-hundredths, nine one-millionth parts, thirty-three forty-thousandth parts, fifteen hundred ten-thousandth parts, sixty-five ten-hundred-and-eighty-fourths, twenty-two ninetieths.

In *Diagram* 2, the seven vertical lines divide the unit into eight eighths; and the two horizontal lines divide it into three thirds. The vertical and horizontal lines together, divide it into twenty-four twenty-fourths.

Diagram 2.



Therefore, the part A*b* = one twenty-fourth of the unit. A*c* = one twelfth, for there are in the unit twelve pieces of that size. But A*c* contains two pieces of the size of A*b*. Therefore, A*c* = also two twenty-fourths of the unit. In the same way, it may be seen that A*s* = one eighth, or three twenty-fourths; also, that A*e* = one sixth, or two twelfths, or four twenty-fourths. Again, A*t* = one fourth, or two eighths, or three twelfths, or six twenty-fourths; for A*t* contains *two* pieces of the size of A*s*, that is two eighths; *three* pieces of the size of A*c*, that is three twelfths; six pieces of the size of A*b*, that is *six* twenty-fourths.

---

* NOTE TO TEACHER. The pupil should be made aware that the decimal point is not required in the numerator or denominator of a fraction, because the right hand end of the horizontal line shews whence places are counted. Decimal figures do sometimes occur in the terms of a fraction, and then of course the decimal point must be inserted; but instances of this kind need not be discussed yet. The decimal point might with propriety be always inserted in both numerator and denominator; but the explanation then necessary would be more perplexing than profitable at this stage.

EXERCISE XV.

Express in figures the following fractions of the square as divided in Diagram 2, *namely:* A*u*, in two ways; A*v*, in six ways; A*w*, in two ways; A*x*, in four ways; A*y*, in two ways; A*g*, in four ways; A*i*, in four ways; A*l*, in three ways; A*m*, in four ways; A*n*, in four ways; A*o*, in two ways; A*p*, in six ways; A*r*, in four ways; A B C D, in seven ways.

The pupil will be assisted by remarking that A*l* = A*e*; A*m* = A*t*; A*n* = A*i*; A*p* = A*v*; A*g* = A*t*.

NUMERATION OF FRACTIONS.

In reading off fractions, written in figures, all that is to be remembered is that the numerator is to be read first, and then the denominator.

EXERCISE XVI.

Read the following fractions. $\frac{16}{45}$; $\frac{131}{614}$; $\frac{504}{913}$; $\frac{12}{3818}$; $\frac{11}{12}$; $\frac{184}{3005}$; $\frac{856}{1114}$; $\frac{1}{10}$; $\frac{3}{100}$; $\frac{14}{1000}$; $\frac{601}{100000}$; $\frac{5146}{10000000}$; $\frac{78425}{100000000}$; $\frac{436554}{1000000000}$; $\frac{29}{29}$; $\frac{18}{15}$; $\frac{15}{13}$; $\frac{64}{11}$; $\frac{614}{416}$; $\frac{416}{614}$; $\frac{461}{641}$.

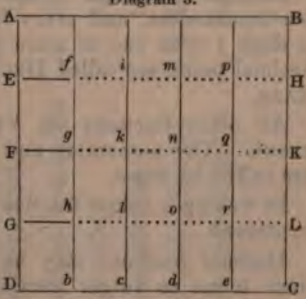
How many equal parts is unity supposed to be divided into, in each of the above fractions?

SIMPLE AND COMPOUND FRACTIONS.

A simple fraction is formed by dividing unity into equal parts, and taking a certain number of them. (See page 12.) If we take a simple fraction, and divide it, instead of dividing unity, we shall obtain a compound fraction.

A compound fraction is a fraction of a fraction.

Diagram 3.



Take a square, as before, for your unit. Draw four vertical lines, dividing the unit into five fifths. Then, take one fifth as A*b*, in *Diagram* 3, and draw three horizontal lines across it, dividing it into four equal parts; namely, A*f*, E*g*, F*h*, G*b*. Each of these, therefore, is the compound fraction $\frac{1}{4}$ of $\frac{1}{5}$. Now continue the three horizontal lines, dotted, quite across the unit, which will then be divided into twenty equal parts like A*f*. A*f* is, therefore, $\frac{1}{20}$ of the unit, as well as being, as we saw just now, $\frac{1}{4}$ of $\frac{1}{5}$.

In the same way it is seen that

A*g* = the compound fraction $\frac{2}{4}$ of $\frac{1}{5}$, and also = the simple fraction $\frac{2}{20}$.

A*k* = the compound fraction $\frac{2}{4}$ of $\frac{2}{5}$, and also = the simple fraction $\frac{4}{20}$.

A*o* = the compound fraction $\frac{3}{4}$ of $\frac{3}{5}$, and also = the simple fraction $\frac{9}{20}$.

And because AH = $\frac{1}{4}$ of the unit, AK = $\frac{2}{4}$, AL = $\frac{3}{4}$, and each of these fourths is divided into *five* equal parts by the four vertical lines;

A*f* = $\frac{1}{5}$ of $\frac{1}{4}$, and also = $\frac{1}{4}$ of $\frac{1}{5}$, and also = $\frac{1}{20}$.
A*i* = $\frac{2}{5}$ of $\frac{1}{4}$, „ = $\frac{1}{4}$ of $\frac{2}{5}$, „ = $\frac{2}{20}$.
A*n* = $\frac{3}{5}$ of $\frac{2}{4}$, „ = $\frac{2}{4}$ of $\frac{3}{5}$, „ = $\frac{6}{20}$.

EXERCISE XVII.

Express in as many ways as you can, both as simple and as compound fractions, the following parts of the unit as it is divided in Diagram 3. A*p*; A*n*; A*q*; A*k*; A*l*; A*r*; A*c*; A*d*; A*e*; K*l*; K*d*; H*g*; B*i*; B*k*; B*r*; H*d*; L*f*; L*k*; L*p*.

Divide a number of units into sevenths, and find the simple fractions which are equal to $\frac{1}{8}$ of $\frac{1}{7}$; $\frac{1}{2}$ of $\frac{5}{7}$; $\frac{3}{10}$ of $\frac{2}{7}$; $\frac{1}{7}$ of $\frac{1}{7}$; $\frac{3}{4}$ of $\frac{1}{7}$; $\frac{2}{5}$ of $\frac{3}{7}$; $\frac{3}{8}$ of $\frac{6}{7}$; $\frac{1}{9}$ of $\frac{3}{7}$; $\frac{2}{3}$ of $\frac{4}{7}$; $\frac{5}{8}$ of $\frac{4}{7}$; $\frac{4}{5}$ of $\frac{4}{7}$; $\frac{4}{6}$ of $\frac{5}{7}$.

Decimal Fractions. Tenths, hundredths, thousandths, ten-thousandths, and every fraction whose denominator is digit 1 with one or more ciphers between it and the decimal point, are called DECIMAL FRACTIONS, or DECIMALS.

All other fractions are VULGAR or COMMON FRACTIONS. The numerator and denominator of a fraction are called its *terms*.

In writing a vulgar fraction both terms must always be expressed.

Decimal fractions may be expressed, either by both their terms as vulgar fractions, or by their numerators only, according to the method we now proceed to explain.

NOTATION OF DECIMAL FRACTIONS.

Places of decimals are counted from the DECIMAL POINT *towards the right.* In the following line of figures, 314·506829 the 5 is in first place of decimals, 0 in second place of decimals, 6 in third place of decimals, 8 in fourth place of decimals, 2 in fifth place of decimals, 9 in sixth place of decimals.

And decimal fractions are expressed by their numerators only, according to the subjoined table.

SECOND NOTATION TABLE:

FOR DECIMAL FRACTIONS.

To be read from decimal point downward.

Places of decimals are counted to the right from

· DECIMAL POINT.

Tenths	are expressed	by digits in *first place of decimals.*
Hundredths	,,	by digits in *second place of decimals.*
Thousandths	,,	by digits in *third place of decimals.*
Ten-thousandths	,,	by digits in *fourth place of decimals.*
Hundred-thousandths	,,	by digits in *fifth place of decimals.*
Millionths	,,	by digits in *sixth place of decimals.*
Ten-millionths	,,	by digits in *seventh place of decimals.*
Hundred-millionths	,,	by digits in *eighth place of decimals.*
Thousand-millionths	,,	by digits in *ninth place of decimals.*
&c.	*&c.*	*&c.*

Hence $\frac{5}{10}$ is written with numerator only, thus, ·5
$\frac{6}{100}$ „ „ thus, ·06
$\frac{9}{1000}$ „ „ thus, ·009
$\frac{4}{10000}$ „ „ thus, ·0004
$\frac{8}{100000}$ „ „ thus, ·00008
$\frac{7}{1000000}$ „ „ thus, ·000007

$\frac{3}{10}$ and $\frac{5}{100}$ .... = ·35
$\frac{4}{10}$, $\frac{6}{100}$, and $\frac{9}{100}$ .... = ·469
$\frac{5}{100}$ and $\frac{8}{10000}$ .... = ·0508
$\frac{3}{1000}$, $\frac{4}{10000}$, and $\frac{7}{1000000}$ .... = ·003407
$\frac{8}{10}$, $\frac{3}{1000}$, $\frac{4}{100000}$, and $\frac{8}{10000000}$ = ·8030408
$\frac{1}{10}$, $\frac{1}{1000}$, and $\frac{1}{1000000}$ ...... = ·101001
$\frac{4}{100}$, $\frac{4}{10000}$, and $\frac{4}{1000000}$ .... = ·040404
$\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, and $\frac{3}{10000}$ .... = ·3333
$\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, and $\frac{1}{10000}$ .... = ·1111
$\frac{7}{100}$ and $\frac{7}{100000000}$ .... = ·07000007

These examples shew that the number of decimal places employed to express a decimal fraction by its numerator only, is the same as the number of ciphers in its denominator.

Hundredths have two ciphers in the denominator, and two places of decimals. Thousandths have three ciphers in denominator, and three places of decimals. Millionths have six ciphers in denominator, and therefore millionths are written in sixth place of decimals.

·08 represents $\frac{8}{100}$; but if the cipher be removed the digit 8 will then stand in the first instead of the second place of decimals, and will, therefore, then represent $\frac{8}{10}$ instead of $\frac{8}{100}$ as it does now. But any number of ciphers on the right of the 8 will make no change in its value. ·08000000 is the same as ·08, because in both expressions the 8 occupies the second place of decimals, and consequently in both it represents the same decimal fraction; namely, $\frac{8}{100}$.

In decimals then, as well as in integers, the cipher is only of service when it stands between a significant figure and the decimal point. See Principle II. page 3.

COMPARISON OF DECIMAL FRACTIONS.

Take a unit (square or line) of large size. Divide it into ten tenths. Divide one of the tenths into ten equal parts. Each of these new parts is $\frac{1}{10}$ of $\frac{1}{10}$, and is equal to $\frac{1}{100}$ of the unit, for as ten of them can be procured from each tenth, one hundred of them can be made from the whole unit. Divide one of these hundredths into ten equal parts. Each of these is $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$ of the unit. By continuing, in this way, to divide each new fraction into its tenths, the truth of the following tables may be proved.

TABLE I.

FOR COMPARISON OF DECIMAL FRACTIONS.

$\frac{1}{10}$	$=\frac{10}{100}$	or $\frac{100}{1{,}000}$	or $\frac{1{,}000}{10{,}000}$,* &c.
$\frac{1}{100}$	$=\frac{10}{1{,}000}$	or $\frac{100}{10{,}000}$	or $\frac{1{,}000}{100{,}000}$
$\frac{1}{1000}$	$=\frac{10}{10{,}000}$	or $\frac{100}{100{,}000}$	or $\frac{1{,}000}{1;000{,}000}$
$\frac{1}{10{,}000}$	$=\frac{10}{100{,}000}$	or $\frac{100}{1;000{,}000}$	or $\frac{1{,}000}{10;000{,}000}$
$\frac{1}{100{,}000}$	$=\frac{10}{1;000{,}000}$	or $\frac{100}{10;000{,}000}$	or $\frac{1000}{100;000{,}000}$
$\frac{1}{1;000{,}000}$	$=\frac{10}{10;000{,}000}$	or $\frac{100}{100;000{,}000}$	or $\frac{1{,}000}{1{,}000;000{,}000}$

TABLE II.

$\frac{1}{100}$	$=\frac{1}{10}$ of $\frac{1}{10}$;	and $\frac{10}{100}$	$=\frac{1}{10}$
$\frac{1}{1{,}000}$	$=\frac{1}{10}$ of $\frac{1}{100}$;	and $\frac{10}{1{,}000}$	$=\frac{1}{100}$
$\frac{1}{10{,}000}$	$=\frac{1}{10}$ of $\frac{1}{1{,}000}$;	and $\frac{10}{10{,}000}$	$=\frac{1}{1{,}000}$
$\frac{1}{100{,}000}$	$=\frac{1}{10}$ of $\frac{1}{10{,}000}$;	and $\frac{10}{100{,}000}$	$=\frac{1}{10{,}000}$
$\frac{1}{1;000{,}000}$	$=\frac{1}{10}$ of $\frac{1}{100{,}000}$;	and $\frac{10}{1;000{,}000}$	$=\frac{1}{100{,}000}$
$\frac{1}{10;000{,}000}$	$=\frac{1}{10}$ of $\frac{1}{1;000{,}000}$;	& $\frac{10}{10;000{,}000}$	$=\frac{1}{1;000{,}000}$

Arithmetical Symbols. The arithmetical symbol for the word "because" is $\because$ and the sign for the word "therefore" is $\therefore$. In future we shall employ these signs.

$\because \frac{10}{100}=\frac{1}{10}$, $\frac{20}{100}=\frac{2}{10}$, $\frac{30}{100}=\frac{3}{10}$; and $\because \frac{34}{100}=\frac{30}{100}$ and $\frac{4}{100}$; $\therefore \frac{34}{100}=\frac{3}{10}$ and $\frac{4}{100}$; $\therefore \frac{34}{100}$ will be written without its denominator, thus, ·34. See examples of notation of decimal fractions on page 18.

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* NOTE TO TEACHER. The actual division of the unit, whether square or line, will be possible only to a very limited extent; hundredths, or at most thousandths, being probably the smallest fractions that will be thus found. But what must be secured is the pupil's perfect comprehension of the relation of the successive decimal fractions; and this may be tested by requiring him to carry out each line of Table I. a few steps further, and to continue either of the Tables one or two lines further down, or to reverse the left-hand half of Table II. This latter Table will shortly be employed to expound the law of "carrying."

Again, $\because \frac{100}{1000} = \frac{1}{10}$, $\frac{900}{1000} = \frac{9}{10}$; and $\because \frac{973}{1000} = \frac{900}{1000}$ and $\frac{70}{1000}$ and $\frac{3}{1000}$, $\therefore \frac{973}{1000} = \frac{9}{10}$ and $\frac{7}{100}$ and $\frac{3}{1000}$; $\therefore$ $\frac{973}{1000}$ will be written, without denominator, thus, ·973. See page 18.

Also, $\frac{8459}{10,000} = \frac{8000}{10,000}$ and $\frac{400}{10,000}$ and $\frac{50}{10,000}$ and $\frac{9}{10,000}$; $\therefore \frac{8459}{10,000} = \frac{8}{10}$ and $\frac{4}{100}$ and $\frac{5}{1000}$ and $\frac{9}{10,000}$; $\therefore \frac{8459}{10,000}$ is written thus: ·8459; and $\frac{314156}{1,000,000}$ thus: ·314156; and $\frac{843}{100,000}$ thus: ·00843; and here it is to be noticed that the two ciphers, which are added to bring the 8 into its proper place, make up with the three significant figures, the same number of decimal places as there are ciphers in the denominator of the fraction. See page 18.

EXERCISE XVIII.

Express in figures, without denominators, the following decimal fractions. Four tenths; five one-hundredths; six one-thousandths; seven ten-thousandths; eight one-millionths; nine ten-millionths; three one-hundred-millionths; eight one-thousand-millionths; twenty-four one-hundredths; sixteen one-thousandths; three hundred and ninety-four one-thousandths; seventeen ten-thousandths; five hundred and forty-three ten-thousandths; two thousand, two hundred and fifteen ten-thousandths; thirty-four one-hundred-thousandths; seven hundred and nineteen one-hundred-thousandths; one thousand and five one-hundred-thousandths; ninety-three thousand and twelve one-hundred-thousandths; ninety-seven thousand and sixteen one-hundred-thousandth parts; fourteen one-millionth parts; seven hundred and six one-millionth parts; six thousand and forty-seven one-millionths; thirty-nine thousand eight hundred and fifteen one-millionths; four hundred and eighteen thousand eight hundred and seventy-six one-millionth parts; forty nine ten-millionth parts; six hundred and fourteen ten-millionth parts; nine thousand three hundred and five ten-millionth parts; forty-three thousand two hundred and twenty-four ten-millionth parts; eight hundred and seventy-nine thousand four hundred and eleven ten-millionth parts; four million nine hundred and fourteen thousand three hundred and twenty-seven ten-millionth parts.

Restore denominators to the above decimal fractions.

Write answers to the following questions. When a decimal fraction is written without its denominator, what decides the number of places to be employed? If there be not in the numerator significant figures enough to fill these places, where are ciphers added? On which side of digits in decimal places are ciphers useless? Why are they useless there? What is the principle on which ciphers are used, both in integers and decimals? How many tenths make a unit? How many ten-thousandths make a tenth? How many one-hundred-thousandths make one one-hundredth? Make the arithmetical signs for 'because,' and 'therefore.' What is meant by the 'terms' of a fraction? Which of these must be written for a vulgar fraction? Which for a decimal fraction?

NUMERATION OF DECIMAL FRACTIONS.

By reversing the table for Notation of Decimal Fractions, (p. 17,) we obtain the

SECOND NUMERATION TABLE:

FOR DECIMAL FRACTIONS.

To be read from decimal point downward.

· Decimal Point. Digits to the right of decimal point stand for decimals.

Digits in *first place of decimals*	represent	**Tenths.**
Digits in *second place of decimals*	,,	**Hundredths.**
Digits in *third place of decimals*	,,	**Thousandths.**
Digits in *fourth place of decimals*	,,	**Ten-thousandths.**
Digits in *fifth place of decimals*	,,	**Hundred-thousandths.**
Digits in *sixth place of decimals*	,,	**Millionths.**
Digits in *seventh place of decimals*	,,	**Ten-millionths.**
Digits in *eighth place of decimals*	,,	**Hundred-millionths.**
Digits in *ninth place of decimals*	,,	**Thousand-millionths.**
&c. *&c.*		*&c.*

In reading off decimal fractions their denominators should always be named, whether written or not.

Every decimal fraction may be read in two ways.

1. The value of each significant figure in the numerator may be named separately.

2. The whole of the digits in the numerator may be read in the name of the right hand one.

Thus: ·8432 may be read either *separately*, as $\frac{8}{10}$, $\frac{4}{100}$, $\frac{3}{1000}$, and $\frac{2}{10000}$; or, ·8432 may be read *collectively*, as ten-thousandths, $\frac{8432}{10000}$.

The latter method is the best, and therefore should be commonly employed, but the former is sometimes required.[*]

EXERCISE XIX.

Read, or write in words, first taking the digits separately and then collectively, each of the following decimal fractions:

·415
·00103
·045061
·8643
·95
·00504
·75401
·63005
·09040507
·01201201
·75
·625
·125
·375
·1354
·875
·3125
·00958
·12345678
·900900909

Write the above with their denominators restored.

Read, or write in words, in the same manner, the following numbers, consisting of integers and decimals:

31·145
1605·32
347·2418
8431·5608
12002·34125
17·5201
24634·314
7416·008
51353·0808
6347·8434
234·3618
64·8431

DIVISION OR PARTITION OF AN INTEGER.

The division of a unit is the cutting of it into equal parts, as explained on page 12.

The division of an integer, or number of units, is the distribution of it into equal lots of units.[†]

Halves of an Integer. Take two equal strips of paper, or other convenient units, and place them before you at a little distance from each other. The two units are now distributed into two equal portions. Each of these portions is one half of the two units, and consists of one unit.

∴ *One half of two = one unit.*[‡]

Take other two similar units. Put one of them with each of the former. Here are now two twos, or four, divided into two equal lots, each containing two units.

∴ *One half of two twos = two units.*

Dispose similarly of a third two, and a fourth two, and a fifth two, and so on until you have divided ten twos. And as you proceed, you will remark that

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[*] NOTES TO TEACHER. Decimal fractions should always be read with their denominators whether so written or not: for example, ·2 should always be read "*two tenths* or *twenty one-hundredths*," and so on; and never (as is the practice with some arithmeticians) "*decimal two*." A still more objectionable habit is that of reading such an expression as ·842, as "*decimal eight hundred and forty-two*." The true character of the decimal fraction is constantly kept before the pupil's mind by supplying its denominator in reading it.

[†] See subsequent remarks on the true idea of Division, note under elementary rules.

[‡] The pupil must always read in words every arithmetical symbol he meets with.

$\frac{1}{2}$ of three twos = 3·
$\frac{1}{2}$ of four twos = 4·
$\frac{1}{2}$ of five twos = 5·
$\frac{1}{2}$ of six twos = 6·
$\frac{1}{2}$ of seven twos = 7.
$\frac{1}{2}$ of eight twos = 8·
$\frac{1}{2}$ of nine twos = 9·
$\frac{1}{2}$ of ten twos = 10·

So that the half of an integer is found by distributing it into two equal lots of units. And this is done by separating every two of its units into two portions of one unit each, and then collecting those units into two separate lots.

PRINCIPLE III. *There are as many units in the half of a number as there are twos in the whole of it.*

If, after distributing, as above directed, all the twos that can be got from a number, a single unit should remain, that unit must be divided into two halves, (page 12,) and one of these halves must be put with each of the two equal lots of units.

Thus, to find the half of eleven, which consists of five twos and one unit:

First. Distribute the five twos into two lots of 5· each.

Second. Put with each of these lots one half of remaining unit.

$\therefore \frac{1}{2}$ of 11· = $5\frac{1}{2}$[*]

This result quite agrees with Principle III. For 11· consists of five twos and one unit; that is, of five twos and the half of two. And the half of eleven consists of five units and the half of a unit. Therefore, there are as many units in the half of 11· as there are twos in the whole of it.

Thirds of an Integer. The third of an integer is found by distributing all its threes into three equal lots of units, and putting with each of those lots $\frac{1}{3}$ of each unit remaining after all the threes are divided.

Fourths, Fifths, Sixths, &c., of an Integer.

The *fourth* part of an integer is found by distributing its units into *four* equal lots, of units and fourths.

The *fifth* part of an integer is found by distributing its units into *five* equal lots, of units and fifths.

The *sixth* part of an integer is found by distributing its units into *six* equal lots, of units and sixths.

The *seventh* part of an integer is found by distributing its units into *seven* equal lots, of units and sevenths.

And so on with other parts of an integer. The name of the part indicates how many of the units shall be distributed at once;

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[*] NOTE TO TEACHER. The decimal point is not required when the units are followed by a vulgar fraction.

for, to obtain the third of an integer, we take its units three at a time; to obtain its fifth, we take its units five at a time; to obtain the tenth, we take the units ten at a time.

The name of the parts into which the integer is to be divided determines also the fractions into which each of the remaining units is to be cut.

EXTENSION OF PRINCIPLE III. *There are as many units in the third of a number as there are threes in the whole of it; as many units in the fourth as fours in the whole; as many units in the tenth as tens in the whole; &c. &c.*

Concrete and Abstract Numbers. A number which expresses one particular kind of unit is a concrete or applicate number.

A number which expresses no particular kind of unit is an abstract number.

Thus, 24· is an abstract number, because no particular kind of unit is expressed by it; it represents 24· of any or every sort of unit. But 24· shillings, or 24· yards, or 24· pounds is an applicate or a concrete number, because the 24· is applied to the expression of 24· units of a stated kind; namely, shillings, or yards, or pounds.

EXERCISE XX.

QUESTIONS, AND EXAMPLES IN ABSTRACT NUMBERS.

1· What is meant by the division of a unit?
2· What does the division or partition of an integer mean?
3· How is the half of a unit found?
4· How is the half of an integer procured?
5· How many units in the half of an integer?
6· What is the half of nineteen twos?
7· What is the half of 6· of 9· of 7· of 14· of 15· of 20· of 23·?
8· How is the third of an integer found?
9· What is the number of units in the third of an integer?
10· What is the third of nine threes?
11· How many units and thirds of a unit in one third of 7· of 8· of 10· of 12· of 14· of 16· of 20· of 22· of 25·?
12· What is the third of two units?
13· What is the third of 5· of 4· of 6· of 9· of 11· of 13· of 15·?
14· Find the fourth of 12· of 16· of 15· of 18· of 19· of 21·
15· What is the fifth of 7· of 9· of 10· of 14· of 16· of 20·?
16· What is the sixth of 8· of 10· of 11· of 12· of 14· of 17· of 19·?
17· Find $\frac{1}{7}$ of 12· $\frac{1}{8}$ of 14· $\frac{1}{9}$ of 16· $\frac{1}{10}$ of 50· $\frac{1}{5}$ of 25· $\frac{1}{5}$ of 27.
Find $\frac{1}{4}$ of 26· $\frac{1}{9}$ of 27· $\frac{1}{8}$ of 23· $\frac{1}{4}$ of 5· $\frac{1}{6}$ of 7· $\frac{1}{8}$ of 10·
18· By Principle III. How many units are there in one tenth of a number?
19· If I distribute the units of a number into nine equal lots, what part of the integer is each of those lots?
20· What number of units will there be in each of those lots?

21· In dividing an integer, what is it decides how many equal lots the units shall be distributed into?

22· What else is decided by the same thing?

23· How many units are distributed at a time in finding the twelfth of an integer?

24· If I take the units of an integer nine at a time, and distribute each nine into units, what part of the integer shall I find?

EXAMPLES IN CONCRETE OR APPLICATE NUMBERS.

25· What is one half of 10· pence, of 9· pence, of 5· pence, of 11· pence?

26· What is $\frac{1}{3}$ of 5· pounds of beef? What is $\frac{1}{4}$ of 7· yards of carpet?

27· If 14· pounds of sugar were divided into three equal parts, how much sugar would be in each lot?

28· Five sticks of equal length will reach, end to end, seven feet; how long is each stick?

29· Find $\frac{1}{5}$ of 8· ounces; $\frac{1}{7}$ of 9· gallons; $\frac{1}{6}$ of 10· minutes.

30· Find $\frac{1}{10}$ of 14· pounds; $\frac{1}{4}$ of 11· shillings; $\frac{1}{11}$ of 29· sheets of paper.

31· Share 5· apples of equal size equally among 3· boys.

32· Fourteen pounds of cherries are to be divided equally among 3· boys: what weight for each?

33· Nineteen hundred-weights of soap are packed in 7· equal-sized boxes: how much in each box?

THE PRINCIPLE OF DECIMAL NOTATION.

Because, by Principle III., there are as many units in one tenth of a number as there are tens in the whole of it, then

∵ there is *one* ten in 10· ∴ 1· is $\frac{1}{10}$ of 10·
∵ there are *ten* tens in 100· ∴ 10· is $\frac{1}{10}$ of 100·
∵ there are *a hundred* tens in 1,000· ∴ 100· is $\frac{1}{10}$ of 1,000·
∵ there are *a thousand* tens in 10,000· ∴ 1,000· is $\frac{1}{10}$ of 10,000·
∵ there are *ten thousand* tens in 100,000· ∴ 10,000· is $\frac{1}{10}$ of 100,000·

And Table II. for comparison of decimal fractions, page 19, shews that

·1 = $\frac{1}{10}$ of 1·	·001 = $\frac{1}{10}$ of ·01	·00001 = $\frac{1}{10}$ of ·0001
·01 = $\frac{1}{10}$ of ·1	·0001 = $\frac{1}{10}$ of ·001	·000001 = $\frac{1}{10}$ of ·00001
&c.	&c.	&c.

Hence, in a line consisting of digit 1 repeated in adjoining places of integers and decimals, as

1111111·111111

each digit has one-tenth of the value of its left-hand neighbour, and tenfold the value of its right-hand neighbour.

This is true also of each of the following lines, which, as well as the former line, may be extended to any number of places in either direction.

2222·22 &c.
3333·33 &c.
4444·44 &c.
5555·55 &c.
6666·66 &c.
7777·77 &c.
8888·88 &c.
9999·99 &c.

For, 2· = $\frac{1}{10}$ of 20· 20· = $\frac{1}{10}$ of 200· 200· = $\frac{1}{10}$ of 2000· and so on.
And, ·2 = $\frac{1}{10}$ of 2· ·02 = $\frac{1}{10}$ of ·2 ·002 = $\frac{1}{10}$ of ·02 " "
Also, 2· = tenfold ·2 ·2 = tenfold ·02 ·02 = tenfold ·002 "
And, 20· = tenfold 2· 200· = tenfold 20· 2000· tenfold 200· "

The other lines may be proved in the same way.

Ten, therefore, may be called the governing number of our method of arithmetical notation, which, for this reason, is called the Decimal System of Notation. The word "decimal" is derived from the Latin word "decem," *ten*.

PRINCIPLE IV. *In the decimal system of arithmetical notation, each digit has one tenth of the value it would have in the next place to the left, and tenfold the value it would have in the next place to the right.*

From this principle we derive the two rules for carrying, the use of which rules will be shewn hereafter.

First rule for carrying. In decimal arithmetic, every *ten* of any place may be carried as *one* to the next place to the left.

Second rule for carrying. In decimal arithmetic, every *one* of any place may be carried as *ten* to the next place to the right.

Decimal Arithmetic is not the Arithmetic of Decimal Fractions only, but also that of Integers, when they are expressed by the Decimal System of Notation.

ELEMENTARY PROCESSES OF ARITHMETIC.

Numbers can be changed in only two ways. They can be increased, and they can be diminished.

The increasing of a number, by adding or joining one or more other numbers to it, is called Addition.

When equal numbers are added together, that kind of addition is called Multiplication.

The diminishing of a number, by taking a smaller number out of it, is called Subtraction.

The distribution of a number into its parts is called Division.

Addition is the uniting of two or more numbers into one.

Multiplication is the addition or repetition of equal numbers.

Subtraction is the taking of a smaller number out of a larger one.

Division is the distribution of a number into its parts.

ADDITION.

Addition is the increasing of a number by uniting one or more other numbers with it.

Arithmetical Symbol. The sign of addition is a cross, composed of two equal lines, one vertical and the other horizontal, intersecting each other in the middle, thus: + ; it is called, in reading, "plus," which is the Latin word for "more."

Thus, the expression 6· + 2· + 1· is read "six plus two plus one;" that is to say, "six and two more and one more;" and it means that 6· and 2· and 1· are to be added together, or collected into one number, namely, 9·

Sum. *The sum of numbers is found by adding them together.*

EXERCISE XXI.

EXAMPLES TO BE WORKED MENTALLY.

1· Add 3· & 5· & 7·
2· Add 4· & 6· & 3·
3· Add 7· & 5· & 9·
4· Add 9· & 6· & 5·
5· Add 10· & 5· & 4·
6· Add 11· & 3· & 8·
7· Add 12· & 8· & 3·
8· Add 3· & 5· & 13·
9· Add 5· & 16· & 9·
10· Add 7· & 12· & 11·

11· What are 3· + 4· + 9· + 5· = ?
12· What are 5· + 3· + 4· + 7· = ?
13· What are 9· + 4· + 3· + 8· = ?
14· What are 8· + 5· + 7· + 3· = ?
15· What are 11· + 3· + 6· + 5· = ?
16· What are 13· + 9· + 2· + 11· = ?
17· What are 14· + 6· + 9· + 3· = ?
18· What are 16· + 3· + 8· + 5· = ?
19· What are 18· + 14· + 3· + 2· = ?
20· What are 20· + 11· + 9· + 8· = ?

21· Find the sum of 3· & 9· & 5· & 7· & 6·
22· „ „ „ of 9· & 12· & 13· & 6· & 14·
23· „ „ „ of 8· & 10· & 10· & 10· & 10·
24· „ „ „ of 14· & 13· & 10· & 5· & 2·
25· „ „ „ of 21· & 10· & 8· & 10· & 7·
26· „ „ „ of 7· & 5· & 4· & 3· & 6·
27· „ „ „ of 18· & 4· & 5· & 16· & 23·

28· Find the sum of 6· & $\frac{1}{2}$ of 18· *

29· " " " of $\frac{1}{2}$ of 12· & $\frac{1}{3}$ of 12· also of $\frac{1}{8}$ of 16· & $\frac{1}{10}$ of 30·

30· " " " of $\frac{1}{5}$ of 20· & $\frac{1}{8}$ of 24· " " $\frac{1}{4}$ of 24· & $\frac{1}{2}$ of 10·

31· " " " of $\frac{1}{7}$ of 21· & $\frac{1}{8}$ of 32· " " $\frac{1}{6}$ of 42· & $\frac{1}{8}$ of 40·

32· " " " of $\frac{1}{5}$ of 15· & $\frac{1}{4}$ of 16· " " $\frac{1}{8}$ of 32· & $\frac{1}{9}$ of 27·

33· " " " of $\frac{1}{3}$ of 9· & $\frac{1}{6}$ of 12· " " $\frac{1}{5}$ of 25· & $\frac{1}{11}$ of 44·

34· " " " of $\frac{1}{2}$ of 8· & $\frac{1}{9}$ of 18· " " $\frac{1}{6}$ of 18· & $\frac{1}{8}$ of 16·

EXAMPLES IN CONCRETE NUMBERS.

35· Add 9· apples, 3· apples, 7· apples, and 12· apples.

36· Add 13· pence, 11· pence, 6· pence, and 2· pence.

37· Add $\frac{1}{4}$ of 8· pence, $\frac{1}{6}$ of 18· pence, and $\frac{1}{12}$ of 12· pence.

38· Henry had 10· shillings, Amelia had half as much as Henry, and Julia had 3· shillings more than Amelia; how much had all three?

39· The tail of Tom's kite was 30· feet long, the kite was $\frac{1}{5}$ of the length of the tail, and the string was 110· feet long; what was the length of kite, tail, and string together?

40· What is the joint weight of 7· pounds of coffee, 4· pounds of tea, 9· pounds of cheese, 2· pounds of currants, and 8· pounds of soap?

41· Little Mary's market-basket weighs 5· pounds when empty, what weight had she to carry when her basket contained 5· pounds of mutton, 6· pounds of flour, 2· pounds of treacle in a jug weighing 1· pound, and half a six-pound Dutch cheese?

ADDITION OF NUMBERS TOO LARGE TO BE ADDED MENTALLY.

First. Arrange the numbers carefully in a vertical column, so that the decimal point in each line shall stand directly under the decimal point in the line above; units being under units, tens under tens, hundreds under hundreds, tenths under tenths, hundredths under hundredths, &c. Draw a horizontal line under the column of numbers, and place a decimal point below it, in a line with those above it.

Second. Find the **sum** of the digits in the right-hand place.

Third. Carry one for every ten of this sum to the next place to the left. Set down the rest of the sum under its own place.

Fourth. Proceed to the next place to the left. Add its digits and the number carried. Carry to the left, one for every ten, and set down the rest.

---

* NOTE TO TEACHER. The pupil is supposed to find these fractions of the Integers by *trial* with strokes, or peas, or other units.

Fifth. Work every column similarly. What is carried from the last column must be set on the left of the **sum.**[*]

EXAMPLE WORKED OUT. INTEGERS ONLY. Find the sum of 86143· 9459· 574· and 74298·

These numbers, when arranged, stand thus:

86143·
9459·
574·
74298·

Then, sum of right-hand column $= (8 + 4 + 9 + 3)$ units;
$= 24$ units, $= 2$ tens and 4 units.
Set down 4 *(units) and carry* 2 *(tens) to tens' column.*

Sum of tens' column with 2 tens carried $= (2 + 9 + 7 + 5 + 4)$ tens;
$= 27$ tens, $= 2$ hundreds and 7 tens.
Set down 7 *(tens) and carry* 2 *(hundreds) to hundreds' column.*

Sum of hundreds' column with 2 hundreds carried $= (2 + 2 + 5 + 4 + 1)$ hundreds;
$= 14$ hundreds, $= 1$ thousand and 4 hundreds.
Set down 4 *(hundreds) and carry* 1 *(thousand) to thousands' column.*

Sum of thousands' column with 1 thousand carried $= (1 + 4 + 9 + 6)$ thousands;
$= 20$ thousands, $= 2$ ten-thousands.
Set down 0 *(thousands) and carry* 2 *(ten-thousands) to ten-thousands' column.*

Sum of ten-thousands' column with 2 (ten-thousands) carried $= (2 + 7 + 8)$ ten-thousands;
$= 17$ ten-thousands, $= 1$ hundred-thousand and 7 ten-thousands.
Set down 7 *(ten-thousands) and carry* 1 *(hundred-thousand) to hundred-thousands' column.*

As there are no digits in hundred-thousands' place to be added,
Set down 1 *in hundred-thousands' place.*

---

[*] NOTE TO TEACHER. The word "sum" should never be employed in any other than this its true sense. A slovenly practice exists of designating as "sums" arithmetical operations of every kind. We hear of "sums in addition," "sums in division," and even of "*subtraction sums*"! This practice should be abandoned, and instead of the loose phrase, "doing a sum," the pupil should be trained to use the correct expression, "working an example." Nomenclature furnishes potent aid in the pursuit of any science, but to do so it must be both *clearly understood* and *accurately applied.*

The example is now completed, and stands thus:

86143·
9459·
574·
74298·
170474· = *sum of the above four numbers.*

EXERCISE XXII.

EXAMPLES FOR PRACTICE, INTEGERS ONLY; *the numbers being already arranged.*

Find the sum of each of the following sets of numbers.

Ex. 1·	Ex. 2·	Ex. 3·	Ex. 4·
314·	84321·	34596·	341695·
8021·	3479·	59483·	593462·
3456·	543·	74396·	741684·
842·	41638·	35743·	395832·
· = sum of the four numbers.	= · &c.	= · &c.	= · &c.

Ex. 5·	Ex. 6·	Ex. 7·	Ex. 8·	Ex. 9·
314862·	471305·	341685·	4316842·	314806·
79643·	984216·	923164·	943654·	47054·
42138·	643573·	5427·	2465·	596·
945491·	594316·	643015·	71389·	3495·
314753·	843659·	96·	692·	450058·

Ex. 10·	Ex. 11·	Ex. 12·	Ex. 13·	Ex. 14·
41844328·	546396·	43186·	396184·	714834215·
59·	89543·	46285·	6984·	2349364·
127645·	946318·	1659097·	71905·	59642536·
4109·	1385·	24364·	2693·	1367305·
189·	27·	9476834·	543614082·	58216·

Method of Proof. After working an example in addition of Integers, the accuracy of the work may be tested by taking these two steps.

First. Omit one of the numbers, and find the sum of the others.

Second. Add this last-found sum to the number omitted. The number thus found is evidently the sum of all the numbers, and should be the same as that found by the first addition.

The number omitted in proving is usually the top one, which is then separated from the others by a horizontal line drawn under it.

Ex. 15· arranged, worked & proved.

314825·

459764·
936495·
83957·
9421·
721368·

Thus working and proving Ex. 15· we first add all six numbers, and find their sum = 2;525,830·

Then, cutting off top number, we find the sum of the lower five numbers = 2;211,005·

Then, adding this number and the top one, we find their sum = 2;525,830·

This is again the sum of all six numbers; and, agreeing with the sum previously found, proves the work correct.

2525830· = *sum of all six numbers.*

2211005· = *sum of lower five numbers, top one being omitted.*

2525830· = *sum of all six numbers. Proof.*

Work and prove the following examples, describing each of the three lines, found in the course of the work.

Ex. 16·	Ex. 17·	Ex. 18·	Ex. 19·	Ex. 20.
314236·	341684·	6148326·	314659·	741638·
964598·	74365·	439654·	821368·	215496·
23165·	309·	1630·	754983·	653843·
3614·	25416·	74368·	64215·	461587·
164358·	413973·	504·	89541·	703846·
946·	6410·	549691·	116437·	9135·
43684·	342154·	24306·	8954·	22·

Ex. 21·	Ex. 22·	Ex. 23·	Ex. 24·	Ex. 25·
3148629·	54016348·	8416·	341·	7143068·
5964357·	6943520·	93·	8464·	956184·
6419803·	4586·	29·	97105·	98429·
7206415·	13406582·	114·	641564·	6273·
943·	691887·	6008·	3950418·	81369·
69008·	40653·	41596·	95635·	532084·
57·	986·	8216935·	7412·	8216958·

EXERCISE XXIII.

Examples for Practice, *in which the numbers are to be arranged by the pupil.*

1· What are 641534· + 846· + 30015· + 16· + 1305064· + 95· = ?

2· What are 1356421· + 64· + 301· + 421645· + 88416· + 365· = ?
3· What are 794· + 843168· + 3148· + 91· + 3601· + 2012 = ?
4· What is the sum of 642· 8436· 91462· and 841· = ?
5· Add together 11416· 346421· 19· 354· and 12·
6· How many will 841· 935· 1642· and 96· amount to?
7· Find the sum of 3014· 190013· 2213· and 161435·
8· Add together 39146· 18462· 39604· 81462· and 3164·
9· Find the sum of 43714· 84217· 50406· 71436· and 492·
10· What number will 45642· 31498· 54036· and 3014· make?
11· What are 3142394· + 54316· + 19316· + 74174· + 808080· = ?
12· What are 794165· + 15437· + 84253· + 591· + 6164· = ?

EXERCISE XXIV.

EXAMPLES FOR PRACTICE, *in which both the notation and the arrangement are to be performed by the pupil.*

1· *Add together:* twenty-nine; six hundred and forty-two; four thousand, five hundred and twelve; eight hundred and sixteen; nineteen thousand and sixty; fifteen thousand and fifty-three; *and* fifteen hundred.

2· *Add together:* twelve hundred and sixty-one; ten hundred and sixty-six; twenty thousand, four hundred and fifteen; nineteen millions, nineteen thousand and nineteen; twenty-two millions, eighteen hundred and eleven; *and* sixty-two hundred and twenty-six.

3· *Add together:* three hundred and forty millions; six hundred and twenty-nine millions, eight hundred and thirty-nine thousand, four hundred and one; two thousand and twenty-two; twenty millions, eight hundred and sixteen; forty-five millions, fifty-four thousand and fifty-four; *and* thirty-two thousand, nine hundred and twenty-one.

4· *What is the sum of:* fifteen hundred millions, sixteen hundred thousand and twelve; forty-four millions, nine hundred and twelve thousand, three hundred and four; thirteen hundred and thirteen; twenty-nine thousand and fifteen; *and* six hundred and forty millions, nine hundred and five?

5· *How many will:* thirty-three millions, seven hundred and ninety-two thousand, six hundred and forty-five; eleven millions, nine hundred and eleven thousand and twenty-five; sixteen thousand and sixty; thirty-two; *and* one thousand and fifty; *make?*

6· *Find the sum of:* twelve; ninety thousand, four hundred and six; eight thousand and fifteen; nine; *and* six hundred and twenty-three.

7· *What number do the following amount to:* four thousand, three hundred and ninety-five; thirty-nine; five millions, fifty thousand and sixteen; four hundred and twenty-eight; ninety-four thousand, three hundred and forty-five; *and* one hundred and eighteen?

8· *Find the sum of:* nine hundred and forty-three; twenty-one thousand, four hundred and thirty-six; nineteen; three thousand, five hundred and seventy-five; nine hundred and thirty-two; ninety-five; *and* one million, nine hundred and forty-six thousand, six hundred and three.

Addition of Decimal Fractions.

Decimals are added in precisely the same way as Integers. But, after arranging the numbers, drawing the horizontal line, and placing under it the decimal point, one other preparatory step must be taken. It is this: ***Write below the line a cipher under each decimal place.****

EXAMPLE WORKED OUT. INTEGERS AND DECIMALS. Find the sum of: 314·27 58·184 3629·357 457·426 and 5·8.

The example prepared for addition stands thus:

```
 314·27
  58·184
3629·357
 457·426
   5·8
--------
    ·000
========
```

Sum of right-hand column, = 17 thousandths; = 1 hundredth and 7 thousandths. *Set down* 7 *(thousandths)* instead of the cipher, *and carry* 1 *(hundredth).*

Sum of hundredths' column, with 1 hundredth carried, = 23 hundredths; = 2 tenths and 3 hundredths. *Set down* 3 *(hundredths)* instead of the cipher, *and carry* 2 *(tenths).*

Sum of the tenths' column, with 2 tenths carried, = 20 tenths; = 2· and 0 tenths. *Let the cipher remain in tenths' place, and carry* 2·

Proceed with the addition of integers as in former examples, and you will find the sum of the five numbers to be 4465·037

Method of Proof, the same as in Addition of Integers.

EXERCISE XXV.

Find the sum of each of the following sets of numbers, and prove the work; describing (as in Exercise xxii., *Example* 15·*) each of the three lines found in the course of the process.*

---

* NOTE TO TEACHER. This little device is adopted that each figure of the **SUM** may, as soon as written, express its own true value. If the ciphers be not employed, each successive decimal figure would in reality occupy the first place of decimals until the next-found figure were interposed; and hence, no one of them would be true until the tenths' place were filled. For, after adding the thousandths' column, the **SUM-LINE** would stand thus: ·7 *representing seven tenths instead of seven one-thousandths, as intended.* And, after adding the hundredths' place we should have ·37 *expressing three tenths and seven one-hundredths, instead of three one-hundredths and seven one-thousandths.* In the preparation of this little work, the author's ruling idea is that in teaching any arithmetical process, the result of every successive step should invariably be made to intelligibly and accurately declare its own meaning and value.

Ex. 1·	Ex. 2·	Ex. 3·	Ex. 4·	Ex. 5·
349·183	74·631	3145·85	71493·16	1643·2005
25·0004	·95	637·421	42·5985	95·416
4516·36	3428·008	45·504	589·359	983·4
248·54937	547·947	8956·936	7·6483	64·52

Ex. 6·	Ex. 7·	Ex. 8·	Ex. 9·	Ex. 10·
5436·954	9164·832	349·410	6436·845	91486·384
347·61007	45·9106	658·83	9054·61	8274·645
9639·85	·045	4937·4	969·85	·943
546·53	416·95	24595·5014	3425·004	·0968
92·8615	32·	7016·0008	769·65	164·95
4361·954	1964·	95·15	2050·493	·536

Ex. 11·	Ex. 12·	Ex. 13·	Ex. 14·	Ex. 15·
·41683	·9465	41·368	461·38	3148·65
·054	·594	·842	·426	24·96
·6938	·3069	·91	48·245	463.218
·546	·5426	·645	146·95	·964
·934	·6948	·32	37·842	·3546
·0015	·436	469·7	549·36	·4158

EXERCISE XXVI.

NOTATION AND ADDITION, INTEGERS AND DECIMAL FRACTIONS.*

1· *Find the sum of:* Thirty-nine, and twenty-four one-hundredths; six hundred and twenty-four, five tenths, seven one-hundredths, and three ten-thousandths; nine hundred and twelve, seven one-hundredths, and nine one-millionths; *and* twenty-four thousand and six, fourteen one-thousandths, and eight ten-thousandths.

---

* Note to Teacher. These examples should be dictated to the pupils, and worked by them on the black board; and should afterwards be gone over by each pupil on his own slate or paper.

2· *Find the sum of:* Forty-two millions, eight hundred and fifty thousand, six hundred and three, nine tenths, four one-hundredths, nine one-thousandths, eight ten-thousandths, and two one-millionths; sixty-four, and twelve one-hundredths; one hundred and nineteen, eight one-hundredths, and four one-thousandths; twenty-two hundreds and twenty-two, nine tenths, and six one-hundred-thousandths; *and* sixty-three, and seven one-thousandths.

3· *Add together:* thirteen millions, ninety-nine thousand, sixteen hundred and four, and twenty-nine one-hundredths; twenty-three thousand, seven hundred and eleven, and four hundred and twenty-five ten-thousandths; eighty-four millions, seventeen thousand, five tenths, nine one-hundredths, and six one-millionths; *and* eighteen hundred and six, and seven thousand, six hundred and twenty-five hundred-thousandths.

4· *What number will the following amount to:* sixteen thousand, sixteen hundred and sixteen, sixteen one-hundredths, and eight one-thousandths; ninety-four millions and nineteen, seven one-hundredths and nine ten-thousandths; four hundred and twenty-two thousand and fifty, eleven one-hundredths, three one-thousandths, and six ten-thousandths; fifteen, and two hundred and fourteen one-thousandths; three hundred and nineteen thousand and twelve, and five tenths; *and* two thousand, one hundred and twelve?

5· *Find the sum of the following numbers:* two thousand, one hundred and forty-six, and three hundred and eighty-nine one-thousandths; two hundred and fourteen, and six thousand three hundred and eighty-nine ten-thousandths; twenty-one, and forty-six thousand, three hundred and eighty-nine one-hundred-thousandths; two, and one hundred and forty-six thousand, three hundred and eighty-nine one-millionths; *and* two million, one hundred and forty-six thousand, three hundred and eighty-nine ten-millionths.

ADDITION OF CONCRETE OR APPLICATE NUMBERS.

See Page 24·

Concrete or applicate numbers can only be added together when they express the same sort of units.

24· shillings is a concrete number, and 32· bushels is another concrete number; but they cannot be added together into one sum. For, although the abstract numbers 24· and 32· amount to 56·, yet 24· shillings and 32· bushels neither make 56· shillings nor 56· bushels. We can go no further towards combining these two lots of unlike units than to write them with the sign of addition between them, thus: 24· shillings + 32· bushels.

Principle V. *Unlike units, cannot be compared in any way.*

It is impossible to say which is the longer, a *mile* or a *day*, for one is a unit of distance, and the other is a unit of time. From this impossibility of comparing unlike units arises the fun of such questions as, "How far is it from Westminster Bridge to Christmas Day?" and similar amusing absurdities.

Large and small units of the same kind are not to be included in the foregoing remark. Pounds sterling, shillings, sixpences, groats, threepenny-pieces, pence, half-pence, farthings, crowns, florins, half-crowns, and half-sovereigns, are all units (of different value) of British money. These can be added, because a certain number of units of one value are equal to one unit of a higher value. A certain number of pence are equal in value to a pound sterling, and a certain number of florins are equal in value to a certain number of crowns, and so on. But no number of *days* would ever amount to one *mile*, or to any number of *miles*, nor would any number of *miles* ever amount to a number of *days*. Hence, our different coins are units of *like kind, but of unequal value;* but *days* and *miles* are units *unlike in kind;* so are *hundredweights* and *yards*, *hours* and *men*, *shillings* and *ounces;* and it is these units, *unlike in kind*, which cannot be added or compared in any way.

These considerations will be of much use to us hereafter.

SIMPLE AND COMPOUND ADDITION.

All the foregoing examples have been in Simple Addition, because the numbers added have been abstract numbers.

The sum of concrete numbers, consisting of units of the same kind and value, is also found by Simple Addition.

The sum of unequal units of like kind is found by Compound Addition, which will be hereafter taught.

EXERCISE XXVII.

SIMPLE ADDITION OF CONCRETE NUMBERS, INTEGERS AND DECIMALS, TOO LARGE TO BE ADDED MENTALLY.

1· *Find the sum of:* 184· shillings; 309·5 shillings; 6104·25 shillings; and 35906·75 shillings.

2· *What is the sum of:* 364·385 hundredweights; 1346·625 h. w.; 53·975 h. w.; 5042·95 h. w.; *and* 12·5 h. w.?

Note. Hundredweights are written thus, cwts.; pounds sterling, that is sovereigns, thus, £; pounds weight, thus, lb.; ounces, thus, oz.

3· A butcher bought a bullock weighing 7·5 cwts.; another, weighing 6·25 cwts.; a third, weighing 4·75 cwts.; and a sheep, weighing 1·25 cwts.; what weight of meat did he buy in all?

Henry has in his drawer 25· marbles, 16· apples, 3· peg-tops, 4· sheets of paper, 5· sticks of slate-pencil, and seven steel pens; John has 13· sheets of paper, 4· sticks of slate-pencil, 21· steel pens, 11· apples, 1· peg-top, and 54· marbles; Charles has 20· apples, 14· sticks of slate pencil, 22· marbles, 2· peg-tops, 11· sheets of paper, and 13· steel pens; Arthur's box contains 4· peg-tops, 3· slate-pencils, 13· marbles, a pocket-comb, 19· sheets of paper, 9· apples, two oranges, half-a-dozen pears, and 6· steel pens. What is the sum of their effects?

Here are concrete numbers of various kinds. The units of each kind must be added into a separate sum.

Sum of the marbles = (25· + 54· + 22· + 13·) marbles
= 114· marbles.
Sum of the apples = (16· + 11· + 20· + 9·) apples
= 56· apples.
Sum of peg-tops = (3· + 1· + 2· + 4·) peg-tops
= 10· peg-tops.
Sum of sheets of paper = (4· + 13· + 11· + 19·) sheets of paper
= 47· sheets of paper.
Sum of steel-pens = (7· + 21· + 13· + 6·) steel-pens
= 47· steel-pens.
Sum of slate-pencils = (5· + 4· + 14· + 3·) slate-pencils
= 26· slate-pencils.

Therefore, the whole sum of these four young gentlemen's effects is 114· marbles, 56· apples, 10· peg-tops, 47· sheets of paper, 26· slate pencils, 47· steel pens, 1· pocket comb, 2· oranges, and 6· pears.

In the following examples the pupil must himself select like units for addition.

5· A labourer transplanted on Monday, 61· young oaks, 25· birches, and 75· larches; on Tuesday, 105· oaks, 12· birches, and 30· larches; on Wednesday, 50· larches, 28· oaks, 35· birches, 20· laurels, and 21· rhododendrons; on Thursday, 45· laurels and 152· oaks; on Friday, 68· rhododendrons, 35· silver firs, 30· larches, and 10· birches; and, on Saturday, 79· oaks, 35· rhododendrons, and 50· laurels. What was the exact sum of his week's work?

6· A poulterer bought from a farmer's wife, eleven geese, four turkeys, 29· lb. of butter, 42· eggs, 12· lb. of cheese, and six ducks; from another he purchased twelve fowls, two turkeys, eight geese, and 100· eggs; a third supplied him with twelve ducks, fourteen pounds of cheese, one turkey, two geese, and 40· lb. of butter. What was the exact amount of stock he laid in?

7· Mrs. Gillies ordered from her butcher, Mr. Carver, 24· lb. of beef, twelve pounds of veal, 2· lb. of suet, and a leg of mutton, weighing 9· lb.; Mrs. Floyd also ordered from him 9· lb. of mutton, 8· lb. of veal, 3· lb. of suet, and 30· lb. of beef; Mrs. Green purchased 12· lb. of mutton, and 14· lb. of beef; and then Mr. Carver found he had remaining 96· lb. of beef, 28· lb. of mutton, 5· lb. of suet, and 40· lb. of veal. How much of each kind of meat did he sell to the three ladies? How much had he of each kind at first? What weight had he *in all* at first? What weight did he sell altogether? What weight had he remaining in all?

ADDITION OF VULGAR FRACTIONS.

Vulgar fractions can only be added when they have equal denominators.

This is for the same reason that unlike units cannot be collected into one sum. (See page 35· and Principle v.) If I have $\frac{1}{4}$ of an apple, and $\frac{1}{3}$ of the same apple, I have *two pieces*, but they are neither two fourths nor two sixths. These two unlike parts cannot be described by any fractional expression having 2 for its numerator. Before I can add them, I must find some smaller parts into which both the one fourth and the one sixth can be cut. Those smaller parts are twelfths. For, dividing any unit into twelfths, I find that by taking three of those twelfths I obtain one fourth of the unit, and by taking two of the twelfths I obtain one sixth of the unit. *(Prove this by so dividing a unit-square.)* Hence $\frac{1}{4}=\frac{3}{12}$; $\frac{1}{6}=\frac{2}{12}$; and after being changed into these smaller parts the fractions can be added.

The sum of vulgar fractions, which have equal denominators, is found by adding their numerators.

$$\therefore\quad \frac{3}{15}+\frac{2}{15}+\frac{9}{15}+\frac{11}{15}=\frac{3+2+9+11}{15}=\frac{25}{15}$$

$$\text{and}\quad \frac{1}{4}+\frac{1}{6}=\frac{3}{12}+\frac{2}{12}=\frac{3+2}{12}=\frac{5}{12}$$

The method of thus preparing unlike fractions for addition will be explained after Multiplication.

EXERCISE XXVIII.

EXAMPLES IN ADDITION OF VULGAR FRACTIONS, HAVING THE SAME DENOMINATOR.

1· Find the sum of $\frac{4}{5}$, $\frac{3}{5}$, $\frac{2}{5}$, & $\frac{1}{5}$; and of $\frac{9}{10}$, $\frac{3}{10}$, $\frac{8}{10}$ & $\frac{7}{10}$
2· „ „ „ $\frac{7}{8}$, $\frac{2}{8}$, $\frac{4}{8}$, & $\frac{6}{8}$; and of $\frac{4}{11}$, $\frac{5}{11}$, $\frac{10}{11}$ & $\frac{3}{11}$
3· „ „ „ $\frac{9}{12}$, $\frac{5}{12}$, $\frac{7}{12}$, & $\frac{3}{12}$; and of $\frac{6}{7}$, $\frac{3}{7}$, $\frac{4}{7}$ & $\frac{2}{7}$
4· „ „ „ $\frac{1}{2}$, $\frac{3}{2}$, $\frac{6}{2}$, & $\frac{5}{2}$; and of $\frac{15}{19}$, $\frac{11}{19}$, $\frac{13}{19}$, & $\frac{6}{19}$

5· Add together $\frac{5}{10}$, $\frac{2}{10}$, $\frac{7}{10}$, $\frac{3}{10}$, & $\frac{11}{10}$; also $\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$ & $\frac{3}{8}$
6· „ „ $\frac{6}{15}$, $\frac{3}{15}$, $\frac{4}{15}$, $\frac{12}{15}$, & $\frac{13}{15}$; also $\frac{7}{45}$, $\frac{9}{45}$, $\frac{6}{45}$ & $\frac{12}{45}$

7· What are $\frac{3}{47}+\frac{5}{47}+\frac{2}{47}+\frac{6}{47}+\frac{1}{47}=?$
8· „ „ $\frac{4}{21}+\frac{3}{21}+\frac{6}{21}+\frac{8}{21}+1\cdot=?$
9· „ „ $\frac{5}{39}+1\cdot+\frac{3}{39}+\frac{15}{39}+\frac{6}{39}=?$
10· „ „ $\frac{2}{17}+\frac{5}{17}+\frac{6}{17}+\frac{4}{17}+1\cdot=?$
11· „ „ $\frac{6}{13}+2\cdot+\frac{9}{13}+\frac{5}{13}+1\cdot=?$
12· „ „ $\frac{4}{107}+1\cdot+\frac{3}{107}+\frac{95}{107}+\frac{33}{107}=?$

Arithmetical Symbol. To express that one quantity is greater or less than another, we place between them the sign of inequality, which consists of two lines forming an acute angle, thus > or thus < the opening being always turned towards the greater of the two quantities.

Hence, when the greater quantity stands first, the opening of the lines is turned towards the left > and the sign is then to be read "is greater than," or "are greater than." But, when the smaller of the two quantities stands first, the opening of the lines is turned towards the right, and the sign then represents the words "is (or are) less than." So that

$\frac{5}{6} > \frac{6}{8}$ means "*five sixths are greater than six eighths ;*" and
$\frac{11}{12} < \frac{13}{14}$ " "*eleven twelfths are less than thirteen fourteenths.*"

Proper and Improper Fractions. "A fraction is any part or parts of a unit." (See page 12·)

The greatest number of halves, thirds, fourths, fifths, or of any other parts that we can form by dividing one unit can only amount to unity.

∴ one unit $= \frac{2}{2}$ or $\frac{3}{3}$ or $\frac{4}{4}$ or $\frac{15}{15}$ or $\frac{100}{100}$ &c. (See page 12·)

Hence, any fraction whose numerator is equal to its denominator is equal to unity.
" any fraction whose numerator is less than its denominator is less than unity.
" any fraction whose numerator is greater than its denominator is greater than unity.

Fractions of this last kind, namely, those greater than unity, are formed by dividing more than one unit into the same kind of parts. By cutting two units into fifths (for example) we might obtain $\frac{6}{5}$, $\frac{7}{5}$, $\frac{8}{5}$, $\frac{9}{5}$, $\frac{10}{5}$, each of which fractions is greater than unity.

Therefore, a fraction is $=$, $>$, or $<$ unity,
according as its numerator is $=$, $>$, or $<$ its denominator.

A proper fraction is less than unity.
An improper fraction is equal to or greater than unity.
A proper fraction has its numerator less than its denominator.
An improper fraction has its numerator equal to or greater than its denominator.

EXERCISE XXIX.

Recapitulatory questions.[*]

What is meant by "unit"? What is an integer? Explain the use of the decimal point. Which are the "significant figures," and why are they called "significant figures"? What is the use of the cipher? Where is the cipher useless, and why? How is the same digit made to represent different numbers? What is the art of expressing numbers in digits called? Repeat the first notation table. Repeat the second notation table. Repeat the first numeration table, and the second. Where do notation tables and numeration tables begin? Declare the exact value represented by each digit in the following numbers, namely: 840201·5436 ; 701000·506; 303030·01 ; and 11209500·65004

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[*] NOTE TO TEACHER. "*The secret of all successful study is the constant habit of reviewing.*" If the pupil cannot at once answer any one of the questions, or perform any operation enjoined in this exercise, he should search the past pages of the book until he meet with the information which may have escaped his memory.

What is the first principle of Arithmetic?

What is the second principle of Arithmetic?

What does the word local mean?

What is meant by saying that digits have a local value?

What advantage arises from giving digits a local value?

What is a fraction? How do you procure halves of a unit, how fourths, ninths, tenths of a unit? Describe the two steps necessary to get four elevenths of a unit. Draw a line on your slate, and find six eighths of that line. Draw another line, same length, and find seven elevenths of it. Draw for your unit a square, and find three eighths of one half of it. What kind of fraction is this? Draw other three squares, and find two ninths of the first; seven eighths of one third of the second; and two fifths of three elevenths of the third. Why are two numbers necessary for the notation of a vulgar fraction? How are those two numbers placed? What is each of those numbers called? Perform the notation of six thirteenths; twenty-two forty-fifths; three hundred and twenty-two nine hundred and twelfths. Read in words $\frac{13}{14}$; $\frac{160}{614}$; $\frac{3}{17}$; $\frac{19}{26}$; $\frac{541}{837}$; $\frac{116}{1048}$; $\frac{215}{11004}$; $\frac{36}{841}$; and $\frac{194}{211}$. Explain how each of those fractions of a unit would be procured. What two things does the denominator of a fraction tell you? What does the numerator tell you? *What is a simple fraction? What is a compound fraction? What fractions are called decimal fractions?* Which of the two terms of a decimal fraction is generally omitted? How many places of figures are required to express a decimal fraction? What is the correct way of reading ·415*; first, taking the figures collectively; and, second, taking the figures separately.

Write answers to the following questions.

1· Why is ours called the Decimal System of Arithmetical Notation?
2· What is the principle of this system?
3· What is meant by "Decimal Arithmetic"?
4· Repeat the two rules for carrying.
5· In how many ways can numbers be changed?
6· What is Addition? What is Subtraction?
7· What kind of Addition has a particular name, and what is that name?
8· What is Division of Integers?
9· Describe the sign of Addition. Why is it called "*plus*"?
10· *Put into arithmetical characters*† *the following sentences.*

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* NOTE TO TEACHER. The two incorrect methods to be avoided are "Decimal four, one, five," and "Decimal four hundred and fifteen." See note, page 22·

† NOTE TO TEACHER. It is scarcely possible to over-estimate the value, to the young student, of a thorough acquaintance with the language of Arithmetic, both verbal and symbolical, and an accurate facility in employing it. In this manual, the various terms and symbols are introduced by degrees, as they are required: and the desired familiarity with them will be gradually attained by frequent exercise on examples like the present, in translating ordinary language into Arithmetical, and the reverse. The examples here given are merely intended as specimens, our limits forbidding the insertion of a sufficient number to ensure proficiency.

"Because eight and nine and five are equal to twenty-two;
And because two and four and sixteen are equal to twenty-two;
Therefore, eight and nine and five are equal to two and four and sixteen."

This is done thus :

$$\because\ 8\cdot + 9\cdot + 5\cdot = 22\cdot \qquad \text{And} \because\ 2\cdot + 4\cdot + 16\cdot = 22\cdot$$
$$\therefore\ 8\cdot + 9\cdot + 5\cdot = 2\cdot + 4\cdot + 16\cdot$$

11· *Express in arithmetical language the following lines.*

"The sum of nineteen and one fourth of thirty-two is equal to twenty-seven."

"One half of twenty added to one eighth of sixteen is equal to twelve."

"Sixteen and nine are equal to thirteen and twelve."

"One half of one half of a unit is equal to one fourth of the unit."

12· *Write in words the following lines.*

$\frac{1}{2}$ of $8\cdot + 9\cdot + \frac{1}{4}$ of $16\cdot = \frac{1}{5}$ of $15\cdot + \frac{1}{6}$ of $18\cdot + \frac{1}{4}$ of $12\cdot + \frac{1}{2}$ of $16\cdot$

$6\cdot + 9\cdot + 5\cdot + 4\cdot = \frac{1}{4}$ of $16\cdot + \frac{1}{3}$ of $24\cdot + \frac{1}{6}$ of $72\cdot$

$\frac{1}{4}$ of $\frac{1}{5}$ of $1\cdot = \frac{1}{20}$ of $1\cdot = \frac{1}{2}$ of $\frac{1}{10}$ of $1\cdot = \frac{1}{5}$ of $\frac{1}{4}$ of $1\cdot = \frac{1}{10}$ of $\frac{1}{2}$ of $1\cdot$

$\frac{3}{4}$ of $\frac{1}{5}$ of $1\cdot = \frac{3}{20}$ of $1\cdot$ $\qquad$ $\frac{1}{2}$ of $\frac{2}{10}$ of $1\cdot = \frac{2}{20}$ of $1\cdot = \frac{1}{10}$ of $1\cdot$

13· *Write in words the following.*

$\because\ 24\cdot = \frac{1}{2}$ of $48\cdot$ and $\because\ 6\cdot = \frac{1}{4}$ of $24\cdot$ $\quad \therefore\ 6\cdot = \frac{1}{4}$ of $\frac{1}{2}$ of $48\cdot$

$\because\ 9\cdot + 8\cdot = \frac{1}{2}$ of $34\cdot$ and $\because\ 18\cdot = 9\cdot + 8\cdot + 1\cdot$ $\quad \therefore\ 18\cdot = \frac{1}{2}$ of $34\cdot + 1\cdot$

14· How would you express in words $8\cdot + 6\cdot = 14\cdot$?

Answer. $8\cdot + 6\cdot = 14\cdot$ may be expressed in words in two ways.
First. "*Eight plus six are equal to fourteen.*"
Second. "*The sum of eight and six is fourteen.*"

MULTIPLICATION.*

"Multiplication is the addition or repetition of equal numbers."†

If I find the sum of four sixes, that is, if I add together *six* and *six* and *six* and *six*, I repeat six *four times*, and it is said that I "*multiply six by four*," or take "*four times six.*"

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* NOTES TO TEACHER. The introduction of Multiplication before Subtraction is an innovation which has been ventured on for the following four reasons.

First. Multiplication is only a species of Addition.

Second. Addition of Vulgar Fractions must remain imperfect until Multiplication be taught.

Third. Multiplication must precede Subtraction of Vulgar Fractions.

Fourth. There is *no* reason why Subtraction should come before Multiplication.

† This definition does not include what are called fractional multipliers, (which are not purely multipliers, inasmuch as each of them involves also a divisor,) but this will be provided for further on.

Five words employed in Multiplication must be clearly understood. They are these: Multiplicand, Multiplier, Factor, Product, and Multiple.

The number six repeated nine times amounts to fifty-four.

Here 6· is the Multiplicand, the number that is repeated.
,, 9· is the Multiplier, shewing how many times to repeat 6·
,, 54· is the Product of 6· and 9·
,, 6· and 9· are Factors of the Product 54·
,, 54· is a Multiple of 6· and also a Multiple of 9·

The **Multiplicand** is the number which is to be repeated.

The **Multiplier** is the number expressing the repetitions of the multiplicand.

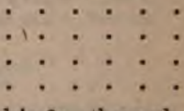
The **Product** is the number produced by repeating the Multiplicand.

The **Factors** of the Product are the Multiplicand and Multiplier.

The Product is a **Multiple** of each of the Factors.

Take 24· peas, pebbles, or other convenient units. Place them in four horizontal lines of six units each, under each other. There will be six vertical lines of four units each.

Counting the horizontal lines shews that *four times six = twenty-four.* Counting the vertical lines shews that *six times four = twenty-four.*
∴ *Four times six = six times four.*



In 4· times 6· = 24· the multiplicand is 6· the multiplier is 4· and the product is 24·

In 6· times 4· = 24· the former multiplier 4· is now the multiplicand, and the former multiplicand 6· is now the multiplier, but the product is still 24· as before.

PRINCIPLE VI. *The Multiplicand and the Multiplier may be exchanged the one for the other, and no alteration will be made thereby in the Product.*

Arithmetical Symbol. The sign of multiplication is a cross formed by two equal sloping lines, cutting each other in the middle, thus, ×. It is called, in reading, "multiplied by," or "times."

6· × 4· means "*six multiplied by four*," or "*four times six*," or, by Principle vi. "*four multiplied by six*," or "*six times four*."
and ∵ 6· + 6· + 6· + 6· means four sixes added together;
and ∵ 6· × 4· „ six repeated four times;
∴ 6· + 6· + 6· + 6· means the same as 6· × 4·

Multiples. *A multiple of a number is found by multiplying it by any number.*

2· × 1· (that is, "once two,") = 2·
∴ 2· *is the first multiple of* 2·
2· × 2· (that is, "twice two,") = 4·
∴ 4· *is the second multiple or double of* 2·

2· × 3· = 6· which is the third multiple or triple of 2·
2· × 4· = 8· „ „ fourth multiple or quadruple of 2·
9· × 3· = 27· „ „ third multiple of 9·
8· × 5· = 40· = fifth multiple of 8·
7· × 6· = 42· = sixth multiple of 7·
9· × 7· = 63· = seventh multiple of 9·
&c. &c. &c.

The first multiple of a number is the number itself.
„ second multiple of a number is found by multiplying it by 2·
„ third multiple „ „ by multiplying it by 3·
„ fourth multiple „ „ by multiplying it by 4·
&c. &c. &c.

And, because by Principle vi., exchanging the Multiplicand and the Multiplier one for the other makes no alteration in the Product,
∴ The 3rd multiple of 2· = 2nd multiple of 3·
„ 9th multiple of 7· = 7th multiple of 9·
„ 8th multiple of 4· = 4th multiple of 8·
And so on.

Composite Numbers. Every multiple, except the first, of any number greater than unity, is a composite number.

Prime Numbers. A prime number is one which is not a multiple of any number but itself and unity.

A composite number has factors greater than unity.

A prime number has no factors except itself and unity.

Hence 8· is a composite number,
But 3· 5· 7· and 11· are prime numbers.

The first twelve multiples of the first twelve numbers
(1· 2· 3· 4· 5· 6· 7· 8· 9· 10· 11· 12·)
must be learned by heart from the Multiplication Table.

The column of second multiples must be first learned, and rehearsed in this way, "twice one are two," "twice two are four," and so on to "twice twelve are twenty-four;" then the column of third multiples, "three times one are three," &c.; then the other columns in succession. The column of first multiples does not require to be learned. It is repeated thus: "once one is one, once two is two," &c.

THE MULTIPLICATION TABLE.

A collection of the first twelve multiples of the first twelve numbers.

1·× 1·= 1·	2·× 1·= 2·	3·× 1·= 3·	4·× 1·= 4·
1·× 2·= 2·	2·× 2·= 4·	3·× 2·= 6·	4·× 2·= 8·
1·× 3·= 3·	2·× 3·= 6·	3·× 3·= 9·	4·× 3·=12·
1·× 4·= 4·	2·× 4·= 8·	3·× 4·=12·	4·× 4·=16·
1·× 5·= 5·	2·× 5·=10·	3·× 5·=15·	4·× 5·=20·
1·× 6·= 6·	2·× 6·=12·	3·× 6·=18·	4·× 6·=24·
1·× 7·= 7·	2·× 7·=14·	3·× 7·=21·	4·× 7·=28·
1·× 8·= 8·	2·× 8·=16·	3·× 8·=24·	4·× 8·=32·
1·× 9·= 9·	2·× 9·=18·	3·× 9·=27·	4·× 9·=36·
1·×10·=10·	2·×10·=20·	3·×10·=30·	4·×10·=40·
1·×11·=11·	2·×11·=22·	3·×11·=33·	4·×11·=44·
1·×12·=12·	2·×12·=24·	3·×12·=36·	4·×12·=48·
5·× 1·= 5·	6·× 1·= 6·	7·× 1·= 7·	8·× 1·= 8·
5·× 2·=10·	6·× 2·=12·	7·× 2·=14·	8·× 2·=16·
5·× 3·=15·	6·× 3·=18·	7·× 3·=21·	8·× 3·=24·
5·× 4·=20·	6·× 4·=24·	7·× 4·=28·	8·× 4·=32·
5·× 5·=25·	6·× 5·=30·	7·× 5·=35·	8·× 5·=40·
5·× 6·=30·	6·× 6·=36·	7·× 6·=42·	8·× 6·=48·
5·× 7·=35·	6·× 7·=42·	7·× 7·=49·	8·× 7·=56·
5·× 8·=40·	6·× 8·=48·	7·× 8·=56·	8·× 8·=64·
5·× 9·=45·	6·× 9·=54·	7·× 9·=63·	8·× 9·=72·
5·×10·=50·	6·×10·=60·	7·×10·=70·	8·×10·=80·
5·×11·=55·	6·×11·=66·	7·×11·=77·	8·×11·=88·
5·×12·=60·	6·×12·=72·	7·×12·=84·	8·×12·=96·
9·× 1·= 9·	10·× 1·= 10·	11·× 1·= 11·	12·× 1·= 12·
9·× 2·= 18·	10·× 2·= 20·	11·× 2·= 22·	12·× 2·= 24·
9·× 3·= 27·	10·× 3·= 30·	11·× 3·= 33·	12·× 3·= 36·
9·× 4·= 36·	10·× 4·= 40·	11·× 4·= 44·	12·× 4·= 48·
9·× 5·= 45·	10·× 5·= 50·	11·× 5·= 55·	12·× 5·= 60·
9·× 6·= 54·	10·× 6·= 60·	11·× 6·= 66·	12·× 6·= 72·
9·× 7·= 63·	10·× 7·= 70·	11·× 7·= 77·	12·× 7·= 84·
9·× 8·= 72·	10·× 8·= 80·	11·× 8·= 88·	12·× 8·= 96·
9·× 9·= 81·	10·× 9·= 90·	11·× 9·= 99·	12·× 9·=108·
9·×10·= 90·	10·×10·=100·	11·×10·=110·	12·×10·=120·
9·×11·= 99·	10·×11·=110·	11·×11·=121·	12·×11·=132·
9·×12·=108·	10·×12·=120·	11·×12·=132·	12·×12·=144·

EXERCISE XXX.

SPECIMEN OF QUESTIONS ON THE MULTIPLICATION TABLE.

1· Repeat the column of fourth multiples.
2· Repeat the same backwards.
3· What multiple of 4· is 32·? What number is 6· one fourth of?

4· What is the product of 6· and 4·?
5· How many fours must be added together to make 36·?
6· By what number must 4· be multiplied to produce 28·?
7· How many elevens in 44·? How many fourths in 11·?
8· What part of 44· is 11·? What part of 44· is 4·?
9· What is the fourth part of 48·?
10· How do you know this? How many units = $\frac{48}{4}$?
11· What is the eighth part of 32·? How many units = $\frac{32}{4}$?
12· How many fours are there in 24· in 25· in 26·?
13· What is the fourth part of 24· of 25· of 26·?
14· By what principle do you find the fourth part of 24· from the multiplication table?

Answer. The multiplication table informs me that there are six fours in 24· Therefore, by Principle iii., there are six units in $\frac{1}{4}$ of 24·

15· What are the factors of 28· 40· 16· 12·?
16· What numbers are 9· and 4· factors of? What No. is 4· $\frac{1}{9}$ of?

Many similar questions should be asked on each column of multiples.

17· There are three pairs of factors for 24· Name them.
18· How many sevens in 29·?

Answer. There are *four sevens* and *one unit* in 29· that is 4· sevens and $\frac{1}{7}$ of seven.

19· What is $\frac{1}{12}$ of 15· $\frac{1}{12}$ of 23· $\frac{1}{12}$ of 39· $\frac{1}{12}$ of 42· $\frac{1}{12}$ of 100·?
(See PRINCIPLE III. page 24·)
20· What is the double of 8· the triple of 9· the quadruple of 12·?
21· Name the 4th multiple of each of the following numbers.
7· 9· 4· 6· 3· 8· 12·
22· What are the third multiples of the first twelve numbers?
23· Name the fourth multiples of the first twelve numbers.
24· Name the fifth multiples of the first twelve numbers.
25· How many units = $\frac{16}{2}$; $\frac{18}{6}$; $\frac{13}{11}$; $\frac{12}{4}$; $\frac{6}{2}$; $\frac{9}{2}$; $\frac{17}{6}$; $\frac{21}{9}$?
26· Arrange the first hundred numbers in two columns, one containing the composite numbers, and the other containing the prime numbers.

MULTIPLICATION OF INTEGERS.

To multiply an Integer by any Integer less than 13·

Place the multiplier under the multiplicand, so that the decimal points shall be in one vertical line.

Under the multiplier draw a horizontal line as long as the multiplicand.

Below this line place a decimal point directly under the other two.

Commence at the right hand of the multiplicand, and multiply each of its figures in succession by the multiplier.

Set down and carry as in addition.

EXAMPLE WORKED OUT.

Multiply nine thousand four hundred and sixty-three by eight.

Example prepared.
9463·
8·
·

8· times 3· = 24·
Set down 4 *(units) and carry* 2 *(tens).*
8· times 6 tens + 2 tens (carried) = 50 tens.
Set down 0 *(tens) and carry* 5 *(hundreds).*
8· times 4 hundreds + 5 hundreds (carried) = 37 hundreds.
Set down 7 *(hundreds) and carry* 3 *(thousands).*
8· times 9 thousands + 3 thousands (carried) = 75 thousands.
Set down 5 *(thousands) and carry* 7 *(ten-thousands).*
Set down 7 *(ten-thousands).*

Example completed.
9463·
8·
75704· = *Product of the Factors* 9463· *and* 8·; = 8*th multiple of* 9463·; = *also* 9463*rd multiple of* 8·

EXERCISE XXXI.

EXAMPLES IN MULTIPLICATION OF ABSTRACT NUMBERS, BY INTEGERS LESS THAN 13·

1· Multiply 64138· by 3·
2· „ 410096· by 4·
3· „ 796543· by 5·
4· „ 59634· by 6·
5· „ 146745· by 7·
6· „ 5043196· by 8·
7· What are 3146598· × 9· = ?
8· „ „ 4563985· × 10· = ?
9· „ „ 65419683· × 11· = ?
10· „ „ 954395· × 12· = ?
11· „ „ 74123456· × 9· = ?
12· „ „ 9261834· × 8· = ?
13· Find the ninth multiple of 64283· and the sixth multiple of 6954318·
14· Find the seventh multiple of 9846321· and the 11th of 947632·
15· What is the double of 3216483·?
16· What is the triple of 47643547·? What number is 495· one half of?
17· What number is quadruple of 76984326·?
18· What is the product of 614· and 12· also of 395· and 7·?
19· Of what number are 2469· and 8· factors?
20· Multiply 93468· by 9· What multiple of 93468· have you found, and what multiple of 9·?
21· Find the 145th multiple of 9·
This is the same as the 9*th multiple of* 145· *See* PRINCIPLE VI., *page* 42· *Always employ the smaller factor as the multiplier.*
22· What number is the sum of 20614· nines?
23· What number is 845· the seventh part of?
Any number is the seventh part of its seventh multiple.
24· Of what number is 4865· the sixth part?
25· 9· is $\frac{1}{486}$ of a certain number. What is that number?
The number must be equal to 486· *nines; that is, it must be the* 486*th multiple of* 9· *or the* 9*th multiple of* 486·

26· Raise 694325· to its eleventh multiple.
27· " 73962· to its ninth multiple.
28· " 11· to its 642nd multiple.
29· " 7· to its three hundred and twelfth multiple.
30· " 6· to its ninety-fifth multiple. What number is 614· one half of?

EXERCISE XXXII.

EXAMPLES IN MULTIPLICATION OF CONCRETE NUMBERS, BY INTEGERS LESS THAN 13·

1· How many pence will pay for 248· ninepenny loaves?

As there will be one ninepence for each loaf, 248· *ninepences will be required to pay for* 248· *ninepenny loaves.*

∴ *the number of pence required* = 9· × 248· = 248· × 9·

Principle vi. page 42.

2· At 154· lb. per bag, how many lb. would 6· bags of potatoes weigh?

Write against each line its exact meaning, thus:

The number of lb. each bag weighs = 154·
The number of bags = 6·
The number of lb. six bags weigh = 924·

3· If 2468· persons contribute to a fund 5· shillings each, how many shillings will be contributed?
4· How many pence would 39· lb. of butter cost at 11· pence a pound?
5· A ton is equal to 2240· lb., what No. of lb. do 8· tons weigh?
6· What weight in lb. would nine carts contain, each holding 2· tons?
7· A yard measure is 3· feet long, how many feet in a cord 396· yards long?
8· A mile is 1760· yards, how many feet in a mile?
9· A foot is 12· inches long, how many inches in 5280· feet?
10· How many inches in a mile?
11· Four farthings are equal to one penny, how many farthings in 2468· pence?
12· Twelve pence make one shilling, how many pence in 439· shillings + 6395· shillings + 116· shillings?
13· How many farthings in one shilling?
14· How many farthings in five shillings, and in twenty shillings?
15· Twenty shillings make £1·, how many pence in £1·?
16· Three fourpences, or groats, make one shilling, how many groats in twenty shillings, or £1·?
17· Five shillings make one crown, how many shillings in 645· crowns?
18· Two sixpences make one shilling, how many sixpences in a crown, in £1·, in 3965· shillings?
19· Eight half-crowns are equal to £1·, how many half-crowns in £504·?
20· Five sixpences make one half-crown, how many sixpences in 784342· half-crowns?

21· Seven days make one week, how many days in a year, which is 52· weeks and 1· day?

22· 12· articles are 1· dozen, how many in 603· dozen?

23· What number of pence are 9· pence $\frac{1}{112}$ of?

24· A man can pay his year's rent by laying by 11· pence every week, what is his rent?

25· If three pints of beer cost 9· pence, how much would that quantity per week cost in seven years, or 365· weeks?

26· A cargo of coals was equally divided among 12· purchasers, each of whom had 87· tons. What was the weight of the cargo?

27· After spending two thirds of his money, Willie had 45· pence left. How many pence had he at first?

Having spent two thirds, the remainder was one third.

28· Two sixths of a piece of cloth were spoiled, and three sixths of it sold. There remained uninjured and unsold 62· yards. What was the original length of the piece?

29· After paying half his debts, a trader found he still owed £3965· What did he owe at first?

30· Nine gold diggers agreed to share equally whatever gold any of them found. The first discovery gave 14· oz. to each; the next gave 23· oz. to each; then a nugget was met with which made the share of each twice as much as he already had. What weight of gold did they find in all?

This example requires addition and multiplication.

31· A coach-wheel, 4· feet in circumference, revolved 13200· times in a certain distance. How many feet in that distance?

32· How many pence would a ton of sugar cost at 5· pence per lb.?

33· How many articles are there in one thousand six hundred and forty-nine dozen?

34· How many feet in 11· miles?

35· In 34216· shillings, how many groats?

36· How many inches in 8· miles?

37· How many farthings in 643821· shillings?

38· A steam engine worked 12· hours a day, and consumed 112· lb. of coals per hour. How many pounds of coals would it consume in five weeks?

39· 112· lb. make 1· cwt. How many lb. in 11· cwt.?

40· What number of pence would 9· cwt. of cheese cost at 7· pence per lb.?

41· A gallon of water usually weighs ten pounds. What would a hogshead of water, containing 63· gallons, weigh?

42· Four quarts make 1· gallon. How many quarts in 1· hogshead?

43· Two pints make one quart; and 4· gills make 1· pint. How many pints in a hogshead? How many gills in a hogshead?

44· At 6· pence per gill, what would a hogshead of brandy cost, in pence?

45· Edward had as many sixpences as there are days in July; Robert had 13· sixpences more than Edward; John had three times as many sixpences as Robert, and 7· sixpences beside. How many pence had the three boys?

MULTIPLICATION OF VULGAR FRACTIONS.

Multiplication is repeated addition of the same quantity. Therefore, multiplying a Vulgar Fraction by an Integer is the same thing as adding together as many such Fractions as there are units in that Integer.

$$\therefore\ \frac{5}{8} \times 6\cdot = \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$$

$$\therefore\ \frac{5}{8} \times 6\cdot = \frac{5+5+5+5+5+5}{8} = \frac{6 \text{ times } 5}{8}$$

$$\therefore\ \frac{5}{8} \times 6\cdot = \frac{5 \times 6}{8} = \frac{30}{8}$$

Therefore, a Vulgar Fraction is multiplied by an Integer by multiplying the numerator by that integer, and retaining the denominator unchanged.

A Decimal Fraction, being only a certain kind of Vulgar Fraction, (usually written without its denominator,) may be multiplied by an Integer in the same way as any other Vulgar Fraction.

The propriety of this will be made clearer by writing the denominator of the Decimal Fraction.

$$\text{For, } \cdot025 \times 7\cdot = \frac{25}{1000} \times 7\cdot = \frac{25 \times 7}{1000} = \frac{175}{1000}$$

$$\therefore\ \cdot025 \times 7\cdot = \cdot175^{*}$$

To Multiply any Fraction by an Integer.

First. Find the product of the numerator and the integer, employing (for convenience) the smaller of the two as multiplier.

Second. Under this product as a numerator, place the denominator of the given fraction.

EXERCISE XXXIII.

MULTIPLICATION OF FRACTIONS BY INTEGERS. EXAMPLES IN WHICH ONE OF THE FACTORS IS LESS THAN 13· ABSTRACT NUMBERS.

In each of the following examples the result will be a fraction. State the kind of fraction in each instance; whether proper or improper.

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* NOTE TO TEACHER. The pupil will probably perceive that decimal fractions are multiplied just as integers are, but for the present it is desirable that he should restore the denominators, and treat all as vulgar fractions.

1· Multiply $\frac{5}{8}$ by 9· and $\frac{7}{14}$ by 12·
2· " $\frac{27}{53}$ by 11· and $\frac{294}{1000}$ by 8·
3· " $\frac{11}{8648}$ by 643· and $\frac{6}{9436}$ by 8416·
4· " $\frac{7}{9}$ by 931562· and $\frac{4187}{6412}$ by 7·
5· Find the product of $\frac{9}{27}$ and 31468·
6· " " of $\frac{26148}{110000}$ and 9·
7· " " of $\frac{15}{25}$ and 39754·
8· " " of ·0453 and 7·
9· " " of ·00846 and 11·
10· " " of ·012385 and 6·
11· Find the 4th multiple of $\frac{7}{11}$
12· " 9th multiple of $\frac{614}{1000}$
13· " 246th multiple of $\frac{3}{8}$
14· " 314875th multiple of $\frac{11}{12}$
15· " 11th multiple of ·045
16· " 9th multiple of ·01438
17· " 7th multiple of ·74316
18· " 8th multiple of ·75
19· " 10th multiple of ·875
20· " 12th multiple of ·14328
21· Of what number is $\frac{7}{8}$ one fifteenth part?
The number must be the 15th multiple of $\frac{7}{8}$
22· $\frac{5}{12}$ is $\frac{1}{81483}$ of a certain number. What is that number?

CONCRETE NUMBERS.

23· How much would 45· yards of linen cost at $\frac{5}{12}$ of one shilling per yard?

Answer. 45 *yards of linen, at* $\frac{5}{12}$ *of a shilling per yard, would cost* 45· *times* $\frac{5}{12}$ *of a shilling; that is,*

$$\frac{5}{12} \text{ of a shilling} \times 45\cdot = \frac{5 \times 45}{12} \text{ of a shilling} = \frac{225}{12} \text{ of a shilling.}$$

24· At $\frac{3}{4}$ of a shilling per lb., what would a ton of candles cost?

25· If a soldier's coat require $\frac{11}{8}$ of a yard of cloth, how much cloth would be required to make coats for a battalion of 2514· men?

26· A cwt. is 112· lb. What would 1· cwt. of Dutch cheese cost, at $\frac{9}{2}$ of a penny per lb.?

To multiply Integers and Decimal Fractions by any Integer less than 13·

Place the multiplier under the multiplicand, so that the decimal points shall be in one vertical line.

Draw a horizontal line below the multiplier.

Beneath this line place a decimal point directly below the others.

Put also beneath this line a cipher* under each decimal figure of the multiplicand.

---

* NOTE TO TEACHER. This should be always enforced. See reason, Note to Addition of Decimals, page 33·

Multiply, set down, and carry, precisely as in Integers.

EXAMPLE WORKED OUT. Multiply 6841·382 by 9·

Example prepared.
```
6841·382
      9·
--------
    ·000
========
```

9· times 2 (one-thousandths) = 18 (one-thousandths = 1 (one-hundredth) and 8 (one-thousandths).

Set down 8 *(one-thousandths) and carry* 1 *(one-hundredth).*

9· times 8 (one-hundredths) + 1 (one-hundredth) carried = 73 (one-hundredths) = 7 (tenths) and 3 (one-hundredths).

Set down 3 *(one-hundredths) and carry* 7 *(tenths).*

9· times 3 (tenths) + 7 (tenths) carried = 34 (tenths) = 3 (units) and 4 (tenths).

Set down 4 *(tenths) and carry* 3 *(units).*

Example completed.
```
6841·382
      9·
--------
61572·438
=========
```

Multiply the integers as before.

61572·438 = *9th multiple of* 6481·382

EXERCISE XXXIV.

EXAMPLES IN MULTIPLICATION OF INTEGERS AND DECIMALS BY INTEGERS LESS THAN 13·; ABSTRACT NUMBERS.

1· What is the double of 641386·159?
2· Find the 4th multiple of 834169·364
3· Multiply 943654·18369 by 5·
4· „ 3416859·758 by 6·
5· „ 957384·3415 by 7·
6· „ 43168·4513 by 8·
7· Raise 943164·218 to its 9th multiple.
8· „ 632145·965 to its 10th multiple.
9· Find the product of 614·82 and 11·
10· „ „ of 9436·915 and 12·
11· What number is triple of 95436·421?
12· „ „ quadruple of 61458·6314?
13· What are 614386·2986 × 12· = ?
14· „ 319763·184 × 11· = ?
15· „ 954638·75 × 10· = ?
16· „ 47354·614 × 9· = ?
17· What No. is 4216·58 the tenth of?
18· What No. is 643·875 „ $\frac{1}{9}$ of?
19· One eighth of a certain No. is 6·84; what is the number?
20· Multiply ·00864 by 9·; and ·04136 by 11·
21· Multiply ·75 by 12·; and ·875 by 8·
22· Multiply ·375 by 6·; and 3·1416 by 7·
23· Multiply 1·7320508 by 12·
24· Multiply ·07958 by 5·; and ·164329 by 4·
25· Double ·61498; and treble ·842163

EXAMPLES IN CONCRETE NUMBERS.

26· A cubic foot of water usually weighs 62·5 lb. How many lb. in 12· cubic feet of water?*
27· A gallon of liquid would exactly fill a vessel measuring 277·2738 cubic inches. How many cubic inches would be occupied by 6· gallons?
28· If one fourth of a ship cost £413·875, what would the whole ship cost?
29· Seven eighths of a sum of money were stolen, and £41·375 left. How much was taken away?
30· A debt was paid in 11· equal instalments, of £1·075: what was the amount of the debt?
31· What would 12· weeks' wages, at £1·875 per week, amount to?
32· What is the value of 10· lb. of tea at 4·75 shillings per lb.?
33· Half a crown is = ·125 of £1· How many pounds are 11· half crowns equal to?

Express the answer in integers and decimals.

CONTINUED MULTIPLICATION.

If we multiply one number by another, and then multiply the product of these by a third number, it is said we have multiplied the three numbers continually together.

The product found by the continual multiplication of three or more numbers is called the **Continued Product** of those numbers.

Thus: the continued product of
6· 9· 8· and 4· is = 6· × 9· × 8· × 4· = 1728·
For 6· times 9· = 54· = *product of* 6· *and* 9·
" 8· times 54· = 432· = *continued product of* 6· *and* 9· *and* 8·
" 4· times 432· = 1728· = *continued product of* 6· *and* 9· *and* 8· *and* 4·

EXERCISE XXXV.

1· Find the continued product of 3· 7· 6· and 5·
2· " " " 9· 9· 8· 7· and 3·
3· Multiply continually together 64·38; 7· 5· and 12·
4· " " " 69349·42; 8· 4· and 6·
5· Find the continued product of 7· and 6· and 5·
6· " " " 7· and 5· and 6·
7· " " " 6· and 7· and 5·
8· " " " 6· and 5· and 7·
9· " " " 5· and 6· and 7·
10· " " " 5· and 7· and 6·

---

* Note to Teacher. This opportunity may be taken to explain to the pupil what a cubic unit is. The proper way to do this is to place before him, for *examination by eye and hand*, a number of cubes, varying in bulk from a cubic inch to a cubic foot. Every school should be furnished with a series of such cubes.

In the last six examples the three numbers multiplied are the same, but differently placed. Yet it is found that the product is the same in all six.

In the same way any four numbers, as 5· 11· 6· 8· may be multiplied together in twenty-four different arrangements, but the product will be always the same; namely (in this instance) 2640·

11· Prove that in whatever order 12· and 3· and 9· and 7· are continually multiplied, the continued product is always the same.

Principle VII. *In whatever order numbers are multiplied together, their continued product is always the same.*

This is an important extension of Principle vi.

Powers of Numbers. The continued products of equal factors are called Powers of each of those factors.

For instance:

8· = 1st power of 8·
8· × 8· = 64· = 2nd power of 8·
8· × 8· × 8· = 512· = 3rd power of 8·
8· × 8· × 8· × 8· = 4096· = 4th power of 8·
8· × 8· × 8· × 8· × 8· = 32768· = 5th power of 8·
&c. &c. &c.

∴ The 1st power of any No. is the No. itself.
The 2nd power of any No. is the product of two such Nos.
The 3rd power of any No. is the continued product of three such Nos.
&c. &c. &c.
The 9th power of any No. is the continued product of nine such Nos.
&c. &c. &c.

Finding the powers of numbers is called Involution.

Involution is the raising or involving of a number to any of its powers.

EXERCISE XXXVI.

EXAMPLES IN INVOLUTION OF INTEGERS.

1· Raise 9· to its 4th power.
2· What is the 6th power of 4·?
3· Involve 6· to its 8th power?
4· Find the 3rd power of 5·
5· „ „ 8th power of 8·
6· „ „ 7th power of 2·
7· Involve 11· to its 7th power.
8· Raise 9· to its 5th power.
9· „ 4· to its 12th power.
10· „ 5· to its 9th power.
11· „ 10· to its 5th power.
12· „ 12· to its 8th power.

Index or Exponent. The number of equal factors which are to be continually multiplied together, in order to obtain the required power of a number, is sometimes pointed out by a small figure called the Index or Exponent of the power. The Index or Exponent is placed at the upper part of the right-hand side of the number which is to be raised to the power indicated by that Index or Exponent. The plural of "Index" is "Indices."

Hence, in $12\cdot^{5}$ the Index or Exponent is 5, shewing that 12· is to be raised to its 5th power.

$9\cdot^{8}$ means "raise 9· to its 8th power."

The Index or Exponent of the power shews how many of the equal factors are to be multiplied together.

To find $6\cdot^{4}$ we must multiply four sixes continually together.

To find $12\cdot^{9}$ we must multiply nine twelves continually together.

The second power of a number is sometimes called the *square* of that number.

The third power of a number is sometimes called the *cube* of that number.

EXERCISE XXXVI. *continued.*

13· Find $9\cdot^{2}$ $9\cdot^{3}$ $9\cdot^{4}$ also $10\cdot^{2}$ $10\cdot^{3}$ $10\cdot^{4}$ $10\cdot^{5}$ $10\cdot^{6}$ $10\cdot^{7}$ $10\cdot^{8}$ $10\cdot^{9}$ $10\cdot^{10}$
14· Find the first six powers of 12· Find the square of 5·
15· What is the fourth multiple of the square of 9·?
Answer. The 4th multiple of $9\cdot^{2} = 9\cdot \times 9\cdot \times 4\cdot = 324\cdot$
16· What is the 7th multiple of the 4th power of 3·?
Answer. The 7th multiple of $3\cdot^{4} = 3\cdot \times 3\cdot \times 3\cdot \times 3\cdot \times 7\cdot = 567\cdot$
17· What is the 8th multiple of the 7th power of 6·?
18· Find $6\cdot^{3}$ $8\cdot^{5}$ $11\cdot^{9}$ $5\cdot^{11}$ $10\cdot^{12}$ $6\cdot^{17}$ and $12\cdot^{14}$
19· Find the 4th power of the 4th multiple of 3·
20· What is the 9th power of the sum of 4· and 7·?

TABLE OF THE FIRST NINE POWERS OF 10·

$10\cdot^{1} =$ 10·
$10\cdot^{2} =$ 100·
$10\cdot^{3} =$ 1000·
$10\cdot^{4} =$ 10,000·
$10\cdot^{5} =$ 100,000·
$10\cdot^{6} =$ 1;000,000·
$10\cdot^{7} =$ 10;000,000·
$10\cdot^{8} =$ 100;000,000·
$10\cdot^{9} =$ 1,000;000,000·

In the above table it will be remarked that the number of ciphers in any power of 10· is equal to the index of that power.

The 1st power of 10· (or $10\cdot^{1}$) is 10·
where Index = 1; No. of ciphers = 1
The 2nd power of 10· (or $10\cdot^{2}$) is 100·
where Exponent = 2; No. of ciphers = 2
The 3rd power of 10· (or $10\cdot^{3}$) is 1000·
where Exponent = 3; No. of ciphers = 3

∴ *Any power of* 10· *is at once expressed by making up the number of ciphers between the significant figure and the decimal point equal to the Index or Exponent of the power required.*

MULTIPLICATION BY ANY POWER OF 10·

PRINCIPLE IV. (page 26·) informs us that, "In the Decimal System of Arithmetical Notation, each digit has one tenth of the value it would have in the next place to the left."

Therefore, to increase the value of any digit ten times, we have only to remove that digit one place to the left.

And if we thus increase ten times the value of every one of a line of digits, we shall have increased ten times the number expressed by that line of digits. That is, we shall have multiplied the number by 10·

∴ 614385·7029 × 10· = 6143857·029

∴ The tenth multiple of a number is found by removing every digit *one place to the left.*

This is, of course, the same in effect as removing the Decimal Point *one place to the right.*

Similarly, we find the 100th multiple of a No. by removing each of its digits *two places to the left.*

Similarly, we find the 1000th multiple of a No. by removing each of its digits *three places to the left.*

&c. &c. &c.

In other words,

Multiplying	by $10^{\cdot 1}$	removes every	digit	*one*	place to the left.
„	by $10^{\cdot 2}$ or 100·	„	„	*two*	places to the left.
„	by $10^{\cdot 3}$ or 1000·	„	„	*three*	places to the left.
„	by $10^{\cdot 4}$ or 10000·	„	„	*four*	places to the left.
	&c.	&c.		&c.	

Principle VIII. *Multiplying by any power of* 10· *removes every digit of the Multiplicand to the left as many places as there are ciphers in the Multiplier.*

When the Multiplicand consists of Integers only, it is multiplied by 10· by inserting *one* cipher between its right-hand figure and the Decimal Point. It is multiplied by 100· by inserting *two* ciphers, and so on.

∴ 6184· × 10· = 61840·	6184· × 10000· = 61840000·
6184· × 100· = 618400·	6184· × 100000· = 618400000·
6184· × 1000· = 6184000·	6184· × 1000000· = 6184000000·

and so on; always inserting between the right-hand figure of the Multiplicand and the Decimal Point *as many ciphers as there are in the Multiplier.*

But, if the Multiplicand contain Decimals, it is multiplied by any power of 10· *by removing the Decimal Point as many places to the right as there are ciphers in the Multiplier.* For, removing the Decimal Point to the *right*, of course, removes every digit to the *left*, because places of digits are counted from the Decimal Point.

∴ 6184·3219 × 10· = 61843·219
6184·3219 × 100· = 618432·19
6184·3219 × 1000· = 6184321·9
6184·3219 × 10000· = 61843219·
6184·3219 × 100000· = 618432190·
6184·3219 × 1000000· = 6184321900·

MULTIPLICATION OF ANY NUMBER BY A FRACTION.

Multiplication of any number is finding the *sum* or amount of as many such numbers as there are units in the multiplier. Therefore, when the multiplier is only a Fraction of a unit, we have to find *that Fraction of the Multiplicand.*

∴ 8· × 4· means, "Find what four eights are equal to."
and 8· × $\frac{1}{4}$ means, "Find what one fourth of eight is equal to."

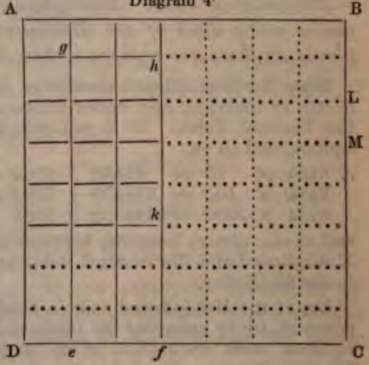
∴ 8· × $\frac{1}{4}$ = $\frac{1}{4}$ of 8· and, by PRINCIPLE VI. (page 42·)
8· × $\frac{1}{4}$ = $\frac{1}{4}$ × 8· ∴ $\frac{1}{4}$ × 8· = $\frac{1}{4}$ of 8·
In the same way $\frac{1}{2}$ × $\frac{1}{2}$ = $\frac{1}{2}$ of $\frac{1}{2}$
$\frac{4}{5}$ × $\frac{3}{7}$ = $\frac{4}{5}$ of $\frac{3}{7}$ = $\frac{3}{7}$ × $\frac{4}{5}$ = $\frac{3}{7}$ of $\frac{4}{5}$

PRINCIPLE IX. *The word "of" between numbers has the same meaning as the sign of multiplication.**

Therefore, to find the product of $\frac{5}{8}$ and $\frac{3}{7}$ we must find the Simple Fraction which is equal to the Compound Fraction $\frac{5}{8}$ of $\frac{3}{7}$ (See pages 15· & 16·)

Take a unit-square, as ABCD (Diagram 4·), and divide it by six vertical lines into seven sevenths. Divide each of those sevenths into eight equal parts by seven horizontal lines drawn quite across the unit.

Diagram 4·



As there are, therefore, in the whole unit $\frac{7}{7}$, like A*e*, each containing eight pieces like A*g*, there are fifty-six of these last pieces in the unit.

∴ A*g* = $\frac{1}{8}$ of $\frac{1}{7}$ = $\frac{1}{56}$

We here remark that *the denominator of the simple fraction is the product of the two denominators of the compound fraction.* And this must always be so, for whatever number of parts you cut each of the first-formed fractions into, *so many times* will the number of pieces be multiplied.

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* NOTE TO TEACHER. This is a very important principle, not only explaining multiplication of fractions, but assisting to elucidate the fundamental truth of the "*Rule of Three*"; viz., that "When four numbers are proportionals, the product of the extremes is equal to the product of the means."

Now, Af is $\frac{3}{7}$ of the unit, and contains eight pieces of the size of Ah.

$\therefore$ Ah is $\frac{1}{8}$ of $\frac{3}{7}$, which, being 3· times as much as $\frac{1}{8}$ of $\frac{1}{7}$, should = the triple of Ag, which is $\frac{1}{8}$ of $\frac{1}{7}$ or $\frac{1}{56}$ $\therefore$ Ah should = $\frac{3}{56}$

The Diagram shews this to be true, for Ah contains three pieces like Ag. So that Ah is = $\frac{3}{56}$ $\therefore$ $\frac{1}{8}$ of $\frac{3}{7} = \frac{3}{56}$

Lastly, Ak is $\frac{5}{8}$ of $\frac{3}{7}$, for it contains five pieces of the size of Ah,

$$\therefore \quad \text{A}k = \text{five times } \frac{3}{56} = \frac{3}{56} \times 5\cdot = \frac{3 \times 5}{56} = \frac{15}{56}$$

And this result is proved by the Diagram, where Ag is $\frac{1}{56}$ of the unit, and Ak contains fifteen pieces like Ag.

$$\therefore \quad \frac{5}{8} \text{ of } \frac{3}{7} = \frac{5 \times 3}{8 \times 7} = \frac{15}{56}$$

Now, by Principle IX. (page 56·) $\frac{5}{8} \times \frac{3}{7} = \frac{5}{8}$ of $\frac{3}{7}$; and
$\frac{3}{7} \times \frac{5}{8} = \frac{3}{7}$ of $\frac{5}{8}$

And, by Principle VI. (page 42·) $\frac{5}{8} \times \frac{3}{7} = \frac{3}{7} \times \frac{5}{8}$

$$\therefore \frac{5}{8} \times \frac{3}{7}, \text{ or } \frac{5}{8} \text{ of } \frac{3}{7}, \text{ or } \frac{3}{7} \times \frac{5}{8}, \text{ or } \frac{3}{7} \text{ of } \frac{5}{8} = \frac{15}{56} = \frac{5 \times 3}{7 \times 8}$$

$\therefore$ $\frac{15}{56}$ is the simple fraction = either of the compound fractions $\frac{5}{8}$ of $\frac{3}{7}$, or $\frac{3}{7}$ of $\frac{5}{8}$. And $\frac{15}{56}$ is also the product of the fractions $\frac{5}{8}$ and $\frac{3}{7}$.

By dividing each of the 56ths of Diagram 4· into *three* equal parts, we might shew the truth of the following statements; viz.,

[*]that $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{1}{7} = \frac{2 \times 7 \times 1}{3 \times 8 \times 7} = \frac{14}{168} = \frac{2}{3} \times \frac{7}{8} \times \frac{1}{7}$

that $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{3}{7} = \frac{3}{168} = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{4} \times \frac{3}{7}$

and many others.

The word "Reduce" in arithmetic implies the changing of the form of a quantity without altering its value.

Thus, if I write £1· as 240· pence, I have not changed the value of the sum of money spoken of, I have merely reduced £1· to pence.

Again, if instead of the compound fraction $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{2}{9}$, I write the simple fraction which is of the same value: namely, $\frac{24}{315}$, I have performed the reduction of the compound fraction to its *equivalent* simple fraction.

To reduce a compound fraction to its equivalent simple fraction.

First. Find the continued product of all the numerators. This product is the numerator of the simple fraction.

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[*] Note to Teacher. It should be pointed out to the Pupil that the last-named Fraction is to be *first* found in the Diagram. First $\frac{1}{7}$, then $\frac{7}{8}$ of $_{7}$; then $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{1}{7}$.

Second. Find the continued product of all the denominators. This product is the denominator of the simple fraction.

EXERCISE XXXVII.

REDUCTION OF COMPOUND FRACTIONS TO THEIR EQUIVALENT SIMPLE FRACTIONS. EXAMPLES IN ABSTRACT NUMBERS.

1· What simple fraction is equal to $\frac{3}{8}$ of $\frac{6}{17}$?
2· Find the simple fraction equivalent to $\frac{3}{14}$ of $\frac{7}{11}$ of $\frac{9}{8}$.
3· Reduce $\frac{6}{7}$ of $\frac{3}{12}$ of $\frac{11}{9}$ of $\frac{5}{8}$ to its equivalent simple fraction.
4· Express as a simple fraction $\frac{1}{8}$ of $\frac{3}{14}$ of $\frac{9}{11}$ of $\frac{5}{7}$ of $\frac{29}{12}$.
5· Shew from Diagram 4· as already divided,
that $\frac{1}{3}$ of $\frac{3}{8}$ of $\frac{1}{7} = \frac{1}{56}$; and that $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{7} = \frac{2}{56}$
6· Prove also, from Diagram 4· as already divided
that $\frac{5}{8}$ of $\frac{2}{7} = \frac{10}{56}$; that $\frac{4}{5}$ of $\frac{3}{8}$ of $\frac{5}{7} = \frac{12}{56}$; and
that $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{2}{7} = \frac{2}{56}$.
7· Shew also that $\frac{5}{12}$ of $\frac{3}{8}$ of $\frac{4}{7} = \frac{5}{56}$; and that $\frac{7}{20}$ of $\frac{5}{7}$ of $\frac{4}{8} = \frac{7}{56}$.

Integers expressed as Fractions. Since the denominator of a fraction expresses the number of pieces into which the unit is cut, and integers express units undivided; therefore any integer is the numerator of a fraction whose denominator is digit 1

Therefore 6· may be written $\frac{6}{1}$;

$$19\cdot = \frac{19}{1}; \quad 235\cdot = \frac{235}{1}; \quad \&c.$$

EXERCISE XXXVII. *continued.*

8· What simple fraction is equivalent to $\frac{3}{4}$ of $\frac{12}{13}$ of 19·?
9· Reduce to its equivalent simple fraction $\frac{3}{11}$ of $\frac{7}{16}$ of $\frac{1}{9}$ of 64·
10· Express as simple fractions $\frac{7}{15}$ of $\frac{14}{3}$ of $\frac{11}{12}$ of 22· and
$\frac{8}{19}$ of $\frac{12}{11}$ of 45·
11· What are $\frac{4}{10}$ of $\frac{2}{10}$; $\frac{4}{10}$ of $\frac{2}{100}$; $\frac{4}{10}$ of $\frac{2}{1000}$; $\frac{4}{10}$ of $\frac{2}{10000}$ = ?
12· What are $\frac{3}{100}$ of $\frac{2}{100}$; $\frac{3}{100}$ of $\frac{2}{1000}$; $\frac{3}{100}$ of $\frac{2}{10000}$; $\frac{3}{100}$ of $\frac{2}{100000}$ = ?
13· What are $\frac{3}{1000}$ of $\frac{2}{1000}$; $\frac{3}{1000}$ of $\frac{2}{10000}$; $\frac{3}{1000}$ of $\frac{2}{100000}$; $\frac{3}{1000}$ of $\frac{2}{1000000}$ = ?

To Multiply Integers or Vulgar Fractions by Fractions.

First. If any of the factors be integers, put them into the fractional form, by putting under each the denominator 1

Second. Find the continued product of all the numerators. This is the numerator of the product-fraction.

Third. Find the continued product of all the denominators. This is the denominator of the product-fraction.

EXERCISE XXXVIII.

EXAMPLES IN MULTIPLICATION OF INTEGERS AND FRACTIONS BY FRACTIONS. ABSTRACT NUMBERS.

1· Find the product of $\frac{5}{8}$ & $\frac{3}{9}$, also of $\frac{4}{5}$ & $\frac{3}{17}$.
2· „ „ of $\frac{3}{7}$, $\frac{6}{8}$, $\frac{4}{11}$ & $\frac{5}{14}$; also of $\frac{3}{9}$, $\frac{1}{18}$, & $\frac{5}{4}$.
3· What are $\frac{3}{11} \times \frac{6}{14} \times \frac{12}{9} \times 8\cdot = ?$
4· „ $\frac{7}{15} \times \frac{4}{11} \times \frac{3}{8} \times \frac{10}{5} \times 12\cdot \times \frac{2}{10} = ?$
5· „ $\frac{1}{16} \times \frac{5}{11} \times 3\cdot \times \frac{6}{7} \times \frac{14}{9} = ?$
6· „ $\frac{5}{14} \times \frac{11}{9} \times 16\cdot \times \frac{4}{7} \times 3\cdot = ?$
7· What are $5\cdot \times \frac{4}{10}$; $5\cdot \times \frac{4}{100}$; $5\cdot \times \frac{4}{1000} = ?$
8· „ $50\cdot \times \frac{4}{10}$; $50\cdot \times \frac{4}{100}$; $50. \times \frac{4}{1000} = ?$
9· „ $500\cdot \times \frac{4}{10}$; $500\cdot \times \frac{4}{100}$; $500\cdot \times \frac{4}{1000} = ?$
10· „ $5000\cdot \times \frac{4}{10}$; $5000\cdot \times \frac{4}{100}$; $5000\cdot \times \frac{4}{1000} = ?$
11· „ $\frac{5}{10} \times \frac{4}{10}$; $\frac{5}{10} \times \frac{4}{100}$; $\frac{5}{10} \times \frac{4}{1000}$; $\frac{5}{10} \times \frac{4}{10000} = ?$
12· „ $\frac{5}{100} \times \frac{4}{10}$; $\frac{5}{100} \times \frac{4}{100}$; $\frac{5}{100} \times \frac{4}{1000}$; $\frac{5}{100} \times \frac{4}{10000} = ?$

Because, multiplying any number by a fraction is the same thing as taking *that* fraction of the multiplicand; and because, by PRINCIPLE IV. (page 26·) "*Every digit has one-tenth of the value it would have in the next place to the left, and ten-fold the value it would have in the next place to the right;*" therefore, to multiply a number by $\frac{1}{10}$, that is, by ·1, we have only to remove every one of its digits one place to the right.

∴ 6184·3219 × ·1 = 618·43219 = $\frac{1}{10}$ of 6184·3219

Now, multiplying this product by ·1 would be finding $\frac{1}{10}$ of it. But this would be $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$, of the original Multiplicand 6184·3219 And ∵ 618·43219 × ·1 = 61·843219

∴ also, 6184·3219 × ·01 = 61·848219
(Each digit being removed *two* places to the right.)
Similarly, 6184·3219 × ·001 = 6·1843219
(Each digit being removed *three* places to the right.)
6184·3219 × ·0001 = ·61843219
(Each digit being removed *four* places to the right.)
6184·3219 × ·00001 = ·061843219
(Each digit being removed *five* places to the right.)
&c., &c., &c.,

Hence, multiplying by digit 1 in any place of Decimals, merely removes every digit of the Multiplicand to the right as many places as the multiplying digit is to the right of the Decimal Point. And, consequently, the number of

Decimal Figures in the Product is equal to the *sum* of the numbers of Decimal Figures in the Multiplicand and in the Multiplier.

Combining these examples with those given under PRINCIPLE VIII. (page 55·) we find that

6184·3219 × 100000· = 618432190·
6184·3219 × 10000· = 61843219·
6184·3219 × 1000· = 6184321·9
6184·3219 × 100· = 618432·19
6184·3219 × 10· = 61843·219
6184·3219 × 1· = 6184·3219
6184·3219 × ·1 = 618·43219
6184·3219 × ·01 = 61·843219
6184·3219 × ·001 = 6·1843219
6184·3219 × ·0001 = ·61843219
6184·3219 × ·00001 = ·061843219

And so on, extending the table either upwards or downwards.

PRINCIPLE X. *Multiplying by digit* 1, *standing in any but the units' place, merely removes every digit of the multiplicand to the right or left as many places as the multiplying digit is from the units' place.*

EXERCISE XXXIX.

EXAMPLES IN MULTIPLICATION OF INTEGERS AND DECIMALS, BY 1 IN ANY PLACE OF INTEGERS OR DECIMALS.

ABSTRACT NUMBERS.

1· Multiply 84·316 by ·001, by ·01; by ·1, by 10· and by 100·
2· „ 613·54 by 100· and ·01 by 1000·
3· „ ·004 by ·001 and ·01 by ·01
4· „ 1· by ·01
5· What are ·01 × ·001 × 48· = ?
6· Find the continued product of 64·8 and 100· and ·01
7· What is the 10th multiple of ·01 and of ·1 ?
8· Multiply ·0001 by 10000· and 6432· by ·001
9· „ 64· by ·000001
10· „ 3·1416 by 10000·
11· Find $\frac{1}{100}$ of 914321·642
12· Raise $\frac{1}{10}$ to its fifth power, and $\frac{1}{10000}$ to its fourth power.
13· What is the 100th Multiple of ·00675 ?
14· What is the 100th Multiple of ·06421385 ?
15· Raise ·01 to its 8th power.
16· Find ·001^{4}; ·00001^{2}; and ·0001^{3}
17· Multiply ·1 by ·1 and ·6 by ·1 also ·09 by ·1
18· Multiply 3416·59 by 10000· by 1000· by 100· by 10· by 1· by $\frac{1}{10}$ by $\frac{1}{100}$ by $\frac{1}{1000}$ by $\frac{1}{10000}$ and by $\frac{1}{100000}$ and arrange the ten products as in the above table.
19· Find the continued product of 25· 1000· 348·5 and $\frac{1}{100}$
20· Find the continued product of 64· 81· and ·001

EXERCISE XL.

EXAMPLES IN REDUCTION OF COMPOUND FRACTIONS, AND IN MULTIPLICATION OF INTEGERS AND FRACTIONS BY FRACTIONS. CONCRETE OR APPLICATE NUMBERS. RECAPITULATORY QUESTIONS.

1· What part of a shilling is $\frac{1}{4}$ of $\frac{1}{2}$ of a shilling?

2· A man bought $\frac{3}{5}$ of a cheese, and found $\frac{1}{4}$ of the piece decayed; what part of the whole cheese was that?

3· A shilling is $\frac{1}{20}$ of 1·£, a groat is $\frac{1}{3}$ of a shilling, and 3· farthings are $\frac{3}{16}$ of a groat. What part of a shilling are three farthings, and what part of 1·£?

Three farthings are $\frac{3}{16}$ *of* $\frac{1}{3}$ *of a shilling* = *of a shilling?*
and " " $\frac{3}{16}$ of $\frac{1}{3}$ of $\frac{1}{20}$ of 1·£ = of 1·£.?

4· At $\frac{3}{8}$ of a shilling per yard, what would 27· yards of calico cost?

The cost of one article × *any No., shews the cost of that No. of articles.*

$$\therefore \textit{Cost of } 27\cdot \textit{ yds.} = \frac{3}{8}s. \times 27\cdot = \frac{3 \times 27}{8} = \frac{81}{8} \textit{ of a shilling.}$$

5· At the same rate, what would $\frac{3}{4}$ of a yard cost?

6· At $\frac{5}{12}$ of a £. per yard, what would $\frac{7}{8}$ of a yard of cloth cost?

7· What part of a crown is $\frac{3}{8}$ of sixpence?

Write answers to the following questions.

8· What is multiplication?

9· Explain the words "Multiplicand, Multiplier, Product, Factor," and "Multiple."

10· Shew, by arranging 63· units, that 9· times 7· = 7· times 9·

11· What principle does this fact illustrate?

12· Describe the sign of multiplication. Make it.

13· How is a multiple found? What is the quadruple of any number?

14· "Four times eight are thirty-two." Express this arithmetically.

15· Thirty-two here has *three* names. What are they? (Page 43·)

16· What numbers are called composite?

17· What are those numbers called which are *not* composite?

18· Which of the following numbers are composite: 29· 84· 32· 64· 39· 81· 61· 22· 35· 92· 108· 113· 144·?

19· What is the multiplication table?

20· How would you find the double of any number?

21· In multiplying one number by another, which of them would you employ as multiplier, and why?

22· How do you multiply a fraction by an integer?

23· What multiple of 6· is 6· times 5·?

24· What is meant by continued multiplication?

25· What is the continued product of equal factors called?

26· How do you find the ninth power of 6·?

27· What is the Index or Exponent? Where is it placed? What does it point out?

28· Express arithmetically that the fifth power of two is equal to thirty-two.

29· What is the number of ciphers in any power of ten equal to?

30· How may any power of ten be at once expressed?
31· By what easy method can you find the tenth multiple of a number?
32· What other multiples can be found in a similar way?
33· When you multiply by any power of 10· how does the product differ from the multiplicand?
34· What does multiplying by a fraction mean?
35· What word between numbers has the same meaning as × ?
36· What does the word "Reduce" mean in arithmetic?
37· How is a compound fraction reduced to a simple fraction?
38· By what means may the truth of this reduction be proved?
39· How is an integer put into the form of a fraction?
40· How is one fraction multiplied by another?
41· If any number be multiplied by digit 1 in any place of decimals, how will the product differ from the multiplicand?
42· What general principle does this fact form part of?

EXTENDED SIGNIFICATION OF THE WORD "UNIT."

SIMPLE AND COMPLEX UNITS.

On the first page of this book, it is said that "a unit is any single undivided thing." This is the true meaning of the word. But it is frequently convenient to consider a *collection* of things as *one*, and to call such lot or collection (for the occasion) a unit.

To distinguish, then, between these two kinds of units, we will call the true unit, (that is, "any single undivided thing,") a **Simple Unit.**

Any collection of units, (when such collection is to be regarded as one,) we will call a **Complex Unit.**

Thus, *a pair, a couple, or a brace*, is a complex unit, consisting of two simple units. A *dozen* is a complex unit, containing twelve simple units. A *gross* is a larger complex unit, comprising twelve dozens, or one hundred and forty-four simple units.

A hundred soldiers constitute a *Company*. Ten Companies, or a thousand soldiers, make up a *Regiment*. A number of Regiments of the same kind form a *Brigade*. Two or more Brigades make a *Division*. And, sometimes, there are several Divisions to make up an *Army*.

Hence, Armies, Divisions, Brigades, Regiments and Companies, (when spoken of as single things,) are complex units, made up of the simple units, individual soldiers.

The practice of treating collections of things as complex units is very convenient; for it enables us to express large quantities in fewer words or figures than if the whole number of simple units were always to be named. If it were not agreed, for example, to call every collection of 1760· yards one mile, we should be obliged to describe any distance, however great, in yards, which would be an extremely unwieldy and inconvenient method. For instance, a distance which is now shortly expressed as 22· miles, could then only be expressed as 38720· yards.

MULTIPLICATION BY A COMPOSITE MULTIPLIER.

Explanations of arithmetical processes are also frequently made more simple and clear by considering collections of numbers as complex units. For example, considering *seven* as a complex unit, it becomes evident that

∵ 6· + 6· = 12· ∴ 6· *sevens* + 6· *sevens* = 12· *sevens*,
which is the same thing as saying that
6· *sevens* × 2· or *twice six sevens* = 12· *sevens*,
or 7· × 12· = 7· × 6· × 2· or, (by Prin. vii. p. 53·) = 7· × 2· × 6·

Again, ∵ 3· + 3· + 3· + 3· = 12·
∴ 3· *sevens* + 3· *sevens* + 3· *sevens* + 3· *sevens* = 12· *sevens*;
that is, 4· times 3· *sevens* = 12· *sevens*,
or, 7· × 12· = 7· × 4· × 3· or, 7· × 3· × 4·

Corresponding Factors. When a composite number (see page 43·) has more than one pair or set of factors, those of the same pair or set are called corresponding factors of the composite number.

12· has two pairs of factors, one pair being 4· and 3·; and the other pair 6· and 2·
∴ 4· is the factor of 12· corresponding to 3· and 3· to 4·
6· is the factor of 12· corresponding to 2· and 2· to 6·

36· has three pairs of factors: first, 6· and 6·; second, 9· and 4·; third, 12· and 3·
∴ 6· is the factor of 36· corresponding to 6·
9· is the factor of 36· corresponding to 4· and 4· to 9·
12· is the factor of 36· corresponding to 3· and 3· to 12·

Now, we have seen above, that
7· × 12· = 7· × 6· × 2· or = 7· × 4· × 3·

That is, *the product of* 7· *and* 12· *is equal to the continued product of* 7· *and either pair of corresponding factors of* 12·

This fact may be proved very clearly by adding 12· *sevens* in a particular arrangement.

First. To shew, by addition, that
7· × 12· = 7· × 6· × 2· or = 7· × 2· × 6·

7· + 7· + 7· + 7· + 7· + 7· = 42·
7· + 7· + 7· + 7· + 7· + 7· = 42·
14· + 14· + 14· + 14· + 14· + 14· = 84·

Here it is seen that the sum of 6· *sevens* = 42·
And 2· *forty-twos* = 84· = 12· times 7·

It is also seen that the sum of 2· *sevens* = 14·
And 6· *fourteens* = 84· = 12· times 7·
∴ 7· × 12· = 7· × 6· × 2· or = 7· × 2· × 6·

Second. To shew, by addition, that

$$7{\cdot} \times 12{\cdot} = 7{\cdot} \times 4{\cdot} \times 3{\cdot} \quad \text{or} = 7{\cdot} \times 3{\cdot} \times 4{\cdot}$$

$7{\cdot} + 7{\cdot} + 7{\cdot} + 7{\cdot} = 28{\cdot}$	Here we see that 4· *sevens* = 28·
$7{\cdot} + 7{\cdot} + 7{\cdot} + 7{\cdot} = 28{\cdot}$	And 3· *twenty-eights* = 84· = 12· times 7·
$7{\cdot} + 7{\cdot} + 7{\cdot} + 7{\cdot} = 28{\cdot}$	We also see that 3· *sevens* = 21·
$21{\cdot} + 21{\cdot} + 21{\cdot} + 21{\cdot} = 84{\cdot}$	And 4· *twenty-ones* = 84· = 12· times 7·

$$\therefore \; 7{\cdot} \times 12{\cdot} = 7{\cdot} \times 4{\cdot} \times 3{\cdot} \quad \text{or} = 7{\cdot} \times 3{\cdot} \times 4{\cdot}$$

If we take any other Multiplicand instead of 7· and any other Composite Multiplier instead of 12· we shall come to a like result. For instance, we may prove, as above, that

$$\begin{aligned} 19{\cdot} \times 24{\cdot} &= 19{\cdot} \times 12{\cdot} \times 2{\cdot} \text{ or } = 19{\cdot} \times 2{\cdot} \times 12{\cdot} \text{ or} \\ &= 19{\cdot} \times 8{\cdot} \times 3{\cdot} \text{ or } = 19{\cdot} \times 3{\cdot} \times 8{\cdot} \text{ or} \\ &= 19{\cdot} \times 6{\cdot} \times 4{\cdot} \text{ or } = 19{\cdot} \times 4{\cdot} \times 6{\cdot} \end{aligned}$$

PRINCIPLE XI. *The product of any number and a composite multiplier is equal to the continued product of that number, and any set of corresponding factors of the composite multiplier.*

Compare this with Principles vi. and vii.

To multiply any number by a composite multiplier, having only two factors, each of which is less than 13·[*]

First. Multiply the multiplicand by any factor of the multiplier.

Second. Multiply the product by the corresponding factor of the multiplier.

EXAMPLE WORKED OUT. Multiply 3142·586 by 24·

24· has three pairs of factors less than 13· Therefore, this example may be worked in six ways.

First.	Second.
3142·586	3142·586
6·	4·
18855·516 = 6· times 3142·586	12570·344 = 4· times 3142·586
4·	6·
75422·064 = 24· times 3142·586	75422·064 = 24· times 3142·586
Third.	Fourth.
3142·586	3142·586
8·	3·
25140·688 = 8· times 3142·586	9427·758 = 3· times 3142·586
3·	8·
75422·064 = 24· times 3142·586	75422·064 = 24· times 3142·586

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[*] NOTE TO TEACHER. The reason of this limitation is, that the pupil can, as yet, only decide whether a Number is or is not Composite by seeking it among

Fifth.	Sixth.
3142·586	3142·586
12·	2·
37711·032 = 12· times 3142·586	6285·172 = 2· times 3142·586
2·	12·
75422·064 = 24· times 3142·586	75422·064 = 24· times 3142·586

EXERCISE XLI.

EXAMPLES IN MULTIPLICATION OF INTEGERS AND DECIMAL FRACTIONS BY COMPOSITE MULTIPLIERS, HAVING TWO FACTORS, EACH BEING AN INTEGER LESS THAN 13·

Each line of the work to be described as in foregoing example, and each example to be worked in as many ways as possible.

1· Multiply 3312·4254 by 25·
2· " 64304·713 by 28·
3· " 173954·35 by 27·
4· " 64321·98761 by 21·
5· " 983165·2183 by 32·
6· Multy. 5436431·952 by 45.
7· " 704062·5 by 88·
8· " 613434·54719 by 96·
9· " 4571361·7364 by 144·
10· " 3146·425 by 132·
11· Find the 45th multiple of 43421·3243
12· What is the product of 614386·15 and 54·?
13· Raise 24· to its fifth power.
14· Find $36{\cdot}^{2}$ $42{\cdot}^{3}$ $81{\cdot}^{4}$ $96{\cdot}^{5}$ $24{\cdot}^{4}$
15· What is the continued product of 16· 48· 35· 144· and 72·?
16· What are 3425·16 × 88· × 77· × 35· = ?
17· " 14396·218 × 63· × 18· × 21· = ?
18· " 7439·154 × 81· × 108· × 32· = ?
19· " 349618·23 × 33· × 55· × 14· = ?

MULTIPLICATION BY A SINGLE DIGIT IN ANY PLACE.

Suppose we be required to multiply 64832· by 20· Now, 20· is a Composite Number, having the Corresponding Factors 2· and 10·

$$\therefore\ 64832{\cdot} \times 20{\cdot} = 64832{\cdot} \times 2{\cdot} \times 10{\cdot}$$

PRINCIPLE XI. page 64·

But, "Multiplying by 10· merely removes every Digit one place to the left." PRINCIPLE VIII. page 55·

Therefore, to multiply 64832· by 20· we must multiply by 2· and, as we proceed, set down every figure of the Product *one place to the left.* To multiply by 200· we multiply by 2·, and set every figure of the Product *two places to the left.* And so on.

Again, because $\cdot 2 = \frac{1}{10}$ of 2· (PRINCIPLE IV. p. 26·)

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the Multiples contained in the Multiplication Table, where every factor is less than 13· When, by learning an extended Multiplication Table, or by any other means, he shall become competent to detect the factors of higher Composite Numbers than those here treated of, he will extend to them the application of Principle xi.

Therefore, (by PRIN. IX. p. 56·) ·2 = 2· × $\frac{1}{10}$ = 2· × ·1
∴ 64832· × ·2 = 64832· × 2· × ·1

But, "Multiplying by ·1 merely removes every Digit *one* place to the right." PRINCIPLE X. page 60·

Therefore, to multiply 64832· by ·2 we must multiply by 2· and, as we proceed, set down every figure of the Product *one place to the right.* To multiply by ·02 we multiply by 2·, and set every figure of the Product *two places to the right.* And so on. We see then, that

Multiplying by *tens* removes Digits of Product *one* place to the *left.*
" by *hundreds* " " *two* places to the *left.*
" by *thousands* " " *three* places to the *left.*
&c. &c. &c.
" by *tenths* removes Digits of Product *one* place to the *right.*
" by *hundredths* " " *two* places to the *right.*
" by *thousandths* " " *three* places to the *right.*
&c. &c. &c.

PRINCIPLE XII. *In multiplying by any Digit standing in any but the units' place, each figure of the product must be moved to the right or left as many places as the multiplying digit is from the units' place.*

64832· × 2· = 129664·	64832· × ·2 = 12966·4
64832· × 20· = 1296640·	64832· × ·02 = 1296·64
64832· × 200· = 12966400·	64832· × ·002 = 129·664
64832· × 2000· = 129664000·	64832· × ·0002 = 12·9664
&c. &c. &c.	&c. &c. &c.
·64832 × 2· = 1·29664	·64832 × ·2 = ·129664
·64832 × 20· = 12·9664	·64832 × ·02 = ·0129664
·64832 × 200· = 129·664	·64832 × ·002 = ·00129664
·64832 × 2000· = 1296·64	·64832 × ·0002 = ·000129664
&c. &c. &c.	&c. &c. &c.

Here, we again remark what was noticed on page 60·, namely, that "When Decimals are multiplied by Decimals, the Product has as many Decimal places as the Multiplicand and Multiplier together have."

We notice, also, that in multiplying by Units, every figure of the Product falls under the corresponding figure of the Multiplicand. That is to say, Multiplying by a Digit in Units' place neither removes the figures of the Product to the right nor to the left.

To multiply any number by a single Digit (or by 11 *or* 12*) in any place of Integers or Decimals.*

First. Arrange the Multiplicand and Multiplier as usual, decimal point under decimal point, and draw horizontal line.

Second. Place beneath this line a decimal point under the other two.

Third. To the right of this decimal point, write as many ciphers as there will be decimal places in the product.

Fourth. Multiply. And as you find each figure of the product, set it down as many places to the right or left as your multiplier is from the units' place.

Observe that 11 or 12 will be reckoned as in the place of their right-hand Digit. For 11· = eleven units, 110· = eleven tens; 1·2 = twelve tenths; and so on.

FIRST EXAMPLE WORKED OUT. INTEGERS × INTEGERS. Multiply 3894628· by 800·

Example completed.

3894628·
800·

Here, the multiplying digit 8 is *two places to the left of Units.* Therefore, insert two ciphers in the first and second places of Integers of the product.

Then, multiply by 8, setting each figure of product two places to the left.

3115702400· = *Product of the factors* 3894628· *and* 800· = *also* 800*th multiple of* 3894628· *and* = *also* 3894628*th multiple of* 800·

Principle vi. page 42· See also page 43·

SECOND EXAMPLE WORKED OUT. INTEGERS × DECIMALS. Multiply 3894628· by ·008

Example prepared.

3894628·
·008
·000

Here, the multiplying Digit being *three places to the right of Units*, each Digit of Product must go *three places to the right.* Hence, the Product will have *three places of Decimals.*

Therefore, place three ciphers to the right of Decimal Point of Product.

Example completed.

3894628·
·008

Then multiply by 8, setting each Digit of Product three places to the right: of course substituting significant figures for the preparation-ciphers, when necessary.

31157·024 = *Product, &c.* = *also* $\frac{8}{1000}$ *part of* 3894628· (PRINCIPLE IX.), *and* = *also* 3894628*th multiple of* $\frac{8}{1000}$

From these two Examples we observe that, *when the Multiplicand consists of Integers only, the first Digit produced by the Multiplication falls under the Multiplying Digit.*

THIRD EXAMPLE WORKED OUT. DECIMALS × INTEGERS. Multiply ·3894628 by 800·

Example prepared.
·3894628
800·
·00000

Here, the multiplying Digit being *two places to the left of Units*, each Digit of Product will go *two places to the left*. And as the Multiplicand has *seven* Decimal Places, the Product will have *five*.

Therefore, place *five* ciphers to the right of Decimal Point.

Then multiply by 8 and set each Digit of Product two places to the left, substituting as before.

Example completed.
·3894628
800·
311·57024 = *Product, &c.*

FOURTH EXAMPLE WORKED OUT. DECIMALS × DECIMALS. Multiply ·3894628 by ·008

Example prepared.
·3894628
·008
·0000000000

Here, since the multiplying Digit 8 is *three places to the right of Units*, each Digit of Product must go *three places to the right*. So that, as the Multiplicand has *seven* Decimal Places, the Product will have *ten*.

Therefore, place *ten* ciphers to the right of Decimal Point.

Then multiply by 8 and set each Digit of Product three places to the right, substituting as before.

Example completed.
·3894628
·008
·0031157024 = *Product, &c., = also* $\frac{8}{1000}$ *of* $\frac{3894628}{10000000}$

FIFTH EXAMPLE WORKED OUT. INTEGERS × VULGAR FRACTIONS. Multiply $\frac{684}{3569}$ by 900·

This is, of course, the same as 900· × $\frac{684}{3569}$ PRINCIPLE VI. p. 42·

$$\frac{684}{3569}\times 900\cdot=\frac{684\cdot\times 900\cdot}{3569}=\frac{615600}{3569}=900\cdot\times\frac{684}{3569}$$

SIXTH EXAMPLE WORKED OUT. VULGAR FRACTIONS × VULGAR FRACTIONS. Multiply $\frac{384}{11000}$ by $\frac{900}{659}$

$$\frac{384}{11000}\times\frac{900}{659}\text{ or }\frac{900}{659}\times\frac{384}{11000}=\frac{384\cdot\times 900\cdot}{11000\cdot\times 659\cdot}=\frac{345600}{724900}*$$

EXERCISE XLII.

EXAMPLES IN MULTIPLICATION OF ANY NUMBER BY A SINGLE DIGIT, (OR BY 11 OR 12) IN ANY PLACE OF INTEGERS OR DECIMALS. ABSTRACT NUMBERS.

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* NOTE TO TEACHER. It would be premature to say anything yet of the "lowest terms" of a Vulgar Fraction.

Before working any of the following Examples, read them all down, and state in what place the First Digit produced by multiplying in each will be put. If the Product will have any Decimal Places, state how many.

1· What are 314822·×20·?
2· „ „ 9165438·×·05?
3· „ „ 6432·154×40·?
4· „ „ 9562·36×·04?
5· „ „ 1483·6954×·0012?
6· „ „ 9543615·9×200·?
7· „ „ 621543·75×11000·?
8· „ „ 9483·6254×·00011?
9· What are 59684213·×9000·?
10· „ „ ·6483×·0012?
11· Multiply ·0004368 by 10000·
12· „ ·046829 by ·009
13· „ 84135·875 by 1·2
14· „ 6948·35 by 1·1
15· „ ·005 by 6438·62
16· „ 79614·585 by 1200·
17· Find the Continued Product of ·04, ·09, and ·006
18· „ „ „ of 38·45, ·012, and ·011
19· „ „ „ of 3·1416, ·9, 11· and 400·
20· „ „ „ of 642·15, ·006, ·004, and 12·
21· Multiply 6432·185 by 40· by ·04 by 400· by ·004 by 4000· and by ·0004
22· Find the fourth power of ·5, and of ·05
23· Raise 614·83 to its 20th *multiple*, and 40· to its 5th *power*.
24· Involve $\frac{6}{8}$ to its 6th power.
25· Multiply $\frac{37}{429}$ by 1200· by 900· by 80· and by 70·
26· What is the Continued Product of $\frac{11}{20}$, $\frac{50}{94}$, and $\frac{12}{700}$?
27· Involve to its eighth power ·6, also ·06 and 60·
28· Find $\frac{6}{1000}$ of 3482·168
By PRINCIPLE IX. p. 56· $\frac{6}{1000}$ of 3482·168 = 3482·168 × $\frac{6}{1000}$ = 3482·168 × ·006
29· What are $\frac{12}{100}$ of 94326·845?
30· Required eleven ten-thousandth parts of 642·
31· Find $\frac{8}{100}$ of 71426·938; and $\frac{11}{10000}$ of 946·354

CONCRETE NUMBERS.

32· How many articles are there in 6847359· scores?
33· How many shillings are there in £94325·?
34· Reduce £642· 19·*s* to shillings.
Here you multiply the pounds by 20· and then add the 19·s
35· How many pounds weight in 400· Tons?
See EXERCISE XXXII. EXAMPLE 5·
36· What space in cubic inches would 400· gallons of water occupy?
See EXERCISE XXXIV. EXAMPLE 27·
37· How many feet are there in 500· miles?
38· Sixteen ounces make 1·lb of grocers' goods. How many ounces in 1· cwt.?
Multiply by 16· as a Composite number.
39· Fourteen lb. make 1· stone. How many pounds in 543· stone?
14· is a Composite number.
40· What would 2000· sheep cost at 53· shillings each?
41· How many pence would 120· bags contain, each holding £6429·
Reduce the pounds to shillings, then reduce the shillings to pence, then multiply by 120· For the number of pence in the bags is equal to the continued product of 6429· and 20· and 12· and 120·

42· How many farthings in £20000·?

43· Reduce to farthings £49· 8·s

44· Sixty minutes make one hour. Reduce 643·25 hours to minutes.

45· Twenty Cwts. make one Ton. Reduce 9348·5 Tons to Cwts.

46· Gunter's chain, used in measuring land, contains 100· links. How many links in 364·38 chains?

47· Each of those links is 7·92 inches long. How many inches in a chain?

48· A mile is equal to 80· chains. How many inches in a mile?

49· What is $\frac{6}{10000}$ of a Cwt. expressed in lb. and decimals of a lb.?

By Principle ix. $\frac{6}{10000}$ *of* 1· *Cwt.* = 1· *Cwt.* × $\frac{6}{10000}$ = 112 *lb.* × ·0006 *(See* p. 18·*)*

50· A prize worth £9432·875 was distributed among the officers and crew of the victorious ship. The whole was divided into 100· equal shares, and 9· of these shares were assigned to the Captain. What amount had he?

He had $\frac{9}{100}$ *of* £9432·875 *How much is that?*

51· A person borrowed £643·785 and agreed to pay to the lender £4· a year interest for every £100· lent. How much Interest was due at one year's end?

MULTIPLICATION BY ANY INTEGER, VULGAR FRACTION, OR DECIMAL.

If one could immediately think on any Multiple whatever of a Digit, (its 3468th for instance,) as one recalls the first twelve Multiples of the first twelve numbers after learning them from the Multiplication Table, then any Example whatever in Multiplication might be performed in one line as when the Multiplier is a single Digit. But, as this is quite beyond our powers of mind, we can only find high Multiples of a Number by successive steps.

We have already shewn what those steps are:

First. When the Multiplier is a Composite Number, having each of its Factors less than 13· Principle xi. page 64·

Second. When the Multiplier is expressed by a single Digit in any place of Integers or Decimals; (or by 1 and 1, or 1 and 2, in adjoining places.) Principle xii. page 66·

It only remains to explain the general method of multiplying by *any Number whatever*, whether Prime or Composite, whatever number of Digits may be employed to express it, and in whatever places those Digits may stand.

To multiply by any such Number, we multiply by its separate parts; first, by its right-hand figure, then by the next, then by the next, and so on. The Multiples thus found we set under each other, and then add them together.

Suppose I have to multiply a Number by 678· that is to say, I am to ascertain what 678· such Numbers would amount to. (See pages 41· and 56·)

Now, considering this number as a Complex Unit, (see page 62·) it is plain that

8· such Nos. + 70· such Nos. + 600· such Nos. = 678· such Nos.

Suppose, then, we have to multiply 942361· by 678·

We find, First, the 8th Multiple of 942361· = 7538888·
" " Second, the 70th Multiple of 942361· = 65965270·
" " Third, the 600th Multiple of 942361· = 565416600·
These, added together, make the 678th Multiple of 942361· .... = 638920758·

To Multiply any Integer, Vulgar Fraction, or Decimal, by any Integer, Vulgar Fraction, or Decimal.

First. Arrange and prepare as in former cases.

Second. Multiply the Multiplicand by the right-hand Digit of the Multiplier.

Third. Multiply the Multiplicand by the next Digit of Multiplier, placing this Product under the former one.

Fourth. Do the same with the other Digits of the Multiplier in succession.

Fifth. Add together all the Multiples thus found.

First Example worked out. Integers × Integers. Multiply 942361· by 678·

Example completed.

942361·
678·
7538888· = 8th *Multiple of* 942361·
65965270·* = 70th *Multiple of* 942361· } See Prin. xii. page 66·
565416600·* = 600th *Multiple of* 942361· }
638920758· = 678th *Multiple of* 942361·

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* Note to Teacher. It is customary to omit the ciphers here. This practice is most injudicious, *for it falsifies every Digit of the line* in which a cipher is suppressed. Legitimate contractions in arithmetic are by all means to be encouraged, not only as useful abbreviations of work, but also on account of the beautiful ingenuity which many of them involve. It is, however, an unwise economy to save labour by the sacrifice of accuracy. See third note on next page.

SECOND EXAMPLE WORKED OUT. INTEGERS × DECIMALS. Multiply 39648· by ·985

$$\begin{array}{r}
\text{Ex. prepared.}\\
39648\cdot\\
\cdot985\\ \hline
\left.\begin{array}{r}\cdot000\\ \cdot00\\ \cdot0\end{array}\right\}\\ \hline
\cdot000^{*}\\ \hline\hline
\end{array}
\text{See Note p. 33}\cdot
\qquad
\begin{array}{rl}
\text{Ex. completed.} & \\
39648\cdot & \\
\cdot985 & \\ \hline
198\cdot240 & = \frac{5}{1000}\ \textit{of}\ 39648\cdot \\
3171\cdot84 & = \frac{80}{1000}\ \textit{of}\ 39648\cdot \\
35683\cdot2 & = \frac{900}{1000}\ \textit{of}\ 39648\cdot \\ \hline
39053\cdot28 & = \frac{985}{1000}\ \textit{of}\ 39648\cdot \\ \hline\hline
\end{array}
\left.\begin{array}{l} \\ \\ \end{array}\right\}
\begin{array}{l}\text{See Prin.}\\ \text{ix. and x.}\\ \text{also, p. 18}\cdot\end{array}$$

THIRD EXAMPLE WORKED OUT. DECIMALS × DECIMALS. Multiply ·8645 by ·095

Example completed.

$$\begin{array}{ll}
\cdot8645 & \\
\cdot095 & \\ \hline
\cdot0043225 & = \cdot8645 \times \cdot005\ \textit{or} = \frac{5}{1000}\ \textit{of}\ \cdot8645\\
\cdot077805 & = \cdot8645 \times \cdot09\ \ \ \textit{or} = \frac{90}{1000}\ \textit{of}\ \cdot8645\\ \hline
\cdot0821275 & = \cdot8645 \times \cdot095\ \textit{or} = \frac{95}{1000}\ \textit{of}\ \cdot8645\\ \hline\hline
\end{array}$$

FOURTH EXAMPLE WORKED OUT. INTEGERS AND DECIMALS × DECIMALS. Find $\frac{6425}{10000}$ of 943·265

$\frac{6425}{10000}$, being a Decimal Fraction, is written without its Denominator, thus, ·6425 (See page 18·)

And ∵ $\frac{6425}{10000}$ of 943·265 = 943·265 × $\frac{6425}{10000}$ by PRINCIPLE IX.

∴ $\frac{6425}{10000}$ of 943·265 = 943·265 × ·6425

$$\begin{array}{r}
\text{Ex. prepared.}\\
943\cdot265\\
\cdot6425\\ \hline
\cdot0000000\\
\cdot000000\\
\cdot00000\\
\cdot0000\\ \hline
\cdot0000000\\ \hline\hline
\end{array}
\qquad
\begin{array}{rll}
\text{Example completed.} & & \\
943\cdot265 & & \\
\cdot6425 & & \\ \hline
\cdot4716325 & = \cdot0005^{\ddagger} & \textit{of}\ 943\cdot265\\
1\cdot886530^{\dagger} & = \cdot002 & \textit{of}\ 943\cdot265\\
37\cdot73060^{\dagger} & = \cdot04 & \textit{of}\ 943\cdot265\\
565\cdot9590^{\dagger} & = \cdot6 & \textit{of}\ 943\cdot265\\ \hline
606\cdot0477625 & = \cdot6425 & \textit{of}\ 943\cdot265\\ \hline\hline
\end{array}$$

---

* NOTES TO TEACHER. It is intended that, as each Decimal Digit of the Multiple is found, it should be *substituted* for the preparation-cipher occupying its place. (See page 67·) If the pupil work his examples on paper, the preparation-ciphers may be written lightly in pencil.

† The ciphers marked with daggers in this example *not being between a Significant Figure and the Decimal Point are of no service whatever.* (See PRINCIPLE II. page 3·) They *may*, therefore, and, indeed, *should* be omitted. Yet, with remarkable inconsistency, ordinary usage, which (as in first example on page 71·) *omits* ciphers where their presence is most essential, *retains* these!

‡ This example exposes very palpably the absurdity of reading Decimal *Figures* after either of the methods condemned in the notes to pages 22· and

FIFTH EXAMPLE WORKED OUT. INTEGERS AND DECIMALS × INTEGERS AND DECIMALS. Multiply 7849·35 by 2563·94

Ex. prepared.	Example completed.	
7849·35	7849·35	
2563·94	2563·94	
·0000	313·974*	= 7849·35 × ·04
·000	7064·415	= 7849·35 × ·9
·00	23548·05	= 7849·35 × 3·
·0	470961·*	= 7849·35 × 60·
·	3924675·	= 7849·35 × 500·
0·	15698700·	= 7849·35 × 2000·
·0000	20125262·439	= 7849·35 × 2563·94

SIXTH EXAMPLE WORKED OUT. VULGAR FRACTION × VULGAR FRACTION. Find $\frac{614}{983}$ of $\frac{25}{3164}$

$$\frac{614}{983} \text{ of } \frac{25}{3164} = \frac{614}{983} \times \frac{25}{3164} = \frac{614\cdot \times 25\cdot}{983\cdot \times 3164\cdot} = \frac{15350}{3110212}$$

SEVENTH EXAMPLE WORKED OUT. VULGAR FRACTION × INTEGER. Find $\frac{69}{318}$ of 2462·

$$\frac{69}{318} \text{ of } 2462\cdot = \frac{69}{318} \times 2462\cdot = \frac{69\cdot \times 2462\cdot}{318} \text{ (Page 49·)}$$

$$\therefore \frac{69}{318} \text{ of } 2462\cdot = \frac{169878}{318} \textit{ an Improper Fraction. } \text{(p. 39·)}$$

EXERCISE XLIII.

EXAMPLES IN MULTIPLICATION OF ANY NUMBER BY ANY NUMBER; INTEGER, VULGAR FRACTION, OR DECIMAL FRACTION. ABSTRACT NUMBERS. (See p. 24·)

1· Multiply 34168· by 435·
2· " 543·416 by 9462·
3· " 915421·863 by ·064
4· " ·943549 by 64·32
5· " 12184·168 by 4005·
6· " 847693·45 by ·00842
7· What are 43·5 × 64·09 × ·04 = ?
8· " " 7483·62 × 9·463 = ?
9· " " 96541·843 × 643·9 = ?
10· " " ·421386 × 9478·6 = ?
11· " " 614·218 × ·347 = ?
12· " " 98476· × ·00849 = ?

40· "*Decimal nought, nought, nought, five, of* 943·265" and "*Decimal six thousand, four hundred and twenty-five of* 943·265" are two expressions as ridiculous as they are unintelligible.

* NOTE TO TEACHER. The Pupil will remark that a useless cipher is omitted in each of these two lines.

13· Find the product of 6143·28 and $\frac{432}{1000000}$ (See page 18·)
14· Multiply $\frac{64}{319}$ by $\frac{4285}{12642}$, and $\frac{3164}{9458}$ by $\frac{784}{1465}$ (See Exer. xxxviii.)
15· Raise 8143· to its 5th power, and 3148·2 to its 3rd power.
16· Find the continued product of ·642, 95·835, and 42·86
17· Find the continued product of $\frac{14}{29}$, $\frac{642}{1125}$, $\frac{295}{84}$ and $\frac{1239}{6435}$
18· Find the continued product of ·0056, ·00642 and 21·4
19· Raise 9643·215 to its 6429th Multiple.
20· Raise $\frac{14}{39}$ to its 4th power.

The 4th power of any number is the continued product of four such numbers. (Page 53·)

$\therefore$ *4th power of* $\frac{14}{39} = \frac{14}{39} \times \frac{14}{39} \times \frac{14}{39} \times \frac{14}{39} = \frac{14\cdot \times 14\cdot \times 14\cdot \times 14\cdot}{39\cdot \times 39\cdot \times 39\cdot \times 39\cdot}$
$= \frac{38416}{2313441}$ (See page 57·)

21· Of what composite number are 642· and 53· the corresponding factors? (See page 63·)
22· Find the six-hundred-and-forty-fifth Multiple of nine million, six hundred and fifty-two thousand, seven hundred and nine.

CONCRETE NUMBERS.

24· A cwt. is 112·lb. How many lb. in 96432·85 cwt.?
25· How many cubic inches would 6438· gallons of water occupy? (See Exercise xxxiv. Ex. 27·)
26· How many inches are there in 3489· links, each 7·92 inches?
27· What would 693· bales of goods weigh, at 359· lb. each?
28· How much would 465· chests of tea cost, at £97· per chest?
29· What is the united length of 5432· pieces of linen, each 135· yards?
30· How many letters are there in a book consisting of 14· sheets, each sheet forming 24· pages, each page containing 35· lines, and each line, on an average, 48· letters?
31· How many lb. do thirty-four thousand six hundred and thirty-eight, cubic feet, *and* ninety-six thousand three hundred and seventy-five hundred thousandths of a cubic foot of water weigh? (See Exercise xxxiv. Ex. 26·)

USEFUL CONTRACTION, BY ADDING IN "BACK FIGURE."

Multiply 39846257· by 18·

Example completed by usual method.

$$\begin{array}{rl}
39846257\cdot & \\
18\cdot & \\
\hline
318770056\cdot & = 8\cdot \textit{ times } 39846257\cdot \\
398462570\cdot & = 10\cdot \textit{ times } 39846257\cdot \\
\hline
717232626\cdot & = 18\cdot \textit{ times } 39846257\cdot \\
\hline\hline
\end{array}$$

On examining the foregoing process, we see that it consists of two steps.

First. We find the 8th Multiple of the Multiplicand.

Second. To each Digit of this 8th Multiple we add the Digit of the Multiplicand standing in next place to the right.

For the 7 is added into the tens' place of 8th Multiple;
the 5 " " into the hundreds' place; and so on.

The Digit thus added in is called the "*Back Figure,*" because it stands to the right of, or *behind*, the place into which it is added.

And, by thus adding in each back figure as we proceed, *(not waiting to set it down)* we may complete our example in *one line* instead of in *three.*

The same example completed by the contracted method.

39846257·
18·
717232626·

In the first place, no back figure to add.
In the second place, add back figure 7
In the third place, add back figure 5
In the fourth place, add back figure 2 and so on.

EXERCISE XLIV.

CONTRACTED MULTIPLICATION.

Each Example to be worked in one line.

1· Multiply 4831648· by 13·
2· " 96435·895 by 14·
3· " 3596721·63 by 15·
4· " 40586435· by 16·
5· " 7393843· by 17·
6· " 384659·648 by 18·
7· Multiply 64380·842 by 1·9
8· " 7386954·9 by ·014
9· " 26454·986 by ·0018
10· " 17843656·35 by ·13
11· Find 4th power of 18· and of 19·
12· " 9th power of 1·6
13· Multiply continually together 16·, 1·4, 1·7 and ·018
14· Find the Product of ·0514, and ·18
15· Multiply 64381·295 by 130·, by 16000·, and by 1900·

This contraction shews how to multiply in one line by a Multiplier expressed by Digit 1 in any place, and any other Digit in the next place to the right.

The same method may be used with a Multiplier consisting of any number of Digits 1 in adjoining places, having any other Digit on the right. Such Multipliers are 118· 11113· 11700· ·0001116 11·118 &c.

EXAMPLE 16· WORKED OUT. Multiply 34567·892 by 1111·3

First, by ordinary method.

34567·892
1111·3
10370·3676
34567·892
345678·92
3456789·2
34567892·
38415298·3796

Here, we add in
with 2nd fig. of Product, *one* back figure,
with 3rd fig. of Product, *two* back figures,
with 4th fig. of Product, *three* back figures,
with 5th fig. of Product, *four* back figures,
and so on, to 9th fig. of Product.
After that we add
for 10th fig. of Product *three* back figures *only*,
for 11th fig. of Product *two* back figures *only*,
for 12th fig. of Product *one* back figure *only*.

Second, by contracted method.

$$\begin{array}{r} 34567{\cdot}892 \\ 1111{\cdot}3 \\ \hline 38415298{\cdot}3796 \\ \hline\hline \end{array}$$

Working by contraction, then, we say,

For 1st fig.	*six,*			set down 6
„ 2nd „	*twenty-seven,*	+ *one* back figure,	viz. *two,*	„ 9
„ 3rd „	*twenty-six,*	+ *two* back figures,	„ *nine* and *two,*	„ 7
„ 4th „	*twenty-four,*	+ *three* back figures,	„ *eight, nine* and *two,*	„ 3
„ 5th „	*twenty-two,*	+ *four* back figures,	„ *seven, eight, nine* and *two,*	„ 8
„ 6th „	*nineteen,*	+ *four* back figures,	„ *six, seven, eight* and *nine,*	„ 9
„ 7th „	*sixteen,*	+ *four* back figures,	„ *five, six, seven* and *eight,*	„ 2
„ 8th „	*thirteen,*	+ *four* back figures,	„ *four, five, six* and *seven,*	„ 5
„ 9th „	*three,* (carried)	+ *four* back figures,	„ *three, four, five* and *six,*	„ 1
„ 10th „	*two,* (carried)	+ *three* back figures,	„ *three, four* and *five,*	„ 4
„ 11th „	*one,* (carried)	+ *two* back figures,	„ *three* and *four,*	„ 8
„ 12th		*one* back figure,	„ *three* only,	„ 3

EXERCISE XLIV. *Continued.*

Each Example to be worked in one line.

17· Multiply 43186·219 by 1119·
18· „ 3684·2184 by 11·114
19· „ 54632·575 by ·0112
20· „ 9387615·9 by ·001115
21· „ 3418·975 by 111119·
22· What is the Product of 984·542 and 115·?
23· „ „ „ of 37184· and ·00119?
24· „ „ „ of 196·59842 and 111000·?
25· „ „ „ of 6498·439 and 1111100·?
26· „ „ „ of 9·843216 and ·0001117?

These Examples may be proved by the extended method.

Mixed Numbers. An Integer and a Fraction together form a mixed number. $13\frac{5}{8}$, $211\frac{6}{13}$, $5\frac{1}{2}$, $74\frac{7}{9}$ are, then, mixed numbers.

Every mixed number is equal to some Improper Fraction. (See page 39·)

Integers expressed as Fractions. On Page 58· it is explained that an Integer may be put into the form of a Fraction by writing under it 1 as a Denominator.

Integers may also be expressed as Fractions with any Denominator.

For $\because 1\cdot = \frac{2}{2}$ or $\frac{3}{3}$ or $\frac{4}{4}$ or $\frac{5}{5}$ or $\frac{6}{6}$ or, &c. (See p. 12·)
$\therefore 2\cdot = \frac{4}{2}$ or $\frac{6}{3}$ or $\frac{8}{4}$ or $\frac{10}{5}$ or $\frac{12}{6}$ or, &c.
$\therefore 3\cdot = \frac{6}{2}$ or $\frac{9}{3}$ or $\frac{12}{4}$ or $\frac{15}{5}$ or $\frac{18}{6}$ or, &c.

To express an Integer as an improper Fraction with any Denominator.

First. Multiply the Integer by the Denominator. This

Product is the Numerator of the Improper Fraction.

Second. Write the Denominator under this Numerator.

Hence, $19{\cdot} = \frac{19{\cdot} \times 2{\cdot}}{2{\cdot}} = \frac{38}{2}$, or $= \frac{19{\cdot} \times 3{\cdot}}{3{\cdot}} = \frac{57}{3}$, or $= \frac{19{\cdot} \times 4{\cdot}}{4{\cdot}} = \frac{76}{4} =$ &c.

Also, the mixed number $19\frac{3}{4} = \frac{76}{4} + \frac{3}{4} = \frac{79}{4}$

And, „ „ „ $201\frac{5}{16} = \frac{3216}{16} + \frac{5}{16} = \frac{3221}{16}$

To reduce a Mixed Number to its equivalent Improper Fraction.

First. Multiply the Integer by the Denominator of the Fraction.

Second. Add to the Product the Numerator of the Fraction. The *sum* is the Numerator of the Improper Fraction.

Third. Write under this Numerator the Denominator of the given Fraction.

EXERCISE XLV.

EXAMPLES IN REDUCTION OF INTEGERS AND MIXED NUMBERS TO THEIR EQUIVALENT IMPROPER FRACTIONS.

1· Express 21· in 4ths, in 12ths, and in 29ths.

2· How many sixteenths in 3418·?

3· Reduce 3695· to 542nd parts.

4· What No. of fourths $= 3125\frac{3}{4}$?

5· Reduce to equivalent Improper Fractions: $642\frac{11}{15}$, $39\frac{64}{72}$, $610\frac{5}{25}$, $3145\frac{21}{104}$, and $31\frac{65}{211}$

6· Reduce to fourths, 3189·, $63254\frac{1}{4}$, 75693·, and $215\frac{3}{4}$

7· Reduce to fifths $64\frac{4}{5}$, $319\frac{3}{5}$, $18\frac{2}{5}$, $95\frac{1}{5}$

8· Express in ninths $111\frac{4}{9}$, $625\frac{2}{9}$, $712\frac{1}{9}$, $18\frac{7}{9}$

9· How many twelfths in $91\frac{5}{12}$, $64\frac{7}{12}$, $1146\frac{11}{12}$?

10· Reduce to ninety-fifths $439\frac{17}{95}$, $3142\frac{8}{95}$

11· Reduce to equivalent Improper Fractions: $31\frac{5}{26}$, $642\frac{1}{845}$, $35\frac{1}{2}$, $2\frac{1}{4}$, $4\frac{1}{11}$

12· Reduce to equivalent Improper Fractions: $6\frac{1}{5}$, $4\frac{2}{9}$, $8\frac{3}{8}$, $7\frac{1}{4}$, $3\frac{6}{11}$, $8\frac{2}{9}$, $17\frac{3}{11}$, $14\frac{5}{9}$, $18\frac{3}{5}$

Integers and Mixed Numbers are usually reduced to Improper Fractions before being multiplied by Fractions or by Mixed Numbers.

EXAMPLES WORKED OUT. 1· What is the Product of $64\frac{5}{12}$ and $19\frac{1}{2}$?

$64\frac{5}{12} = \frac{773}{12}$; and $19\frac{1}{2} = \frac{39}{2}$, by last Exercise.

$$\therefore\ 64\frac{5}{12} \times 19\frac{1}{2} = \frac{773}{12} \times \frac{39}{2} = \frac{773{\cdot} \times 39{\cdot}}{12{\cdot} \times 2{\cdot}} = \frac{30147}{24}$$

2· Multiply continually together, $34\frac{1}{5}$, $\frac{6}{7}$ and 19·

$$34\frac{1}{5}, \times \frac{6}{7} \times 19{\cdot} = \frac{171}{5} \times \frac{6}{7} \times \frac{19}{1} = \frac{171{\cdot} \times 6{\cdot} \times 19{\cdot}}{5{\cdot} \times 7{\cdot} \times 1{\cdot}} = \frac{19494}{35}$$

EXERCISE XLVI.

EXAMPLES IN MULTIPLICATION OF ANY NUMBER BY A FRACTION OR A MIXED NUMBER.

1· What are $\frac{6}{8} \times 13\cdot \times 5\frac{1}{2} = ?$
2· „ „ $\frac{5}{4} \times \frac{11}{29} \times \frac{3}{4}$ of $\frac{9}{17} = ?$
3· „ „ $3\frac{1}{14} \times 6\frac{1}{2} \times \frac{1}{4}$ of $\frac{5}{6} = ?$
4· Raise $6\frac{1}{2}$ to its 4th power.
5· „ $\frac{1}{2}$ of $\frac{3}{5}$ to its 3rd power.
6· Find the product of $7\frac{1}{4}$ & $35\frac{4}{5}$
7· Find the continued product of $\frac{9}{17}$, $9\frac{1}{5}$, $84\frac{1}{2}$ and $\frac{7}{12}$
8· What are $\frac{4}{5}$ of $149\frac{5}{8} = ?$
9· „ „ $\frac{13}{64}$ of $\frac{3}{5}$ of $\frac{1}{4}$ of $9\frac{1}{8} = ?$
10· Find nine-tenths of $\frac{1}{2}$ of $\frac{17}{112}$
11· „ six twenty-fifths of $341\frac{1}{5}$
12· Mult^y $\frac{1}{4}$ of $\frac{5}{9}$ of $6\frac{1}{4}$ by $\frac{7}{11}$ of $16\frac{1}{4}$
13· „ $649\frac{146}{194}$ by $\frac{4}{5}$ of $\frac{9}{11}$ of 12·
14· Find the 12th Multiple of $\frac{9}{16}$ of $24\frac{5}{12}$
15· What Improper Fraction are $\frac{13}{48}$ and $3\frac{1}{2}$ Factors of?

REDUCTION OF FRACTIONS TO A COMMON DENOMINATOR.

The word "Common" means "belonging to more than one." Hence, when the Denominators of two or more Fractions are the same, it is said they have a Common Denominator.

Thus; $\frac{3}{12}$, $\frac{9}{12}$, $\frac{7}{12}$ and $\frac{11}{12}$ have the Common Denominator 12·
But, $\frac{4}{15}$, $\frac{13}{19}$, $\frac{18}{29}$ and $\frac{16}{45}$ have, no two of them, a Common Denominator.

Fractions cannot be added, unless they have a Common Denominator. (See p. 37· and PRINCIPLE V. p. 35·)

Any Fraction whose Numerator and Denominator are alike is equal to unity. (See pages 12· and 39·)

Therefore, $\frac{3}{3}$ of any thing is the whole of that thing.

$$\therefore \quad \tfrac{3}{3} \text{ of } \tfrac{5}{8} = \tfrac{5}{8}$$

But $\frac{3}{3}$ of $\frac{5}{8} = \frac{5}{8} \times \frac{3}{3} = \frac{15}{24}$ (PRINCIPLE IX. page 56·)

$$\therefore \quad \tfrac{5}{8} = \tfrac{5}{8} \times \tfrac{3}{3} = \tfrac{15}{24}$$

We see, then, that multiplying the Numerator and Denominator of $\frac{5}{8}$ by 3· produces a Fraction exactly equal to $\frac{5}{8}$, although expressed by higher numbers.

Change the multiplier $\frac{3}{3}$ for any other Fraction having Numerator and Denominator equal, as $\frac{4}{4}$, $\frac{5}{5}$, $\frac{13}{13}$, &c., and the reasoning and result will be the same.

$$\tfrac{5}{8} \times \tfrac{4}{4} = \tfrac{20}{32} = \tfrac{5}{8}; \; \tfrac{5}{8} \times \tfrac{5}{5} = \tfrac{25}{40} = \tfrac{5}{8}, \text{ and so on.}$$

The truth of these statements may be proved experimentally by dividing one unit-square into eighths, and another into twenty-fourths, or thirty-second parts, or fortieth parts: when $\frac{5}{8}$ will be found to contain exactly $\frac{15}{24}$ or $\frac{20}{32}$ or $\frac{25}{40}$, and so on.

Principle XIII. *Multiplying the Numerator and Denominator of a Fraction by the same Number makes no alteration in the value of that Fraction.*

This principle enables us to reduce unlike Fractions to others, equal to them, but having a Common Denominator.

For $\frac{5}{8}=\frac{35}{56}$ Multiplying Numerator and Denominator by 7·
And $\frac{4}{7}=\frac{32}{56}$ Multiplying Numerator and Denominator by 8·

Therefore the unlike Fractions $\frac{5}{8}$ and $\frac{4}{7}$, when reduced to a Common Denominator, become $\frac{35}{56}$ and $\frac{32}{56}$

To effect the foregoing reduction, we multiply Numerator and Denominator of *each* Fraction by Denominator of the *other.*

$$\left.\begin{aligned}\text{Again, }\frac{4}{11}&=\frac{4\cdot\times19\cdot\times14\cdot}{11\cdot\times19\cdot\times14\cdot}=\frac{1064}{2926}\\ \frac{15}{19}&=\frac{15\cdot\times11\cdot\times14\cdot}{19\cdot\times11\cdot\times14\cdot}=\frac{2310}{2926}\\ \frac{3}{14}&=\frac{3\cdot\times11\cdot\times19\cdot}{14\cdot\times11\cdot\times19\cdot}=\frac{627}{2926}\end{aligned}\right\}$$

So that, when reduced to a Common Denominator, the unlike Fractions $\frac{4}{11}$, $\frac{15}{19}$, and $\frac{3}{14}$, become $\frac{1064}{2926}$, $\frac{2310}{2926}$, and $\frac{627}{2926}$

In the work of the last example, remark

First. That the Numerator of each Fraction is multiplied continually by the Denominators of the other two.

Second. That the Common Denominator is the Continued Product of all the Denominators.

The Common Denominator is here found three times over, once for each Fraction. This is wasting labour. Once for all would suffice. And the work may be arranged as follows.

Reduce $\frac{4}{11}$, $\frac{15}{19}$ and $\frac{3}{14}$ to a Common Denominator.

$4\cdot\times19\cdot\times14\cdot=1064\cdot=$ *New Numerator for* 1*st Fraction.*
$15\cdot\times11\cdot\times14\cdot=2310\cdot=$ *New Numerator for* 2*nd Fraction.*
$3\cdot\times11\cdot\times19\cdot=\ \ 627\cdot=$ *New Numerator for* 3*rd Fraction.*

$11\cdot\times19\cdot\times14\cdot=2926\cdot=$ *Common Denominator for the three Fractions.*

Result. $\frac{4}{11}$, $\frac{15}{19}$ and $\frac{3}{14}$, expressed with a Common Denominator become $\frac{1064}{2926}$, $\frac{2310}{2926}$, and $\frac{627}{2926}$

To Reduce Fractions to a Common Denominator.

First. Reduce Compound Fractions to Simple ones, and Mixed Numbers to Improper Fractions.

Second. Multiply the Numerator of each Fraction (thus prepared) continually by the Denominators of all the others. This produces the New Numerators.

Third. Find the Continued Product of all the Denominators. This is the Common Denominator.

EXERCISE XLVII.

Reduce to a Common Denominator,

1· $\frac{4}{9}$, $\frac{7}{15}$, $\frac{8}{11}$ and $\frac{9}{10}$
2· $\frac{14}{17}$, $\frac{11}{19}$, $\frac{34}{55}$ and $\frac{17}{18}$
3· $\frac{25}{49}$, $\frac{18}{53}$, $\frac{6}{13}$ and $6\frac{1}{2}$
4· $\frac{4}{5}$ of 24· $\frac{7}{8}$ of $619\frac{5}{8}$ and $\frac{29}{71}$
5· $\frac{7}{8}$ of $\frac{6}{7}$ of 9· and $\frac{11}{16}$ of $\frac{42}{57}$
6· $\frac{14}{15}$ of $\frac{1}{3}$ of $13\frac{5}{8}$, $\frac{116}{219}$ and $\frac{21}{56}$
7· $141\frac{5}{8}$, $65\frac{4}{11}$, 13· and $\frac{1}{2}$ of $\frac{3}{5}$ of 5·
8· $\frac{16}{45}$, $\frac{9}{23}$, $\frac{13}{14}$ of $\frac{13}{14}$ and $\frac{7}{11}$ of 65·
9· $\frac{21}{43}$, $\frac{16}{25}$, $\frac{42}{115}$ and $\frac{30}{791}$
10· $\frac{7}{115}$, $\frac{8}{131}$, $\frac{10}{614}$ and $11\frac{1}{2}$
11· $\frac{5}{6}$, $\frac{4}{7}$, $\frac{3}{8}$, $\frac{4}{9}$, $\frac{3}{11}$, $\frac{6}{15}$ and $\frac{1}{21}$
12· $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$ and $\frac{1}{10}$

To add Fractions having unlike Denominators.

First. Reduce the Fractions to a Common Denominator.

Second. Add the New Numerators together, and under their sum write the Common Denominator.

If there be any Mixed Numbers, do not reduce them to Improper Fractions, but add their Fractions with the others, and add the Integers separately, placing the sum of these on the left of the sum of the Fractions.

FIRST EXAMPLE WORKED OUT. ALL SIMPLE FRACTIONS. Find the sum of $\frac{6}{8}$, $\frac{11}{12}$, $\frac{3}{14}$ and $\frac{6}{15}$

$\frac{6}{8} + \frac{11}{12} + \frac{3}{14} + \frac{6}{15} = \frac{15120}{20160} + \frac{18480}{20160} + \frac{4320}{20160} + \frac{8064}{20160} = \frac{45984}{20160}$

SECOND EXAMPLE WORKED OUT. SIMPLE AND COMPOUND FRACTIONS. Find the sum of $\frac{6}{17}$, $\frac{3}{5}$ of $\frac{8}{9}$ and $\frac{4}{7}$

$\frac{6}{17} + \frac{3}{5}$ of $\frac{8}{9} + \frac{4}{7} = \frac{6}{17} + \frac{24}{45} + \frac{4}{7} = \frac{1890}{5355} + \frac{2856}{5355} + \frac{3060}{5355} = \frac{7806}{5355}$*

THIRD EXAMPLE WORKED OUT. SIMPLE AND COMPOUND FRACTIONS, INTEGERS AND MIXED NUMBERS. Add together $\frac{14}{15}$, 84·, $\frac{3}{8}$ of 9· and $23\frac{1}{18}$

$\frac{14}{15} + 84\cdot + \frac{3}{8}$ of $9\cdot + 23\frac{1}{18} = 84\cdot + 23\cdot + \frac{14}{15} + \frac{27}{8} + \frac{1}{18} = 107\cdot + \frac{2016}{2160} + \frac{6804}{2160} + \frac{120}{2160} = 107\frac{8940}{2160}$*

EXERCISE XLVIII.

1· Add $\frac{6}{8}$, $\frac{1}{5}$, $\frac{4}{9}$ and $\frac{11}{12}$
2· „ $2\frac{1}{2}$, $\frac{13}{14}$, $\frac{11}{18}$, and $\frac{4}{5}$ of $\frac{7}{16}$
3· Find the sum of $\frac{5}{11}$ of 3· $\frac{6}{17}$ of $9\frac{1}{4}$ and $\frac{24}{415}$
4· „ „ „ of $4\frac{1}{2}$, $15\frac{3}{7}$, $29\frac{7}{12}$ and $\frac{117}{144}$
5· „ „ „ of $16\frac{1}{4}$, $32\frac{1}{2}$, $11\frac{3}{4}$, and $12\frac{1}{8}$
6· „ „ „ of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{12}$

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* NOTE TO TEACHER. The reduction of the resulting Improper Fraction to a whole or mixed number cannot be done until the pupil shall have learned Division. This postponement, however, implies no defect in our arrangement; for the *addition* of the quantities is *completed here*, changing the form of the result being no part of *that* process. The two operations, indeed, are so distinct that it is desirable to teach them separately.

7· What are $\frac{3}{5}$ of 45· + $\frac{5}{6}$ of $\frac{1}{4}$ of 19· + $\frac{8}{11}$ = ?
8· „ „ $\frac{4}{5}$ of $\frac{6}{7}$ + $\frac{3}{5}$ of 12· + $\frac{11}{12}$ of 42· = ?
9· „ „ $\frac{5}{12}$ + $\frac{111}{116}$ + $\frac{13}{64}$ + $\frac{1}{2}$ of $\frac{8}{11}$ = ?
10· Add together $\frac{7}{12}$, $\frac{7}{13}$, $\frac{7}{14}$ and $\frac{7}{15}$
11· Find the sum of 16$\frac{1}{2}$, 17$\frac{1}{4}$, 18$\frac{1}{6}$ and 19$\frac{1}{8}$
12· „ „ „ of $\frac{4}{11}$ + $\frac{61}{432}$ + $\frac{842}{1108}$ + $\frac{6}{13}$
13· What is the ninth multiple of $\frac{1}{2}$ + $\frac{1}{4}$ + $\frac{1}{8}$?

Add the three fractions together, and multiply the sum by 9·

SUBTRACTION.

Subtraction is the diminishing of a number by taking a smaller one out of it.

Arithmetical Symbol. The sign of Subtraction is a short horizontal line — placed between the Greater Number and the Less. It is called in reading "Minus," which is the Latin word for "Less."

Thus, the expression 19· — 5· is read, "Nineteen minus five," and means "Nineteen less five," or "Nineteen with five taken out of it." 19· — 5· = 14·

Four words employed in Subtraction must be perfectly understood. They are these: Minuend, Subtrahend, Remainder, and Difference.

In the expression 19· — 5· = 14·

19· is the Minuend; the Number which is to be diminished.

5· is the Subtrahend; the Number to be subtracted.

14· is the Remainder; the Number left after subtracting 5· from 19·

14· is also called the Difference between 19· and 5·

The **Minuend** is the larger Number, from which the lesser is to be taken.

The **Subtrahend** is the smaller number, which is to be taken from the greater.

The **Remainder** or **Difference** is the number left after subtracting.

EXERCISE XLIX.

Examples to be worked mentally.

1· What are 9· — 7· = ?
2· „ „ 16· — 4· = ?
3· „ „ 35· — 11· = ?
4· „ „ 64· + 9· — 62· = ?
5· „ „ 314· — 295· = ?
6· „ „ 100· — 85· = ?
7· „ „ 25· — 10· = ?
8· „ „ 112· — 20· = ?

9· Find the difference of 24· & 14·
10· „ „ of 64· and 39·
11· „ „ of 413· and 350·
12· „ „ of 300· and 199·
13· „ „ of 412· and 32·
14· „ „ of 65· and 43·
15· „ „ of 21· and 49·
16· „ „ of 11· and 65·

17· From 413· take 362·
18· From 1100· take 900·
19· From 42· take 25·
20· Take 17· from 39·
21· Subtract 43· from 64·
22· Subtract 31· from 50·
23. Take 321· from 400·
24· What are 604· — 45· =?

SUBTRACTION OF NUMBERS TOO LARGE TO BE WORKED MENTALLY.

In proceeding to Subtract one Number from another, we place the Lesser under the Greater, place for place, and then, if each Digit of the Subtrahend be less than the Digit standing above it, we find the Difference of the two and set it down, commencing at the right-hand end of the lines and proceeding place by place to the left-hand end.

First Example worked out. Integers and Decimals. Subtract 1435·68 from 7969·89

Example prepared.

```
7969·89
1435·68
───────
    ·00
```

Example completed.

```
7969·89 Minuend.
1435·68 Subtrahend.
───────
6534·21 Difference.
```

8 *(hundredths)* from 9 *(hundredths)* leave 1 *(hundredth) set down* 1 *(hundredth)*
6 *(tenths)* from 8 *(tenths)* leave 2 *(tenths) set down* 2 *(tenths)*
5 *(units)* from 9 *(units)* leave 4 *(units) set down* 4 *(units)*
3 *(tens)* from 6 *(tens)* leave 3 *(tens) set down* 3 *(tens)*
4 *(hundreds)* from 9 *(hundreds)* leave 5 *(hundreds) set down* 5 *(hundreds)*
1 *(thousand)* from 7 *(thousand)* leave 6 *(thousands) set down* 6 *(thousands)*

But should it happen that the Units'-figure of Subtrahend be greater than that of the Minuend, of course it will be impossible to subtract. In such a case, we "borrow" one ten from the Tens' place of the Minuend, convert it into ten units, add those to the Units'-figure of Minuend, and from this *sum*, subtract Units'-figure of Subtrahend.

This must be done in the following example, where the Units'-figure of Subtrahend being 8, and that of Minuend only 5, I say, "Eight from five I cannot. Borrow ten, and say, eight from fifteen."

On arriving at the Tens' place of Minuend, we must remember to reckon the 6 tens as 5 tens only, one ten having been converted into units and used up in the Units' place.

In the following example, it is also necessary in the

Hundreds' place to "borrow" one thousand, which we convert into ten hundreds, making with the 7 hundreds, twelve hundreds, from which we can subtract the 7 hundreds of the Subtrahend.

We must here again remember, on arriving at the Thousands' place, to consider the 2 thousands as only 1 thousand.

SECOND EXAMPLE WORKED OUT. Find the Difference of 4265· and 3748·

Example completed.

4265· *Minuend.*
3748· *Subtrahend.*
517· *Difference.*

Units' place:	"Eight from fifteen = seven; set down 7"
Tens' place:	"Four from five = one; set down 1"
Hundreds' place:	"Seven from twelve = five; set down 5"
Thousands' place:	"Three from three = nought."

Now, in the Tens' place, if, instead of saying, "Four from five," we had said, "Five from six," the remainder would have been *one* as it now is. And in the Thousands' place, it will come to the same thing, whether we say "Three from three," (as above) or "Four from four."

There are, then, two methods of working Subtraction. "Borrowing" is practised in both. But, by the first method, whenever we borrow, we correctly reckon the next figure of the Minuend as *one less* than it is written.

Whilst, by the second method, whenever we borrow, we reckon the next figure of the Subtrahend as *one more* than it is written.

The second method has been heretofore commonly employed; but the first is much to be preferred, as the more natural and simple.

Let us re-examine the last Example. The thing to be done is

From 4 *thousands* + 2 *hundreds* + 6 *tens* + 5 *units*,
Take 3 *thousands* + 7 *hundreds* + 4 *tens* + 8 *units*.

Now, as it is impossible to take 8 units out of 5 units, I "borrow" 1 ten, which I convert into units, thus turning 6 tens and 5 units, into 5 tens and 15 units. In like manner, because I cannot take 7 hundreds out of 2 hundreds, I "borrow" 1 thousand, which I convert into hundreds, and so turn the 4 thousands and 2 hundreds into 3 thousands and 12 hundreds. The borrowings, then, which *we* employ, neither increase nor decrease the value of either

Minuend or Subtrahend. They merely re-arrange the *parts* of those Numbers.* For to take 3748· from 4265· is

From 3 *thousands* + 12 *hundreds* + 5 *tens* + 15 *units*,
Take 3 *thousands* + 7 *hundreds* + 4 *tens* + 8 *units*.
Which leaves 5 *hundreds* + 1 *ten* + 7 *units*.

But, by the second method (the usual one) the process for subtracting 3748· from 4265· would be

From 4 *thousands* + 12 *hundreds* + 6 *tens* + 15 *units*,
Take 4 *thousands* + 7 *hundreds* + 5 *tens* + 8 *units*.
Which leaves, as before, 5 *hundreds* + 1 *ten* + 7 *units*.

Although we thus again arrive at the true result, yet we do so this time by an unnatural and roundabout process, by no means so easy to account for as the former method. For here, in order to find the Difference of 4265· and 3748· we add 1010· to each, so that we do not, in reality, subtract 3748· from 4265· but 4758· from 5275·

If, when we have occasion, in the Units' place, to "borrow," there should be a cipher in Tens' place of Minuend, we must borrow 1 *Hundred*, (if Minuend have any,) and convert it into 9 *tens*, and 10 *units*. If Minuend have neither Tens nor Hundreds, we must borrow 1 *Thousand*, (if any,) and convert it into 9 *hundreds*, 9 *tens* and 10 *units*. And so on.

To Subtract Integers or Decimals from Integers or Decimals.

First. Prepare as for Addition, placing the lesser Number under the Greater.

Second. Commencing at right-hand, Subtract, if possible, each figure of the Subtrahend from that of the Minuend standing above it, and set down the Differences in their proper places.

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* NOTE TO TEACHER. This feature, namely, that no increase is made in either Subtrahend or Minuend, constitutes our System's first claim to preference over the old one.

Its second advantage is that it dispenses altogether with carrying in Subtraction.

But more important than either of those technical recommendations are its clearness and its intelligibility: repeated experiments with very young pupils having proved how much more readily and perfectly they can comprehend the reasoning, and master the process here expounded than they can those of a method which suggests to them such perplexing questions as the following.

1· *How is it that the quantity from which we borrow remains as much as before?*
2· *Why do we borrow from one quantity and return to another?*
3· *How can we borrow anything from a cipher?*
4· *How can it be necessary to increase both quantities in order to find their difference.*

Third. If any figure of Subtrahend be greater than the figure above it, "borrow"* 1 from the next place of Minuend in which there is a significant figure. And then:

Reckon the Minuend figure *from* which you borrow as *one less* than it is written.

Reckon the Minuend figure *for* which you borrow as *ten more* than it is written.

Reckon *every cipher between* these two as a *nine*.

If the Subtrahend contain more Decimal figures than the Minuend, imagine, at starting, a cipher to stand over each of those extra figures. (See Principle II. page 3·)

THIRD EXAMPLE WORKED OUT. Subtract 346· from 10,000· Here, the 1 *ten thousand* of Minuend is borrowed, and converted into 9 *thousands*, 9 *hundreds*, 9 *tens*, and 10 *units*.

Example completed.

10,000· *Minuend.*
346· *Subtrahend.*
9654· *Difference.*

Units' place: 6 (*units*) from 10 (*units*) = 4 (*units*).
Tens' place: 4 (*tens*) from 9 (*tens*) = 5 (*tens*).
Hundreds' place: 3 (*hundreds*) from 9 (*hundreds*) = 6 (*hundreds*).
Thousands' place: 0 from 9 (*thousands*) = 9 (*thousands*).

FOURTH EXAMPLE WORKED OUT. Find the Difference of 314·82 and 95·39675

Here, the Subtrahend has *five* Decimal figures, whilst the Minuend has only *two*. Therefore, imagine three ciphers to be placed at the right-hand of Minuend. Then borrow; converting the 2 *one-hundredths* of Minuend into 1 *one-hundredth*, 9 *one-thousandths*, 9 *ten-thousandths*, and 10 *one-hundred-thousandths*.

Example completed.

314·82 *Minuend.*
95·39675 *Subtrahend.*
219·42325 *Difference.*

5 (*hun-thousths.*) from 10 (*hund-thous.*) = 5 (*hund-thousths*).
7 (*ten-thousths.*) from 9 (*ten-thous.*) = 2 (*ten-thousths*).
6 (*thousandths*) from 9 (*thousths.*) = 3 (*thousandths*).
9 (*hundredths*) from 11 (*hundredths*) = 2 (*hundredths*).
3 (*tenths*) from 7 (*tenths*) = 4 (*tenths*).
5 (*units*) from 14 (*units*) = 9 (*units*).
9 (*tens*) from 10 (*tens*) = 1 (*ten*).
0 from 2 (*hundreds*) = 2 (*hundreds*).

---

* NOTE TO TEACHER. The employment of the word "borrow" is open to objection, inasmuch as *returning* is implied but is not practised, either in the old or the new method. The quantity borrowed, being, however, *used up* on account of the deduction to which the entire Minuend is subject, is fairly accounted for. We have frequently used "convert" as a preferable term to "borrow."

EXERCISE L.

EXAMPLES FOR PRACTICE. INTEGERS AND DECIMALS; *the numbers being already arranged.*

Ex. 1·
From 3986425·
Take 2643214·

Ex. 2·
From 2496·34
Take 958·46

Ex. 3·
From 100406·
Take 36958·

Ex. 4·
From 200000·42
Take 9647·5

Ex. 5·
From 968·5
Take 89·654

Ex. 6·
From 17000·5
Take 965·475

Ex. 7·
From 96483·
Take 8976·25

Ex. 8·
From 245·
Take 16·945

Ex. 9·
From 20012·9
Take 642·

Ex. 10·
From 1000000·
Take 64·

Ex. 11·
From 2016·045
Take 98·0064

Ex. 12·
From 843·2185
Take 198·6437

Ex. 13·
From 1111·111
Take 999·999

Ex. 14·
From 74316·84
Take 69542·36

Ex. 15·
From 100000·
Take 99999·

Ex. 16·
From 600000·
Take 599999·

Ex. 17·
From 6423·584
Take 2586·965

Ex. 18·
From 421·
Take 69·8652

Ex. 19·
From 101·
Take ·9999

Ex. 20·
From 200·
Take ·0009

Ex. 21·
From 160101·2
Take 5049·

Method of proving Subtraction. If we take a certain number out of another, and then restore exactly as many as we took away, it is evident that the original Minuend will remain unaltered.

From 15· take 9· The Difference is 6·
Add 6· to 9· The Sum is 15· which is the original Minuend.
Take 6· from 15· The Remainder is 9· the original Subtrahend.

Taking the Lesser of two Numbers from the Greater leaves their Difference.

Taking the Difference of two Numbers from the Greater leaves the Lesser.

Adding the Difference of two Numbers to the Lesser gives the Greater.

Each of the last two facts furnishes us with a means of testing the correctness of Subtraction. For, we may do so either:

First. *By adding the Difference to the Subtrahend;* when, if the work be correct, the Sum will be = the Minuend.

Second. *By Subtracting the Difference from the Minuend;* when, if the work be correct, the result will be = the Subtrahend.

EXAMPLE WORKED OUT, AND PROVED. INTEGERS AND DECIMALS. Ascertain by how much 314396·85 exceeds 98514·3659

Example completed.

314396·85 *Minuend.*
98514·3659 *Subtrahend.*
215882·4841 *Difference, or Excess of Minuend over Subtrahend.*
314396·85 *Sum of Difference and Subtrahend,* = *Minuend. First Proof.*
98514·3659 *Difference of* 314396·85 *and* 215882·4841 = *Subtrahend. Second Proof.*

The above example is doubly proved. One proof will, of course, suffice. *Either* may be employed, and occasionally, for the sake of practice, *both*, as is done here.

EXERCISE LI.

Work the following examples, proving each by one of the two methods just explained. Describe every line of the process.

Ex. 1·	Ex. 2·	Ex. 3·
From 300104·95	*From* 64·	*From* 91764·548
Take 49565·6	*Take* ·549835	*Take* 9996·4257

	Ex. 4·		Ex. 5·		Ex. 6·
From	2000·	*From*	6429648·	*From*	15438·69
Take	1·6542	*Take*	2854216·	*Take*	9587·43

	Ex. 7·		Ex. 8·		Ex. 9·
From	123456·	*From*	642·5846	*From*	1·
Take	399·5	*Take*	286·9654	*Take*	·9486372

Work over again, and prove, each of the Examples in Exercise L.

Subtracting 642· from 3000· leaves 2358· This fact furnishes us at once with replies to the following *seven* questions.*

1st. What is the Difference of 3000· and 642·?
2nd. By how much does 3000· exceed 642·?
3rd. By how much is 642· less than 3000·?
4th. What must be added to 642· to make up 3000·?
5th. What must be taken from 3000· to leave 642·?
6th. What Number is less than 3000· by 642·?
7th. What Number does 3000· exceed by 642·?

Answers.

1st. The Difference of 3000· and 642· is 3000· — 642· = 2358·
2nd. The Excess of 3000· over 642· is 3000· — 642· = 2358·
3rd. The Defect of 642· from 3000· is 3000· — 642· = 2358·
4th. To increase 642· to 3000· we must add 2358·
5th. To diminish 3000· to 642· we must subtract 2358·
6th. The No. that is 642· less than 3000· is 3000· — 642· = 2358·
7th. The No. that 3000· exceeds by 642· is 3000· — 642· = 2358·

SUBTRACTION OF VULGAR FRACTIONS.

As unlike Fractions (that is to say, those which have unlike Denominators,) cannot be added together until reduced to a Common Denominator, (see pages 37· and 78·) neither can the Difference of such Fractions be ascertained without the same preparation. Indeed, it is not always obvious which of two Vulgar Fractions is the greater, unless they have either Equal Numerators, or a Common Denominator.

When two Fractions have Equal Numerators, the greater Fraction is that which has the Smaller Denominator.

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* NOTE TO TEACHER. See remarks in Preface on varying the form of questions.

It is plain that $\frac{1}{8} > \frac{1}{9}$, because if you cut a unit into *nine* equal parts each must be smaller than if you had cut the same unit into only *eight* equal parts.

$\therefore$ also $\frac{2}{8} > \frac{2}{9}$; $\frac{5}{8} > \frac{5}{12}$; $\frac{6}{7} > \frac{6}{11}$; $\frac{12}{13} > \frac{12}{19}$, &c.

When two Fractions have a Common Denominator, the greater, of course, is that which has the Greater Numerator.

$\frac{6}{15} > \frac{4}{15}$; $\frac{21}{39} > \frac{14}{39}$; $\frac{64}{253} > \frac{31}{253}$, &c.

Hence, to compare Vulgar Fractions which have not equal Numerators, they must be reduced to a Common Denominator.

To find the Difference of any two Vulgar Fractions.

First. If either be a Compound Fraction, simplify it.

Second. Reduce the two Fractions to a Common Denominator.

Third. Find the Difference of the new Numerators.

EXAMPLE WORKED OUT. Find the Difference of $\frac{14}{93}$ and $\frac{11}{65}$

The Difference of $\frac{14}{93}$ and $\frac{11}{65}$ = Difference of $\frac{910}{6045}$ and $\frac{1023}{6045} = \frac{1023}{6045} - \frac{910}{6045} = \frac{113}{6045}$

Having reduced the two Fractions to a Common Denominator, I find that $\frac{11}{65}$ (that is, $\frac{1023}{6045}$) is the greater of them. This, therefore, I place first, as Minuend.

EXERCISE LII.

Find the difference

1· Of $\frac{4}{11}$ and $\frac{15}{43}$
2· Of $\frac{316}{729}$ and $\frac{28}{35}$
3· Of $\frac{31}{45}$ and $\frac{81}{95}$
4· Of $\frac{62}{181}$ and $\frac{104}{215}$
5· Of $\frac{131}{645}$ and $\frac{13}{17}$
6· Of $\frac{1}{2}$ of $\frac{6}{9}$ and $\frac{7}{8}$ of $\frac{3}{5}$
7· Of $\frac{3}{4}$ of $\frac{7}{8}$ and $\frac{3}{5}$ of 11·
8· Of $\frac{64}{85}$ and $\frac{7}{11}$ of $\frac{6}{13}$ of 9·
9· Of $\frac{9}{16}$ and $\frac{3}{8}$ of $\frac{1}{2}$ of $\frac{4}{17}$
10· Of $\frac{6143}{9865}$ and $\frac{318}{642}$
11· Of $\frac{6}{13}$ of 29· and $\frac{8}{19}$ of $\frac{7}{12}$ of 4·
12· Of $\frac{546}{845}$ and $\frac{31}{6428}$
13· Of $\frac{1814}{4818}$ and $\frac{621}{754}$
14· Of $\frac{3652}{9646}$ and $\frac{3148}{7694}$
15· Of $\frac{6421}{9786}$ and $\frac{5437}{7965}$
16· Of $\frac{3}{4}$ of $\frac{5}{9}$ and $\frac{117}{200}$

To subtract a Vulgar Fraction from an Integer.

First. Convert *one* unit of the Minuend into an Improper Fraction of the same Denominator as the Subtrahend-fraction.

Second. From this Improper Fraction, subtract the given Fraction, if possible. If not, convert another unit, and another, and so on, until the Improper Fraction exceeds the given Fraction. Then subtract.

To subtract a Vulgar Fraction or Mixed Number from a Mixed Number; or a Mixed Number from an Integer.

First. If the Fraction of Minuend be *less* than that of Subtrahend, convert units enough to make it equal to or greater than the Subtrahend-fraction.

Second. Subtract Fraction from Fraction, and Integer from Integer.

FIRST EXAMPLE WORKED OUT. A PROPER FRACTION SUBTRACTED FROM AN INTEGER. ONE UNIT BORROWED. What is the Difference of $346\cdot$ and $\frac{29}{84}$

$$346\cdot - \frac{29}{84} = 345\frac{84}{84} - \frac{29}{84} = 345\frac{55}{84}$$

SECOND EXAMPLE WORKED OUT. AN IMPROPER FRACTION SUBTRACTED FROM AN INTEGER. TWO UNITS BORROWED. Take $\frac{64}{35}$ from $398\cdot$

$$398\cdot - \frac{64}{35} = 396\frac{70}{35} - \frac{64}{35} = 396\frac{6}{35}$$

THIRD EXAMPLE WORKED OUT. A PROPER FRACTION SUBTRACTED FROM A MIXED NUMBER. BORROWING NOT REQUIRED. Subtract $\frac{31}{49}$ from $614\frac{32}{37}$

$$614\frac{32}{37} - \frac{31}{49} = 614\frac{1568}{1813} - \frac{1147}{1813} = 614\frac{421}{1813}$$

FOURTH EXAMPLE WORKED OUT. A PROPER FRACTION SUBTRACTED FROM A MIXED NUMBER. BORROWING OF ONE UNIT REQUIRED. Take from $325\frac{6}{7}$, $\frac{14}{15}$

$$325\frac{6}{7} - \frac{14}{15} = 325\frac{90}{105} - \frac{98}{105} = 324\frac{195}{105} - \frac{98}{105} = 324\frac{97}{105}$$

FIFTH EXAMPLE WORKED OUT. A MIXED NUMBER SUBTRACTED FROM A MIXED NUMBER. NO UNIT BORROWED. Take $429\frac{3}{8}$ from $1600\frac{4}{7}$

$$1600\frac{4}{7} - 429\frac{3}{8} = 1600\frac{32}{56} - 429\frac{21}{56} = 1171\frac{11}{56}$$

SIXTH EXAMPLE WORKED OUT. A MIXED NUMBER SUBTRACTED FROM A MIXED NUMBER. BORROWING REQUIRED. From $1896\frac{5}{9}$ take $428\frac{3}{4}$

$$1896\frac{5}{9} - 428\frac{3}{4} = 1896\frac{20}{36} - 428\frac{27}{36} = 1895\frac{56}{36} - 428\frac{27}{36} = 1467\frac{29}{36}$$

SEVENTH EXAMPLE WORKED OUT. A MIXED NUMBER SUBTRACTED FROM AN INTEGER. By how much does $2000\cdot$ exceed $25\frac{11}{12}$?

$$2000\cdot - 25\frac{11}{12} = 1999\frac{12}{12} - 25\frac{11}{12} = 1974\frac{1}{12}$$

EXERCISE LIII.

EXAMPLES IN SUBTRACTION OF VULGAR FRACTIONS AND MIXED NUMBERS. ABSTRACT NUMBERS.

1· From 349· subtract $\frac{16}{45}$
2· Take $3\frac{4}{11}$ from $65\frac{1}{2}$
3· Subtract $14\frac{5}{8}$ from 100·
4· From 100· take $9\frac{3}{13}$
5· From $63\frac{1}{5}$ take $29\frac{5}{8}$
6· From $211\frac{1}{21}$ take $75\frac{4}{15}$
7· Subtract $512\frac{5}{14}$ from 2912·
8· From 3100· take $\frac{48}{13}$
9· Subtract $\frac{8}{33}$ from $37\frac{5}{11}$*
10· Subtract $21\frac{3}{5}$ from $54\frac{2}{3}$
11· Find the Difference of $\frac{3}{8}$ of $\frac{4}{7}$ of 64· and $\frac{3}{8}$ of $12\frac{1}{4}$
12· By how much does $\frac{3}{4}$ of $\frac{6}{7}$ of 45· exceed $\frac{1}{5}$ of $144\frac{1}{7}$?
13· By how much is $\frac{1}{5}$ of 42· less than $\frac{1}{4}$ of 45·?
14· What must be added to $17\frac{1}{8}$ to make up $33\frac{11}{56}$?
15· What must be taken from $364\frac{5}{14}$ to leave $95\frac{3}{8}$?
16· What number is less by $62\frac{7}{11}$ than $415\frac{1}{2}$?
17· What number does 100· exceed by $34\frac{5}{8}$?
18· Whether is $\frac{16}{45}$ of $9\frac{1}{2}$ or $\frac{14}{15}$ of $8\frac{1}{4}$ the greater, and by how much?
19· What is the Difference of $\frac{5}{12}$ of $\frac{3}{7}$ of $9\frac{1}{4}$ and $\frac{7}{8}$ of $\frac{3}{5}$ of 62·?
20· Find the Difference of $64\frac{11}{12}$ and $42\frac{13}{16}$

EXERCISE LIV.

MISCELLANEOUS EXAMPLES IN SUBTRACTION AND MULTIPLICATION; INTEGERS, VULGAR FRACTIONS, DECIMAL FRACTIONS, AND MIXED NUMBERS. ABSTRACT AND CONCRETE NUMBERS.

1· How much remained unpaid of a debt of £678·375 after £349·5 had been discharged?

2· Half-a-crown is 30· pence. How much would be left of it after paying for 3· lb of beef at $4\frac{1}{2}$ pence per lb?

3· Thomas put down a crown (that is 60· pence) to pay for 41· halfpenny oranges. What change should he have?

4· A hedger having built 36·375 chains of a hedge, how much more must he complete to make the whole amount to a mile (80· chains)?

5· If a cask, containing 35· cubic feet, weigh, when full of water, one ton, what is the weight of the cask itself? (See Exercise xxxiv. Ex. 26· p. 52·)

Weight of cask and water = 1· *Ton* = 2240· *pounds.*
Weight of 35· *c. f. of water* = 62·5 *lb.* × 35· = 2187·5 *pounds.*
Weight of cask alone = 52·5 *pounds.*

6· Whether is the 4th power of $\frac{5}{8}$ or the square of $\frac{2}{5}$ the greater, and by how much?

Raise the Fractions to the powers named. Reduce these to a Common Denominator, &c.

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* NOTE TO TEACHER. 33·, being the 3rd Multiple of 11·, the Numerator and Denominator of the Minuend Fraction need only be multiplied by 3·; both Fractions will then have the Denominator 33· and the preparation for finding their Difference will be complete.

7· Of a piece of linen measuring $67\frac{1}{2}$ yards, $\frac{3}{11}$ were damaged by damp. How long was the remaining piece?

8· Out of half-a-crown I paid for 3· twopenny loaves and some apples. I received $6\frac{3}{4}$ pence in change. What did the apples cost?

9· A coach-wheel 5· feet in circumference must revolve 32468· times in performing the journey between two certain towns. After making ten thousand revolutions, how far is the coach from its journey's end?

Whole distance to be travelled = ______ *feet.*
Distance travelled in 10,000· *revolutions* = ______ *feet.*
Distance remaining to be travelled = ______ *feet.*

10· A jar containing $\frac{5}{8}$ of a gallon of water weighs $13\frac{1}{5}$ lb. What is the weight of the jar alone? (See Exercise xxxii. Example 41· p. 48·)

11· After spending $\frac{6}{15}$ of £4· out of $\frac{3}{8}$ of £5· how much was left?

12· What quantity had been removed from a piece of cloth, originally 354·25 yards long, when only 64·875 yards were left?

13· The French weight called a *gramme* weighs 15·5 English grains. What is the Difference between 20· grammes and an English avoirdupois pound of 7000· grains?

CONTINUED SUBTRACTION.

Continued Subtraction is the taking of one Number from another as many times as it can be done.

Thus, if I take 5· from 20· I get a Remainder 15· If I now subtract 5· from this Remainder, leaving 10· and then subtract 5· from 10· leaving 5· and again subtract 5· from this Remainder, nothing will be left: and it will be said that I have continually subtracted 5· from 20·

$$\therefore\ 20\cdot - 5\cdot - 5\cdot - 5\cdot - 5\cdot = 0$$
$$\text{Also, } 20\cdot - 3\cdot - 3\cdot - 3\cdot - 3\cdot - 3\cdot - 3\cdot = 2\cdot$$

Continued Subtraction, however, is not employed for the same purpose as Single Subtraction. The proper object of Subtraction is to find the Difference of two numbers, that is to say, the Remainder after the lesser number has been withdrawn out of the greater.

But, in Continued Subtraction, we do not seek to discover a Remainder, but to ascertain how many times it is possible to subtract the smaller number from the greater.

Therefore, in the first of the two examples above, the answer sought is not the Remainder 0, but 4·, because we find that *four fives* can be taken out of *twenty*.

And in the second example, the answer is not 2·, but "6· and 2· over," for 3· can be taken out of 20· *six times*, and *two will be left.*

The number of times that the smaller number can be taken out of the larger shews, of course, two things.

First. How many of the smaller numbers the larger contains.

Second. How many of the smaller must be taken to make up the larger.

For, since, four fives can be taken out of 20·
∴ 20· contains four fives,
And ∴ four fives must be taken to make up 20·
And, since, from 20· may be taken six threes and a two,
∴ 20· contains six threes and a two,
And ∴ six threes and a two must be taken to make up 20·

If 43· units lay on the table before you, and you desired to find, *by experiment*, how many fives are contained in that number, you would naturally proceed by Continued Subtraction of 5·; whence you would learn that
43· contains eight fives and three over.

But, if you wished to ascertain, by *calculation*, how many fives can be obtained from 43· you would be more likely to adopt another method. You would recur to the Multiplication Table, to find the multiple of five nearest under 43·. You would then subtract this multiple of five from 43· to find the remainder.

For, ∵ 8· times 5· = 40·, & 43· = 40· + 3· or 43· − 40· = 3·
∴ 43· contains eight fives and three over.

Here, then, the same object is attained by Multiplication and Subtraction as we just now achieved by Continued Subtraction.

EXERCISE LV.

RECAPITULATORY QUESTIONS.

1· What is meant by a Complex Unit? Give an instance of a Complex Unit. What part of Multiplication is made clearer by using Complex Units? What is meant by Corresponding Factors? Shew by Addition that 14· × 22· = 14· × 11· × 2· or = 14· × 2· × 11·
2· Repeat Principles VI., VII., and XI.
3· In multiplying by 30· two steps must be taken. What are they?
4· Repeat Principle XII.
5· When Decimals are multiplied by Decimals, how many Decimal places are there in the Product?
6· How do the Digits of the Product fall when the multiplying Digit stands in Unit's place?
7· How are high Multiples of any Number found?
8· What are the "*Successive steps*" necessary to find the 3467th multiple of 6148·3842?

9· In working out this Example, you would prepare by putting under the horizontal line certain ciphers. Where would you put them? How many would you put?

10· Why are those ciphers necessary?

11· What is meant by "*The Back Figure*"?

12· With what multipliers may the "*Back Figure Contraction*" be employed?

13· What is "A Mixed Number"? Give instances of Mixed Numbers.

14· What is every Mixed Number equivalent to?

15· How may an Integer be expressed as a Fraction? (p. 76·)

16· How is a Mixed Number reduced to its equivalent Improper Fraction?

17· What is meant by "A Common Denominator?"

18· When do Fractions require to be reduced to a Common Denominator?

19· Repeat Principle XIII.

20· What does this principle enable us to do?

21· How are Fractions reduced to a Common Denominator?

22· How are Fractions prepared for Addition?

23· Repeat Principle V.

24· What is Subtraction? Describe its Arithmetical Symbol.

25· How is the following expression to be read, and what is its meaning?

$$78\cdot - 42\tfrac{1}{2} = 77\tfrac{2}{2} - 42\tfrac{1}{2} = 35\tfrac{1}{2}$$

26· Explain the words Minuend, Subtrahend, Remainder, and Difference.

27· Name each of those in the foregoing example.

28· When borrowing is necessary in Subtraction, whence do we borrow?

29· What is done with the quantity borrowed?

30· Why is the borrowed quantity never returned?

31· When we borrow from the *next* place to the left, what alterations are made in the value of the two Minuend Figures?

32· When one or more ciphers stand between the Minuend Figure *for* which we borrow and the Minuend Figure *from* which we borrow, what alterations are made in the values of the Minuend Figures, and of the cipher or ciphers between them?

33· As practised on the New System of working Subtraction, does "Borrowing" increase or decrease Minuend or Subtrahend?

34· On the New System, when is carrying required in Subtraction?

35· Why is "Convert" a more appropriate word than "Borrow" in Subtraction?

36· How many modes are there of proving Subtraction?

37· If I subtract $64\frac{1}{4}$ from 20000· I find the Remainder or Difference to be $19935\frac{3}{4}$. I thus find at once the answers to *seven* different questions. Write out those seven questions.

38· Write out the seven answers to them.

39· Of two Fractions having equal Numerators which is the Greater Fraction?

40· Which is the Greater of two Fractions having a Common Denominator?

41· Compare $\frac{14}{35}$ and $\frac{11}{42}$. Two features distinguish the Greater of those two Fractions. What are they?
42· What is Continued Subtraction? Give an instance of it.
43· What is the object of Continued Subtraction?
44· How many things are ascertained at once by a Continued Subtraction?
45· By what other method, beside Continued Subtraction, can you find how many nines there are in sixty-seven?
46· Of what two steps does this last method consist?
47· Which of the two methods is most natural for Experiment, and which for Calculation?

DIVISION.

Division is the distribution of any Number into lots, each containing some stated Number, or a certain portion of that stated Number.

Arithmetical Symbol. The sign of Division is a short horizontal line, having one dot immediately over its centre and other immediately under it: thus, ÷. It is called in reading, "Divided by."

Hence, the expression 30· ÷ 6· is read, "Thirty divided by six," and means that thirty units are to be distributed into lots of six units each.

Four words employed in Division must be clearly understood. They are these: Dividend, Divisor, Quotient, and Measure.

Take *thirty* units. Separate them into lots of *six* units each. There will be *five* lots each containing *six* units.

Again, take *thirty-three* units. Distribute these into lots of *six* units each. There will be *five* lots of six units each, and there will also be *one half* of such a lot.

$$\therefore\ 30{\cdot} \div 6{\cdot} = 5{\cdot} \quad \text{and} \quad 33{\cdot} \div 6{\cdot} = 5\tfrac{1}{2}$$

In the first of these two Examples,

30· is the Dividend, the Number to be distributed.
6· is the Divisor, the Number to be comprised in each lot.
5· is the Quotient, shewing the Number of lots when distributed.

And, in this example, the Divisor 6· is also called a Measure of 30·, because it exactly divides 30· into lots, leaving no remainder or part of a lot.

In the second Example,

33· is the Dividend. 6· is the Divisor. 5½ is the Quotient.

But, here, the Divisor 6· is not a Measure of the Dividend 33·, because 6· does not divide 33· into an exact number of lots, but into *five* lots and *a half* of a lot.

The **Dividend** is the Number to be divided.

The **Divisor** is the Number to be comprised in each of the lots into which the Dividend is to be distributed.

The **Quotient** is the Number of lots into which the Dividend is distributed.

A **Measure** is any exact Divisor of the Dividend.

THE FIRST RESULT OF DIVISION.

"Divide forty by five," means, as already explained, "Distribute forty units into groups of five units each." This being done, there will be eight such groups.

The first object attained by this Division is, then, the same as that of Continued Subtraction; namely, To find how many fives there are in forty.

And, it is now shewn that there are three ways of arriving at the answer to the question, "How many fives in forty?"

First. By Continually Subtracting five, and counting the subtractions.

Second. By Distributing into lots of five each, and counting the lots.

Third. By Subtracting at once the highest possible Multiple of five.

The second method, which is *Pure Division*, can only be employed *experimentally*, that is to say, when we have the real units actually before us, and can effect their distribution by hand.

The first method, Continued Subtraction, is adapted either for *experiment or calculation*.

The third method, Multiplication and Subtraction combined, is the best for *calculation*. It is, therefore, the one always used.

THE DIVISION TABLE.

In order to construct the Division Table, it is only necessary to alter the arrangement of the Multiplication Table, so that each line shall shew, not the same multiple of each of the first twelve numbers, but the first twelve multiples of the same number.

For example, the third line of the Multiplication Table shews the third multiples of 1· of 2· of 3· &c. up to 12·; but the third line of the Division Table shews the first twelve multiples of three.

To exhibit more clearly this difference of arrangement, we print, below, the fourth line of each Table, side by side.

Fourth line of Multiplication Table.	*Fourth line of Division Table.*
Four times 1· are 4·	Once 4· is 4·
Four times 2· are 8·	Twice 4· are 8·
Four times 3· are 12·	Three times 4· are 12·
Four times 4· are 16·	Four times 4· are 16·
Four times 5· are 20·	Five times 4· are 20·
Four times 6· are 24·	Six times 4· are 24·
Four times 7· are 28·	Seven times 4· are 28·
Four times 8· are 32·	Eight times 4· are 32·
Four times 9· are 36·	Nine times 4· are 36·
Four times 10· are 40·	Ten times 4· are 40·
Four times 11· are 44·	Eleven times 4· are 44·
Four times 12· are 48·	Twelve times 4· are 48·

EXERCISE LVI.

Write out and learn by heart the whole Division Table.

SIMPLE EXAMPLES IN DIVISION OF INTEGERS, TO BE WORKED OUT EITHER BY EXPERIMENT OR MENTALLY. ABSTRACT NUMBERS.

1· How many twos in 24·, 28·, 12·, 32·, 66·, 42·, 100·, 88·?

2· Into how many lots of four each can you distribute 16·, 32·, 96·, 48·, 64·, 32·, 80·, 200·, 28·, 36·, 44·, 88·, 76·?

3· What are the Quotients of 33·÷11·, 42·÷7·, 63·÷9·=?

4· What are the Quotients of 84·÷12·, 66·÷3·, 91·÷7·=?

5· How many nines can be obtained from 27·, 81·, 36·, 18·, 63·?

6· Name four *Measures* of 12·, six of 24·, and seven of 36·?

7· How many of the following Numbers is 7· a Measure of: 35·, 42·, 24·, 53·, 29·, 92·, 49·, 94·, 14·, 41·?

CONCRETE OR APPLICATE NUMBERS.

8· How many crowns in 45·s? How many in 39·s?

9· How many sixpenny loaves will 90· pence pay for?

10· A windlass took up, at each revolution, 5· feet of a well-rope. How many times must it turn to raise a bucket from the bottom to the top of a pit 140· feet deep?

11· How many paces of 3· feet must one make in walking a distance of 117· feet?

12· How many lb. of beef at 4· pence per lb. could I pay for with half-a-crown, so as to receive two pence in change?

13· Out of a bag of apples containing eleven dozen, how many boys may receive 5· each, supposing 7· to be assigned to the distributor?

14· How many gallons of water will weigh 1· cwt?

15· From a piece of whip-cord measuring 45· feet, how many top-strings, each 4·5 feet long, can be cut?

16· Out of a crown I received ten-pence change, after paying for the same number of three-penny balls and two-penny tops. How many of each did I purchase?

17· Supposing a loiterer to arrive at school, *daily*, 3· minutes too late in the forenoon, and the same in the afternoon, in how many days will he have lost one hour of his school-time?

18· At 9· pence per day, how long will a man be earning 6· shillings?

19· For how many weeks must one lay by 5· pence a week, in order to accumulate 10· shillings?

20· Tea is weighed by the Avoirdupois lb. of 16· ounces. What would 1· ounce cost at 4·s. per lb.? What at 1·s. per lb.?

21· How many lb. of tea would weigh 64· ounces?

22· How many ounces of tea at three farthings per ounce can be bought for sixpence?

23· In moving over a distance of 300· feet, a coach-wheel made 40· revolutions. What was the circumference of the wheel?

24· If I can walk 20· miles in 5· hours, how long shall I be in walking 8· miles at the same rate?

25· Exeter being 40· miles from Tavistock, how many miles per hour must one travel to make the journey between the two places in 300· minutes?

SECOND RESULT OF DIVISION.

To express one Number "in terms of" another, simply means to state what Multiple or Fraction the first Number is of the second.

Division enables us to do this. For, in order to express 90· in terms of a score, we have only to state the number of scores into which 90· can be distributed. And this number is the Quotient of 90· ÷ 20·; that is to say, $4\frac{1}{2}$.

∵ then, 90· contains 4· twenties and $\frac{1}{2}$ of twenty;
∴ 90·, expressed in terms of a score, is $4\frac{1}{2}$ *(scores)*.

In the same way:

The number of crowns in a guinea = 21· ÷ 5· = $4\frac{1}{5}$.
For, a guinea = 21·s.; and a crown = 5·s.
∴ One guinea "in terms of a crown" is $4\frac{1}{5}$ *(crowns)*.

The second result of Division, then, is, that the Quotient expresses the Dividend in terms of the Divisor.

Now, because in the expression 32· ÷ 8· = 4·, the Quotient 4· represents 4· groups each containing 8· such units as the Dividend is made up of, it is evident that the Quotient 4· has the same absolute value as the Dividend.

This is the case with every Quotient, and arises from the nature of *Pure Division*, which neither increases nor diminishes the Dividend, but merely rearranges it.*

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* NOTE TO TEACHER. This, the essential characteristic of Pure experimental Division, is too frequently lost sight of, and the consequence is utter confusion of *identity* with *coincidence*, the very broad and important distinction between which, we, by starting with the true idea of Division, are enabled to maintain with perfect precision. See page 103·

PRINCIPLE XIV. *Each unit of the Quotient is a Complex Unit, and is equivalent to the whole Divisor.**

THE QUOTIENT EXPRESSED AS A VULGAR FRACTION.

Since 32· ÷ 8· = 4·; and 4· = $\frac{32}{8}$ (see page 77·);
∴ 32· ÷ 8· = $\frac{32}{8}$. And, similarly, 8· ÷ 32· = $\frac{8}{32}$.

PRINCIPLE XV. *The Quotient is at once expressed as a Vulgar Fraction, by placing the Divisor under the Dividend.*

The Fractional form of Quotient is generally preferred, when the object is to represent the Dividend in terms of the Divisor.

When the Divisor is greater than the Dividend, the Quotient can *only* be expressed fractionally.

Thus far, we have shewn, that dividing (for instance,) 32· by 8· ought to furnish a reply to each of the following questions.

First. Into how many groups, of 8· each, can 32· be distributed?

Second. How may 32· be expressed in terms of 8·?

The fractional Quotient $\frac{32}{8}$ accurately answers both of these questions.

For, first, 32· may be distributed into 4· groups of 8· each, and those 4· groups are $\frac{32}{8}$ of any such group.

And, second, because 1· is $\frac{1}{8}$ of 8·, and 32· contains 32· ones, therefore 32· is $\frac{32}{8}$ of 8·

Similarly, if we be forming groups of 32· each, 8· units would form only $\frac{8}{32}$ of such a group.

And, unity being $\frac{1}{32}$ of 32·, 8· units in terms of 32· are $\frac{8}{32}$·

Reciprocals. When two Fractions have the Numerator of each equal to the Denominator of the other, they are called Reciprocal Fractions, or Reciprocals.

Thus, $\frac{3}{8}$ is the Reciprocal of $\frac{8}{3}$; and $\frac{8}{3}$, of $\frac{3}{8}$.

Also, 5· and $\frac{1}{5}$ are Reciprocals, as are 10· and ·1.

And, $\frac{10}{9}$ is the Reciprocal of $\frac{9}{10}$ or ·9.

Again, $3\frac{5}{8}$, being equivalent to $\frac{29}{8}$, the Reciprocal of $3\frac{5}{8}$ is $\frac{8}{29}$.

And the Compound Fraction $\frac{7}{9}$ of $\frac{4}{11}$ has for its Reciprocal $\frac{9}{7}$ of $\frac{11}{4}$.

EXERCISE LVII.

EXAMPLES OF ONE NUMBER IN TERMS OF ANOTHER; OF FRACTIONAL QUOTIENTS; AND OF RECIPROCALS.

To be worked mentally.

1· Express 29· in terms of 3·; also 35·, 44·, and 11·, in terms of 8·

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* NOTE TO TEACHER. The term "Complex Unit," (see page 62·) has its signification here extended, so as to include any kind of Divisor whatever, whether an Integer or a Fraction; any thing, in fact, which is not Unity. We shall recur to this remark.

2· Express 40·s. in florins, in half-crowns, in crowns, and in sovereigns.
3· In 11· dozens, how many scores?
4· A stone of butcher's meat weighs 8· lb. How many such in 45· lb.?
5· A stone, Avoirdupois weight, is 14· lb. How many of these is 1· cwt. equal to?
6· Express a stone of the smaller kind in terms of a stone Avoirdupois weight.

In the following twelve examples, give the Quotients, where possible, both integrally and fractionally.

7· $34\cdot \div 9\cdot = ?$
8· $38\cdot \div 6\cdot = ?$
9· $41\cdot \div 39\cdot = ?$
10· $7\cdot \div 15\cdot = ?$
11· $9\cdot \div 34\cdot = ?$
12· $4\cdot \div 23\cdot = ?$
13· $100\cdot \div 4\cdot = ?$
14· $81\cdot \div 8\cdot = ?$
15· $32\cdot \div 17\cdot = ?$
16· $21\cdot \div 11\cdot = ?$
17· $45\cdot \div 6\cdot = ?$
18· $6\cdot \div 45\cdot = ?$

What are the Reciprocals

19· Of $\frac{3}{4}$, $\frac{5}{9}$, $\frac{6}{11}$, $\frac{13}{14}$, and $\frac{144}{3}$?
20· Of ·9, ·1, ·01, ·001, ·0001 and ·24?
21· Of ·305, ·04035, ·0070504 and ·5642?
22· Of $4\frac{1}{2}$, $17\frac{3}{4}$, 1·5, 3·25, and 43·?
23· Of $\frac{4}{5}$ of $\frac{7}{11}$, $\frac{3}{4}$ of $\frac{5}{7}$ of $4\frac{1}{2}$, and $7\frac{1}{2}$ of 3·25?
24· Of $\frac{7}{12}$ of $\frac{15}{19}$ of 16·, and $\frac{9}{14}$ of ·005 of 6·75?
25· Of what fractions are the following the Reciprocals, $\frac{55}{4}$, $\frac{37}{8}$, $\frac{35}{74}$, $\frac{100}{4}$, $\frac{30}{95}$, $\frac{10000}{64}$, and $\frac{4}{11}$, of $\frac{6}{7}$?

INDIRECT RESULTS OF DIVISION.

The two results already explained of Division are directly expressed by the Quotient, and directly exhibited by actual experimental distribution of the Dividend into portions each equal to the Divisor.

But the Quotient *indirectly* announces other results than the two heretofore discussed.

To elucidate these additional meanings of the Quotient, we must first pause to consider a Vulgar Fraction in a light not yet alluded to.

Since $\frac{1}{8}$ represents one eighth part of 1·, and the eighth part of 2· is evidently twice as much as $\frac{1}{8}$ of 1·;

∴ $\frac{2}{8}$ represents one eighth part of 2·;
,, $\frac{3}{8}$,, one eighth part of 3·;
,, $\frac{7}{8}$,, one eighth part of 7·; and so on.

Here, we remark that the eighth part of an Integer is at once expressed by writing 8 below that Integer.

In like manner:

$\frac{1}{9}$ represents one ninth part of 1·;
$\frac{7}{9}$,, one ninth part of 7·, &c.

Therefore, *any one part of an Integer is at once expressed by writing the Denominator of that part under the Integer.*

This fact constitutes a second EXTENSION OF PRINCIPLE III. (see pages 23· and 24·); and might be enunciated in the same manner, by saying that

There are as many times { $\frac{1}{2}$ *in one half of any Integer*
$\frac{1}{3}$ *in one third* „ „
$\frac{1}{4}$ *in one fourth* „ „
&c. &c. } *as there are units in the whole of that Integer.*

Or thus: *Any Fraction expresses the same part of Unity as its Numerator is of its Denominator.*

For $\frac{2}{8}$ = two eighths of Unity. And 2· = two eighths of 8·

And, lastly, remarking that whilst $\frac{2}{8}$ represents one eighth of its Numerator 2·, Unity is one eighth of the Denominator 8·, we are supplied with the following important fact.

PRINCIPLE XVI. *Any Fraction is the same part of its Numerator that Unity is of its Denominator.*

APPARENT EQUALITY OF UNEQUAL QUANTITIES.

We have already seen that 32· ÷ 8· = $\frac{32}{8}$ or 4·, and it now appears that $\frac{1}{8}$ of 32· = also $\frac{32}{8}$ or 4·.

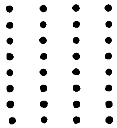
At first sight, it might hence be supposed, that the Quotient of 32· ÷ 8· = $\frac{1}{8}$ of 32·; that is to say, that $\frac{1}{8}$ of 32· = 32· ÷ 8·. This is, however, inaccurate. For, in 32· ÷ 8· = 4· or $\frac{32}{8}$, the Digit 4 represents four Complex Units, or groups, (of eight simple units in each,) and the Fractional Quotient $\frac{32}{8}$ represents $\frac{32}{8}$ of a group of eight. See PRINCIPLE XIV.

Whilst, in $\frac{1}{8}$ of 32· = 4· or $\frac{32}{8}$, the Digit 4 represents four simple units, and the Fraction $\frac{32}{8}$ represents $\frac{32}{8}$ of a simple unit.

To make this still more clear, take thirty-two units and *divide them by eight.*

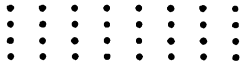
Take other thirty-two units, and distribute these into *eight equal lots.*

The first thirty-two units will be disposed thus:



The vertical lines shew that there are four eights in thirty-two, or that 32· ÷ 8· = 4· *groups of eight.*

The second thirty-two units will be disposed thus:



The vertical lines shew that $\frac{1}{8}$ of 32· = 4· *simple units.*

Not only are $\frac{1}{8}$ of 32· and the Quotient of 32· ÷ 8· unequal quantities, but they are arrived at by dissimilar processes, as will be seen by examining the steps of the foregoing distributions of thirty-two into *eights*, and into *eighths of thirty-two.*

To divide thirty-two by eight, it is only necessary to take the units, eight at a time, and lay each eight aside in one separate group. The division is completed by this simple distribution.

But this being done, the eighth part of thirty-two (which we know to be four simple units,) is no-where exhibited, for the thirty-two units are disposed into *eights*, and not into *fours.*

To obtain one eighth of thirty-two, two steps are necessary:

First. Distribute thirty-two into eights.

Second. Collect a unit from each eight.

This, plainly, forms a group of four units, and exhibits one eighth of thirty-two.

To arrange the whole thirty-two into eight equal groups, the second of the two steps must be performed eight times.

The process, as above described, consists of:

First. *A Pure Division by eight.*

Second. *A Subtraction and an Addition.*

The thing may be otherwise effected, but this suffices to shew, that in order to find eighths of any Integer, something more is requisite than merely dividing by eight.

The diversity of the two processes is also shewn in the difference of treatment of any remaining units.

In dividing 35· by 4·, the 3· remaining units undergo no further division, but are simply laid aside and counted as three fourths of four. And the result of the whole operation is that 35· ÷ 4· = $8\frac{3}{4}$ *groups of four.*

But, in finding the fourths of 35·, we first obtain (by the two steps described above,) four equal groups of eight in each; and, then, we cut each of the 3· remaining units into $\frac{4}{4}$, and, from *each* unit, so cut, we give $\frac{1}{4}$ of a unit to each of the four equal groups. This increases, of course, each of those groups by $\frac{3}{4}$ of a unit. And the result of the entire process is that we have our 35· units at last arranged into *four equal* groups, each containing $8\frac{3}{4}$ *units.*

∴ 35· ÷ 4· = $8\frac{3}{4}$ *groups of four*, or $\frac{35}{4}$ *of a group of four.*
But $\frac{1}{4}$ of 35· = $8\frac{3}{4}$ *units*, or $\frac{35}{4}$ *of a unit.*

The Quotient of 32· ÷ 8·, and $\frac{1}{8}$ of 32· are not equal quantities, but are unequal quantities similarly expressed.

And the first indirect result of Division is this:

The figures of the Quotient express also the number of units in that Fraction of the Dividend which has Unity for its Numerator and the Divisor for its Denominator. In other words: the figures of the Quotient represent also the number of units contained in that Fraction of the Dividend which is expressed by the *Reciprocal* of the Divisor.

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* NOTE TO TEACHER. This, and many similar examples should be performed *experimentally* by the pupil, with pebbles, peas, strips of paper, or any convenient units. Nothing exhibits so clearly the true nature of arithmetical *operations* as actual manipulation of real units, and the practice should be *constantly* maintained throughout the whole elementary course.

IDENTICAL, EQUIVALENT, AND COINCIDENT NUMBERS.

Expressions which represent the same quantity of the same kind of units are **Identical Expressions.**

Thus: 64· and 32· + 15· + 17· are identical expressions. They mean *exactly* the same thing. If 32· units and 15· units and 17· units of the same kind be laid on the table before you, there will be just 64· units of that kind.

Also, 64· is identical with 32· × 2·; for 32· × 2· means two thirty-twos, or sixty-four.

Unequal Numbers which represent exactly the same absolute quantity, but in unlike units or fractions, are **Equivalent Expressions.**

Hence, $\frac{5}{5}$ and $\frac{10}{10}$ are equivalent expressions, for they represent the same value. But they are not identical, because the first speaks of five pieces of a certain size (fifths), and the second speaks of twice as many pieces of half that size (tenths).

Also, 20· shillings are equivalent to 1· sovereign; and $\frac{19}{4}$ to $4\frac{3}{4}$, &c.

When two unequal quantities can be expressed by the same digits similarly placed, the numbers representing those two quantities are called **Coincident Expressions.**

16· ÷ 8· and $\frac{1}{8}$ of 16· are, then, coincident expressions, for each can be represented by the same Digit (namely, 2) in the first place of Integers.

$$\because\ 16\cdot \div 8\cdot = 2\cdot, \text{ and } \tfrac{1}{8} \text{ of } 16\cdot = 2\cdot$$

But the two expressions are neither *identical* nor *equivalent*, for, (as already amply explained,) the Digit 2 in the first expression does not represent the same quantity as the Digit 2 in the second expression.

$$16\cdot \div 8\cdot = 2\cdot \textit{ groups of eight; } \text{ and, } \tfrac{1}{8} \text{ of } 16\cdot = 2\cdot \textit{ units.}$$

The sign of equality may correctly be employed between Numbers which are either identical or equivalent; but it cannot with propriety be inserted between two expressions which are only coincident. There is evident inaccuracy in saying that 16· ÷ 4· = $\frac{1}{4}$ of 16·; or, that, 32· ÷ 8· = $\frac{1}{8}$ of 32· (See p. 102·)

A sign of Numerical coincidence is, therefore, requisite.

Arithmetical Symbol. To indicate coincidence between Numbers, we shall employ the sign of equality with a dot over it, thus, $\doteq$, and shall call it in reading, "coincides with," or, "is coincident with."

Hence, the expression $\frac{1}{8}$ of $32\cdot \doteq 32\cdot \div 8\cdot$, is to be read,
"One eighth of $32\cdot$ coincides with $32\cdot \div 8\cdot$;"
or, "One eighth of $32\cdot$ is coincident with $32\cdot \div 8\cdot$;"
and means that one eighth of thirty-two and the number of eights in thirty-two are similarly expressed in figures; or, that there are *as many units* in one eighth of thirty-two *as there are eights* in $32\cdot$. PRINCIPLE III., pp. $23\cdot$, $24\cdot$, and $101\cdot$.

COINCIDENT RESULTS OF DIVISION AND MULTIPLICATION.

An Integer divided by an Integer.

Since $32\cdot \div 8\cdot \doteq \frac{1}{8}$ of $32\cdot$, and $\frac{1}{8}$ of $32\cdot = 32\cdot \times \frac{1}{8}$, (PRINCIPLES IX. and VI.)

$$\therefore\ 32\cdot \div 8\cdot \doteq 32\cdot \times \tfrac{1}{8}.$$

But, $\frac{1}{8}$ is the Reciprocal of $8\cdot$; therefore, *Dividing an Integer by an Integer produces the same figures as Multiplying by the Reciprocal of the Divisor.*

A Fraction divided by an Integer.

It has been repeatedly explained, that Division is the distributing of the Dividend into lots, each equal to the Divisor.

What then is meant, when we are required to divide a *Proper Fraction by an Integer*, as, for example, $\frac{1}{6} \div 5\cdot$?

It means; form $\frac{1}{6}$ of a unit into as much of a group of $5\cdot$ as possible. That is to say, *ascertain what part $\frac{1}{6}$ of unity is of $5\cdot$*; or, express $\frac{1}{6}$ of a unit "in terms of" $5\cdot$

$\therefore\ \frac{1}{6} \div 5\cdot$ means, "Find what part $\frac{1}{6}$ of a unit is of $5\cdot$"

Now, since Unity $= \frac{1}{5}$ of $5\cdot$, and $\frac{1}{6} = \frac{1}{6}$ of Unity;

$\therefore\ \frac{1}{6} = \frac{1}{6}$ of $\frac{1}{5}$ (of $5\cdot$) $= \frac{1}{6} \times \frac{1}{5}$ (of $5\cdot$) $= \frac{1}{30}$ (of $5\cdot$)

But $\frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$ (of unity)

$$\therefore\ \tfrac{1}{6} \div 5\cdot \doteq \tfrac{1}{6} \times \tfrac{1}{5}.$$

But, $\frac{1}{5}$ is the Reciprocal of $5\cdot$; therefore, *Dividing a Fraction by an Integer produces the same figures as Multiplying by the Reciprocal of the Divisor.*

An Integer or a Fraction divided by a Fraction.

Since Division is the distribution of the Dividend into lots equal to the Divisor, (or into a part of such lot,) it is evident, that in Concrete Numbers, the Dividend and Divisor must represent units of the same kind.

It is impossible to distribute units of length (as feet, yards, *miles, &c.)* into lots of units of weight, or units of time into lots of coins. Such expressions as £184· ÷ 16· men are constantly employed, but they are absurd. £184· ÷ 16· men, means, "*Arrange 184· sovereigns into lots each consisting of 16· men!*"

Further, the Divisor and Dividend must not only express units of the same kind, but units of the same value.

For example: because pounds sterling and pence are units of the same kind, £50· can be divided by 4½d.; but the Dividend and the Divisor must be first reduced to units of the same name and value. Either we may say,

£50· ÷ 4½d. = 1000· *shillings* ÷ ⅜ *of a shilling;*
or = 12000· *pence* ÷ 4½ *pence;*
or = 24000· *half-pence* ÷ 9· *half-pence.*

It is evidently impossible to distribute fifty golden coins into lots of silver or copper coins. The sovereigns must, therefore, be exchanged for their value of copper pence and half pence, or of silver groats and copper half pence, before their division into sums of 4½ pence each can be effected.

Now, inasmuch as the Denominator of a Fraction is merely the namer, pointing out the value of each part expressed by the Numerator, Fractions with a Common Denominator may be regarded as similar units, and those with different Denominators as unlike units.

And, since the Quotient produced by dividing one Integer by another is at once expressed by placing Divisor under Dividend (PRINCIPLE XV.); therefore, when Dividend and Divisor are both Fractions, and have a Common Denominator, the Quotient is expressed fractionally by placing Numerator of Divisor beneath Numerator of Dividend.

$$\therefore\ \tfrac{44}{54} \div \tfrac{11}{54} = \tfrac{44}{11} \text{ (of } \tfrac{11}{54}\text{)}.$$

That is to say, $\frac{44}{54}$ may be distributed into $\frac{44}{11}$ of a lot of $\frac{11}{54}$;
or, into 4· such lots, for $\frac{44}{11} = 4\cdot$

And, $5\cdot \div \frac{3}{8} = \frac{40}{8} \div \frac{3}{8} \doteq 40\cdot \div 3\cdot \doteq \frac{40}{3}$.
Again, $12\cdot \div \frac{3}{4} = \frac{48}{4} \div \frac{3}{4} \doteq 48\cdot \div 3. \doteq 16\cdot$

For, to divide, *experimentally*, 12· by ¼, we first cut sixteen units into *forty-eight fourths;* and, then, distribute these singly.

And to divide 12· by ¾, we distribute forty-eight fourths into sixteen lots, each containing three fourths.*

$$\therefore\ 12\cdot \div \tfrac{1}{4} = \tfrac{48}{4} \div \tfrac{1}{4} = 48 \text{ (\textit{lots of} } \tfrac{1}{4}\text{)};$$
$$\text{But, } 12\cdot \times 4\cdot = 48\cdot \text{ (\textit{units}.)}$$
$$\therefore\ 12\cdot \div \tfrac{1}{4} \doteq 12\cdot \times 4\cdot$$
$$\text{Also, } 12\cdot \div \tfrac{3}{4} = \tfrac{48}{4} \div \tfrac{3}{4} = 16\cdot \text{ (\textit{lots of} } \tfrac{3}{4}\text{)};$$
$$\text{But } 12\cdot \times \tfrac{4}{3} = \tfrac{4}{3} \text{ of } 12\cdot = 16\cdot \text{ (\textit{units}.)}$$
$$\therefore\ 12\cdot \div \tfrac{3}{4} \doteq 12\cdot \times \tfrac{4}{3}.$$

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* NOTE TO TEACHER. This is the only method experimentally practicable. The manner in which the ordinary *contracted process* is derived from it will be hereafter explained.

But, $\frac{4}{3}$ is the Reciprocal of $\frac{3}{4}$; therefore, *Dividing an Integer by a Fraction produces the same figures as Multiplying by the Reciprocal of the Divisor.*

$$\text{Again, } \tfrac{6}{7} \div \tfrac{5}{8} = \tfrac{48}{56} \div \tfrac{35}{56} = \tfrac{48}{35}\ (\textit{of } \tfrac{5}{8})$$
$$\text{But, } \tfrac{6}{7} \times \tfrac{8}{5} = \tfrac{48}{35}\ (\textit{of unity})$$
$$\therefore\ \tfrac{6}{7} \div \tfrac{5}{8} \doteq \tfrac{6}{7} \times \tfrac{8}{5}$$

But, $\frac{8}{5}$ is the Reciprocal of $\frac{5}{8}$; hence, *Dividing a Fraction by a Fraction produces the same figures as Multiplying by the Reciprocal of that Fraction.*[*]

Now, collecting the examples just examined, we find that

$$32^{\cdot} \div 8^{\cdot} \doteq 32^{\cdot} \times \tfrac{1}{8}, \text{ or, } \tfrac{1}{8} \text{ of } 32^{\cdot};$$
$$\tfrac{1}{6} \div 5^{\cdot} \doteq \tfrac{1}{6} \times \tfrac{1}{5}, \text{ or, } \tfrac{1}{5} \text{ of } \tfrac{1}{6};$$
$$12^{\cdot} \div \tfrac{3}{4} \doteq 12^{\cdot} \times \tfrac{4}{3}, \text{ or, } \tfrac{4}{3} \text{ of } 12^{\cdot};$$
$$\tfrac{6}{7} \div \tfrac{5}{8} \doteq \tfrac{6}{7} \times \tfrac{8}{5}, \text{ or, } \tfrac{8}{5} \text{ of } \tfrac{6}{7}.$$

PRINCIPLE XVII. *Dividing by any Number produces the same figures as Multiplying by the Reciprocal of that Number.*[†]

Therefore, to find either "How many eights there are in fifty-six?" or "What is one eighth of fifty-six?" we must divide 56· by 8·, or multiply 56· by $\frac{1}{8}$, and so on with other Divisors.

RELATION OF DIVIDEND AND QUOTIENT.

Since the Quotient expresses how many lots of the same magnitude as the Divisor can be procured from the Dividend, it is evident that, if two unequal Dividends be divided by the same Divisor, the greater Dividend will yield the greater Quotient.

A large bagful of nuts can be divided into a greater number of pints of nuts than a smaller bagful can.

Not only so, but if the one Dividend be double, or triple, or any Multiple whatever of the second, the Quotient derived from the first by any Divisor will be the same Multiple of the Quotient derived from the second by the same Divisor.

Ten cwts. of sugar can be made up into exactly *ten times as many* half pounds as *one* cwt. can.

Also, *twice* 20· contain *twice as many* fives as once 20· contains.

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[*] NOTE TO TEACHER. This is only here shewn *to be the fact.* The *reason of the coincidence* will be explained shortly.

[†] NOTE TO TEACHER. This is one of the most important and comprehensive principles we have yet established. It is the key to every kind of Division, whether of Integers, of Vulgar Fractions, or of Decimal Fractions, *and* is of especial utility in Concrete Numbers.

PRINCIPLE XVIII. *The Dividend and the Quotient increase and decrease together. Multiplying the Dividend (before Dividing,) also multiplies the Quotient: and previously Dividing the Dividend also divides the Quotient of the subsequent Division.*

Hence, ∵	6·	÷ 2· =	3·	= No. of twos in	6·,
∴	60·	÷ 2· =	30·	= No. of twos in	60·,
	600·	÷ 2· =	300·	= No. of twos in	600·,
	6000·	÷ 2· =	3000·	= No. of twos in	6000·,
	&c.	&c.		&c.	&c.
Also	·6	÷ 2· =	·3	= Part of two in	·6,
	·06	÷ 2· =	·03	= Part of two in	·06,
	·006	÷ 2· =	·003	= Part of two in	·006,
	&c.	&c.		&c.	&c.

It is, then, seen, that, *In Dividing by Units, each Figure of Quotient falls under corresponding Figure of Dividend.*

Suppose, then, that we be required to find how many twos are contained in 6666·666.

To discover this, we must divide 6666·666 by 2·, an operation including the whole of the seven Divisions effected above.

For,	No. of twos in	6000·	= 6000·	÷ 2· =	3000·
	" " " in	600·	= 600·	÷ 2· =	300·
	" " " in	60·	= 60·	÷ 2· =	30·
	" " " in	6·	= 6·	÷ 2· =	3·
	And Fraction of two in	·6	= ·6	÷ 2· =	·3
	" " " in	·06	= ·06	÷ 2· =	·03
	" " " in	·006	= ·006	÷ 2· =	·003
∴	No. of twos in	6666·666	= 6666·666	÷ 2· =	3333·333

Here, we exhibit separately the seven Quotients derived from the seven figures of the Dividend. We afterwards collect the seven Quotients into one *Sum*, which is the Quotient of the whole Dividend 6666·666 ÷ 2·

But the thing may be much more compendiously done, by putting each Quotient-figure, as soon as we find it, into its proper place in the general Quotient.

Divisor. Dividend.
2·) 6666·666
3333·333 Quotient = *No. of twos in* 6666·666
≐ *No. of units in ½ of Dividend.*

In like manner, we find how many threes are contained in 693·069, or how many units in one third of that Number, by Dividing 693·069 by 3·.

Divisor. Dividend.

$$3\cdot\overline{)\,693\cdot069}$$
$$\underline{\underline{231\cdot023}}\ \text{Quotient} = \textit{No. of threes in } 693\cdot069$$
$$\doteq \textit{No. of units in } \tfrac{1}{3} \textit{ of } 693\cdot069\cdot$$

The foregoing examples are of the most simple kind that can occur in Division, because in each, every Dividend-figure is an exact Multiple of the Divisor.

When this is not the case, (as it very rarely is,) each *one* that is *over* from any Dividend-figure will be carried as *ten* to next place to the right, added in with whatever figure stands in that place, and then divided. See Second Rule for carrying, page 26·

$700\cdot \div 2\cdot = 350\cdot$

For $700\cdot = 600\cdot + 100\cdot = 6$ hundreds and 10 tens.

Now 6 hunds. $\div\ 2\cdot = 300\cdot$

And 10 tens $\div\ 2\cdot = 50\cdot$

$\therefore\ 700\cdot \div 2\cdot = 350\cdot$ as above.

Similarly, $750\cdot \div 2\cdot = 375\cdot$

For $750\cdot = 6$ hundreds $+$ 14 tens $+$ 10 units

And 6 hunds. $\div\ 2\cdot = 300\cdot$

14 tens $\div\ 2\cdot = 70\cdot$

10 units $\div\ 2\cdot = 5\cdot$

$\therefore\ 750\cdot \div 2\cdot = 375\cdot$ as before.

In dividing an Integer by an Integer, if any units remain, they may be treated in either of two ways.

First. The Quotient of the remaining units may be expressed as a Vulgar Fraction, by simply writing under them the Divisor as a Denominator.

$$8129\cdot \div 5\cdot = 1625\tfrac{4}{5}.$$

Where, finding that 4· units remain, I, at once, express, in a fractional form, the Quotient of 4· ÷ 5· by placing the Divisor 5· below the Remainder 4· See PRINCIPLE XV.

Second. The remaining units may be carried, as so many tens (*of tenths*) to first place of Decimals, and there divided.

$$8129\cdot \div 5\cdot = 1625\cdot8^{*}$$

Where I carry the 4· remaining units to the first place of Decimals, as 40 (*tenth-parts*), and then divide by 5·. See PRINCIPLE IV., and Second Rule for carrying, page 26·

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* NOTE TO TEACHER. The pupil here learns, *incidentally*, how to Reduce a Vulgar Fraction to a Decimal. This operation will, however, be duly *discussed in its proper place.*

DIVISION OF NUMBERS TOO LARGE TO BE WORKED MENTALLY.

SHORT DIVISION. *To Divide Integers or Decimals by any Integer less than* 13·.

First. Write the Divisor on the left of the Dividend, but separated from it by a vertical curve; thus,).

Second. Draw a horizontal line below Dividend, and place under this line a Decimal Point, directly beneath that of Dividend.

Third. Put also, under the horizontal line, *a cipher in every place from Decimal Point to left-hand significant figure of Dividend.*[*]

Fourth. Commencing at left-hand, Divide each figure of Dividend by Divisor, putting down each Quotient-figure under the Dividend-figure from which it is obtained, and carrying every *one*, which may be over in any place, to next right-hand place as a *ten*. (PRINCIPLE IV.)

Fifth. If Dividend contain Integers only, express Quotient of remaining units, either as a Vulgar Fraction or as a Decimal Fraction.[†] (See p. 108·)

Sixth. Efface useless ciphers. (PRINCIPLE II.)

PRINCIPLE XIV., p. 99·, shews that "*Each unit of Quotient is a Complex Unit, and is equivalent to the whole Divisor*. Consequently,

Every *ten* of Quotient = 10· times Divisor.
Every *hundred* of Quotient = 100· times Divisor.
Every *thousand* of Quotient = 1000· times Divisor.
&c. &c. &c.

This explains why, in the following example, although there are many eights in 6000·, yet we have no Quotient-figure in thousands' place. For, Digit 1 in that place would represent 1000· *eights*, which are *not* contained in 6000·.

FIRST EXAMPLE WORKED OUT. INTEGER DIVIDED BY INTEGER.

Find how many *eights* are contained in 6143·, and how many *units* in *one eighth* of 6143·.

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[*] NOTES TO TEACHER. The reason for this proceeding is stated in note to page 33·.

[†] The pupil should be allowed to discover for himself the limited possibility of the latter method. His observations on this point will prepare him for the discussion of Interminate Decimals.

No. of eights in 6143· ≑ No. of units in $\frac{1}{8}$ of 6143·. (PRIN. III.)

Example prepared.	Example completed.	
8·)6143·	Divisor 8·)6143·	or, 8·)6143·
0000·	Quotient = $767\frac{7}{8}$	Quotient = 767·875

Eights in 6 (*thousds.*)	= 0 (*thous. of eights,*)	and 6 (*thous.*)	over.		Carry 6 (0 *hunds.*)
Eights in 61 (*hunds.*)	= 7 (*hunds. of eights,*)	and 5 (*hunds.*)	over.	Set down 7	Carry 5 (0 *tens.*)
Eights in 54 (*tens*)	= 6 (*tens of eights,*)	and 6 (*tens*)	over.	Set down 6	Carry 6 (0 *units.*)
Eights in 63 (*units*)	= 7 (*eights,*)	and 7 (*units*)	over.	Set down 7	Carry 7 (*units.*)
Eights in 7 (*units*)	= $\frac{7}{8}$ (*of eight,*)			Set down $\frac{7}{8}$.	

This gives us Quotient as Integer and Vulgar Fraction.

To obtain it as Integer and Decimal, we carry, from fourth line of above work, 70 (*tenths*), and say,

Eights in 70 (*tenths*)	= 8 (*tenths of 8·*)	and 6 (*tenths*)	over.	Set down 8	Carry 7 (0 *hundths.*)
Eights in 60 (*hundths.*)	= 7 (*hundths. of 8·*)	and 4 (*hundths.*)	over.	Set down 7	Carry 4 (0 *thousths.*)
Eights in 40 (*thousths.*)	= 5 (*thousths. of 8·*)			Set down 5	

∴ No. of eights in 6143· = $767\frac{7}{8}$ *eights*, or = 767·875 *eights*.
And No. of units in $\frac{1}{8}$ of 6143· = $767\frac{7}{8}$ *units*, or = 767·875 *units*.

SECOND EXAMPLE WORKED OUT. DECIMAL DIVIDED BY INTEGER. Find what part of 4· is contained in ·0004954 of a unit; and, also, what part of a unit is equal to $\frac{1}{4}$ of ·0004954 of a unit.

Example prepared.	Example completed.
4·)·0004954	4·)·0004954
·0000	·00012385 of 4· in ·0004954 of 1·; ≑ $\frac{1}{4}$ of ·0004954 of 1·.

EXERCISE LVIII.

EXAMPLES IN DIVISION OF INTEGERS AND DECIMALS, BY INTEGERS LESS THAN 13·. ABSTRACT NUMBERS.

Observe: First. *When Dividend contains no Decimals, Quotient of remaining units is to be given both as a Vulgar Fraction and as a Decimal.*

Second. *After working each Example, write against the Quotient (as in above examples) its true value, and its coincident value.*

Ex. 1·	Ex. 2·	Ex. 3·	Ex. 4·
2·)8431684·	2·)541·36843	2·)·0005463	3·)421368·
Ex. 5·	Ex. 6·	Ex. 7·	Ex. 8·
3·)650·1762	3·)·0020487	4·)7139658·	4·)735642·
Ex. 9·	Ex. 10·	Ex. 11·	Ex. 12·
4·)·000054	5·)4134000·	6·)1·2039	6·)8473614·

Ex. 13·	Ex. 14·	Ex. 15·	Ex. 16·
5·)·134627	6·)·0010032	5·)31·0924	7·)34163584·
Ex. 17·	Ex. 18·	Ex. 19·	Ex 20·
7·)314·80064	7·)·1843254	8·)540000·	8·)1·084536
Ex. 21·	Ex. 22·	Ex. 23·	Ex. 24·
8·)·0789205	9·)6318·423	9·)300000·6	9·)·0061308
Ex. 25·	Ex. 26·	Ex. 27·	Ex. 28·
10·)7438284·	10·)415·3435	10·)·07958	11·)·6958435
Ex. 29·	Ex. 30·	Ex. 31·	Ex. 32·
11·)352·253	11·)·00682187	12·)9413865·	12·)1439·655

Express as Decimal Fractions the Quotients

33· Of 1· ÷ 2·
34· Of 1· ÷ 4·
35· Of 1· ÷ 5·
36· Of 1· ÷ 8·
37· Of 1· ÷ 10·
38· Of 2· ÷ 4·
39· Of 2· ÷ 5·
40· Of 2· ÷ 8·
41· Of 2· ÷ 10·
42· Of 3· ÷ 4·
43· Of 3· ÷ 5·
44· Of 3· ÷ 6·
45· Of 3· ÷ 8·
46· Of 3· ÷ 12·
47· Of 4· ÷ 5·
48· Of 4· ÷ 8·
49· Of 4· ÷ 10·
50· Of 5· ÷ 8·
51· Of 5· ÷ 10·
52· Of 6· ÷ 8·
53· Of 6· ÷ 10·
54· Of 6· ÷ 12·
55· Of 7· ÷ 8·
56· Of 7· ÷ 10·
57· Of 8· ÷ 10·
58· Of 9· ÷ 10·
59· Of 9· ÷ 12·
60· Of 13· ÷ 8·

REDUCTION OF AN IMPROPER FRACTION TO ITS EQUIVALENT WHOLE OR MIXED NUMBER.

Since $\frac{5}{5}$ = 1·, there are as many Units in $\frac{59}{5}$ as there are fives in 59·.

∴ No. of Units in $\frac{59}{5}$ (≐ 59· ÷ 5· and) = 11$\frac{4}{5}$ Units.

To Reduce an Improper Fraction to a whole or Mixed Number.

Divide Numerator by Denominator.

EXERCISE LIX.

EXAMPLES IN REDUCTION, TO WHOLE OR MIXED NUMBERS, OF IMPROPER FRACTIONS HAVING DENOMINATOR LESS THAN 13·.

1· How many units in $\frac{645}{9}$, $\frac{3004}{11}$, $\frac{632089}{9}$, $\frac{3000}{12}$?
2· Reduce to whole or Mixed Numbers, $\frac{614}{12}$, $\frac{35}{9}$, $\frac{84}{11}$, $\frac{63}{7}$, $\frac{95}{5}$.
3· Express as a whole or Mixed Number, the 614th Multiple of $\frac{3}{7}$ of $\frac{4}{5}$ of 19$\frac{1}{2}$·

Reduce to whole or Mixed Numbers,

4· One third of $\frac{1}{2}$ of $7\frac{1}{18}$ of 63·
5· $\frac{7}{18} \times \frac{4}{13} \times \frac{19}{25}$ of $645\frac{1}{2}$
6· The Product of $\frac{14}{15}$ and 6·25
7· The Cube of $\frac{6}{7}$ times $3\frac{1}{2}$
8· The 613th Multiple of $6\frac{4}{5}$
9· The Sum of $\frac{8}{9}$, $\frac{4}{7}$, $\frac{5}{18}$ and $\frac{7}{8}$
10· The Difference of $\frac{514}{7}$ & $\frac{13}{45}$
11· The Square of the 18th Multiple of $\frac{7}{11}$
12· Five eighths of $653\frac{1}{5}$
13· Seven twelfths of $5391\frac{6}{13}$

Method of Proof of Division. When we divide any Dividend by 5· the Quotient expresses (secondarily,) the number of Units in $\frac{1}{5}$ of the Dividend.

Therefore, 5· times that Quotient will re-produce the Dividend; for 5· times $\frac{1}{5}$ of anything is the whole of that thing.

∴ *Dividend = Quotient × Divisor.*

To prove Division, therefore, we multiply the Quotient by the Divisor, and if the work be correct, the Product will be equal to the Dividend.

EXAMPLE WORKED OUT AND PROVED. Divide 64332· by 5·, and prove the work.

Example completed, by first method.

Divisor 5·)64332· *Dividend.*
$12866\frac{2}{5}$ *Quotient ≐ $\frac{1}{5}$ of Dividend.*
5· *Divisor, employed as Multiplier.*
64332· = 5· *times Quotient = Dividend. Proof.*

Example completed, by second method.

Divisor 5·)64332· *Dividend.*
12866·4 *Quotient ≐ $\frac{1}{5}$ of Dividend.*
5·
64332· = 5· *times Quotient = Dividend. Proof.*

PRINCIPLE XIX. *The Quotient and Divisor are Corresponding Factors of the Dividend.*

Corollary I. *Hence, a Fraction multiplied by its Denominator gives its Numerator.*

For $\frac{7}{8}$ is Quotient of 7· ÷ 8· (PRINCIPLE XV.);
∴ $\frac{7}{8} \times 8\cdot = 7\cdot$ (PRINCIPLE XIX.)

This may be shewn also in other two ways.

First. ∵ $\frac{7}{8} = \frac{1}{8}$ of 7· (PRINCIPLE XVI.); ∴ $\frac{7}{8} \times 8\cdot = 8\cdot$ times $\frac{1}{8}$ of 7· = 7·.

For 8· times $\frac{1}{8}$ of anything = the whole of that thing.

Second. $\frac{7}{8} \times 8\cdot = \frac{56}{8}$, and $\frac{56}{8} = 7\cdot$ (EXERCISE LIX.)
∴ $\frac{7}{8} \times 8\cdot = 7\cdot$ as before.

The fact enunciated in this Corollary to Principle XIX. becomes serviceable whenever we have to multiply any Mixed Number by the Denominator of its Vulgar Fraction.

For instance, in proving the foregoing Example in Division, (as worked by first method,) the first step is to multiply $\frac{2}{5}$ by 5·. But, knowing that "*A Fraction multiplied by its Denominator yields for Product its Numerator*," we have nothing further to do with this Fraction ($\frac{2}{5}$) than simply to carry its Numerator (2·).

Now, because 642· contains 107· sixes, 642· ÷ 6· = 107·
∴ 642· = 107· times 6·
But 107· times 6· = 6· times 107· PRINCIPLE VI.
∴ 642· ÷ 107· = 6·, whilst 642· ÷ 6· = 107·

Here, 6· is Quotient when 107· is Divisor, and 107· is Quotient when 6· is Divisor.

Corollary II. to PRINCIPLE XIX. *The Divisor and Quotient may exchange places, if Dividend be retained unaltered.*

Compare this with Principle VI., page 42·.

Seeing, then, that Dividend is Product of Divisor and Quotient; that Dividend ÷ Quotient = Divisor;
and that Dividend ÷ Divisor = Quotient, we say,

PRINCIPLE XX. *Any Number divided by one of its Factors gives its Corresponding Factor.**

EXERCISE LX.

EXAMPLES IN DIVISION OF INTEGERS AND DECIMALS BY INTEGERS LESS THAN 13·.

Every example to be proved.

1· How many eights are contained in two *million*, seven hundred and thirty-seven *thousand*, three hundred and forty-six, *and* seventy-eight *one-hundredth parts*?
2· What Number is contained eight times in 2737346·78?
3· What is the twelfth part of 772317·6?
4· What part of 111136· is 8·?
5· What Number must 7· be multiplied by to produce 241797·78?
6· Find the Factor of 37638·172 corresponding with 11·
7· Of what Number is ·00079528 the 4th Multiple?
8· Of what Number is 69463419·7972 the 11th Multiple?
9· What Multiple of 12· is 43421148·?
10· Find what Number which, being multiplied by 10·, will produce 61483·5429.

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* NOTE TO TEACHER. As Principles IX. and VII. will enable us to establish the truth that, "*When four Numbers are Proportionals, the Product of the Extremes is equal to the Product of the Means*," so by the aid of Principle XX. we shall be enabled to deduce the Rule for finding a Fourth Proportional to three given Numbers.

11· *Six* is one Factor of 8431584·, what is the Corresponding Factor? How do you prove it?
12· What Multiple is 75884256· of 9·?
13· By what No. must 5· be multiplied to produce 170·792315?
14· What Multiple of 3· is 18944569·0356?
15· How many times 2· make 416859·634?
16· How many *nines* must be added together to make 3618729·?
17· What Multiple of 5· are *nine times* 314725·?
18· Find *half the Sum* of 714·38, 3140·2, and twenty thousand and twelve.
19· Find *half the Sum* and *half the Difference* of 914· and 419·
20· Add *half the Sum* of 746· and 343· to *half their Difference*. Subtract *half the Difference* from *half the Sum*.
21· *Half the Sum* of two Nos. + *Half their Diff.* = the No.?
Half the Sum of two Nos. — *Half their Diff.* = the No.?
22· The *Sum* of two Nos. is 15307· and the *Difference* is 3021·; what are those Nos.?
23· The 4th Multiple of the *Sum* of two Nos. is 32172·, and the 9th Multiple of their *Difference* is 66123·; find the Nos.
24· Which is the greater, and by how much, The third part of the Sum of the Square and the Cube of 614387·, or the Fourth Power of the Difference of the Ninth and Twelfth Multiples of 8715463·?

LONG DIVISION.

On page 96· three methods are described for finding how many times a Divisor is contained in a Dividend.

In Exercises LVIII., LIX., and LX., we have proceeded on the third of those three methods; namely, by Subtracting from each Dividend-figure the highest possible Multiple of Divisor.

This is the method always adopted for *calculation:* and in employing it we separate the Dividend, as we go along, into a series of *Minuends*, from each of which we subtract the highest possible Multiple of the Divisor.

And, since we have no higher Digit than 9 to put in any place of Quotient, the *ninth* is the highest Multiple of Divisor that we can ever have occasion to ascertain. For, knowing the *ninth* Multiple, we at once obtain the *ninetieth* the *nine-hundreth*, and so on, by removing Digits of *ninth* Multiple *one*, *two*, or *more* places to the left, by PRIN. X.

Now, when (as is the case in the last three Exercises,) the required Multiples are at once known from the Multiplication Table, the greatest Number we can have to subtract is 108· (*units*, or *tens*, or *hundreds*, &c.); and the greatest possible Remainder is 11· (*units*, or *tens*, or *hundreds*, &c.); for the greatest Minuend is no more than *119· (units*, or *tens*, or *hundreds*, &c.).

Hence, with Divisor an Integer not exceeding 12·, the entire work, Finding the Multiples, Subtracting, and Carrying, for each separate Quotient-figure, can be performed mentally; and nothing needs be "set down" but the Quotient-figures as they are successively found. Hence, Examples thus worked are said to be done by Short Division.

We shall presently find that with *any* Divisor expressed by a single Digit, or by 11 or 12, the work may be performed by Short Division.

In Long Division, all the Minuends, Multiples, and Remainders, are written out at full length; and this is quite necessary when we have a large Divisor.

PLACES OF QUOTIENT-FIGURES.

In Short Division by Integers, we have seen that "In Dividing by Units, each Quotient-figure falls under corresponding Figure of Dividend."

Now, any Integer whatever may be considered and treated as Units. For, if we had a Digit capable of expressing, for instance, *Three hundred and forty-nine*, then that Number would be expressed by that *Single Digit in the Unit's place.*

Therefore, what was true of Units is true also of Integers; Hence,

In Dividing by any Integer, each Quotient-figure falls in the same place as right-hand figure of the Minuend from which it is derived.

LONG DIVISION BY INTEGERS.

To Divide Integers or Decimals by any Integer greater than 12·.

First. Place the Divisor on the left of the Dividend, as in Short Division, and draw, on right of Dividend, a vertical curve, thus (to separate the Quotient from the Dividend.

Second. For your first Minuend, take the least number of left-hand figures of Dividend that express, by themselves, a Number not less than Divisor.

Third. Judge, as nearly as you can, what Multiple of Divisor is nearest less than this Minuend. Place Corresponding Factor of this Multiple as left-hand figure of Quotient, *in same place as right-hand figure of first Minuend* (see p. 107·), filling with ciphers all places to Decimal Point. (PRINCIPLE II. p. 3·)

Fourth. Multiply Divisor by Quotient-figure. Place the Product (which we shall call Multiple-Subtrahend) under first Minuend (filling up with ciphers to Decimal Point).

Fifth. At this stage, pause, and ascertain whether you have not chosen too high a Quotient-figure. *If your Multiple-Subtrahend be greater than your first Minuend, the Quotient-figure will be too high;* therefore remove it and its Multiple-Subtrahend, and try a Quotient-figure *one lower.* Proceed thus until you find a Quotient-figure which gives a Multiple-Subtrahend less than Minuend.*

Sixth. Draw a horizontal line under Multiple-Subtrahend, and Subtract.

Seventh. Again pause, and see whether Remainder (without its filling-up ciphers) be less, or not less than *Divisor.* If *Remainder be not less than Divisor, the Quotient-figure will be too low;* therefore remove it and its Multiple-Subtrahend, and try until you find the Quotient-figure whose Multiple-Subtrahend will leave a Remainder less than Divisor.*

Eighth. To this Remainder, "take down," in its proper place, next Dividend-figure. This furnishes you with a second Minuend, with which work as before; remembering that, *in Dividing by Integers, each Quotient-figure stands in same place as right-hand figure of its Minuend.*

Ninth. Express Quotient of remaining Units, (when Dividend contains no Decimals,) either as a Vulgar Fraction, (PRINCIPLE XV. p. 99·,) or as a Decimal Fraction, adding as many ciphers as may be neccessary to form further Minuends, when all the Dividend-figures have been "taken down."

FIRST EXAMPLE WORKED OUT. Divide 33855237· by 468·

In this Example the Divisor (468·) consists of only three places of figures, but the *three* left-hand figures of Dividend (338) do not contain the Divisor. We must, consequently, take for our first Minuend the *four left-hand figures* of Dividend, (3385). And because the right-hand figure of this Minuend occupies fifth place of Integers, the first Quotient-figure will also stand in fifth place of Integers. After placing the first Quotient-figure, then, the Example will appear thus:

```
Divisor.  Dividend.   Quotient of first Minuend.
   468·)33855237·(70000·
```

---

* NOTE TO TEACHER. Some masters allow their pupils to make for themselves a Table of the first nine Multiples of the Divisor, by referring to which Table it is at once seen what Quotient-Figure should be employed. Such aid *may be useful* at first, but should be dispensed with after a few examples.

And the Example completed will stand as follows:

```
Divisor.  Dividend.   Quotient.
 468·)33855237·(72340·25
      32760000·
      ---------
       1095000·
        936000·
       --------
        159200·
        140400·
        -------
         18830·
         18720·
         ------
          117·0
           93·6
          -----
           23·40
           23·40
           -----
```

1st Minuend 3385 (*ten-thousands*).
1st Quotient-figure, 7(0000·)
1st Multiple-Subtrahend = 468· × 7(0000·) = 3276(0000·)
3276 (*ten-thous.*) < 3385 (*ten-thous.*)
∴ *Quotient-figure not too high.*
1st Remainder = 109 (*ten-thous.*)
109· < Divisor,
∴ *Quotient-figure not too low.*
2nd Minuend 1095 (*thous.*). Proceed as before.
5th Minuend (117·) being less than Divisor, let cipher remain in Units' place of Quotient, and "take down" 0, as no more Dividend-figures are left.
6th Minuend 1170 (*tenths.*)

CONTRACTION BY OMISSION OF CIPHERS.

It is the usual practice to omit all the filling-up ciphers, so that the finished Example stands thus:

```
Divisor.  Dividend.   Quotient.
 468·)33855237·)72340·25
      3276
      ----
       1095
        936
       ----
        1592
        1404
        ----
         1883
         1872
         ----
          1170
           936
          ----
           2340
           2340
           ----
```

Whenever ciphers are thus suppressed, it must be carefully borne in mind that each Digit of Multiple-Subtrahends, of Remainders and of Minuends, is to be understood as belonging to same place of Integers or Decimals as the Dividend-figure standing immediately over it.*

---

* **Note to Teacher.** We have heretofore studiously eschewed this practice of suppression, from experience of its pernicious effects, when introduced prematurely, or without due preparation. (See notes to pp. 71· and 72·). Ninety-nine per cent. of young persons instructed in Arithmetic on the old methods will be found to read untruly every figure of the above work. As a security against error, the pupil (although he has had such ample practice in the true placing and reading of figures,) should work out at least ten Examples of Exercise LXI. *without suppression;* and in those Examples wherein he is permitted to abbreviate the work by omitting ciphers, he should be frequently called on *to read accurately* those parts of the process affected by the omission.

Method of Proof, the same as in Short Division; namely, Multiplying Quotient by Divisor to reproduce Dividend. (PRINCIPLE XIX. page 112·)

SECOND EXAMPLE WORKED OUT AND PROVED. Divide 1·737813 by 462·.

Here, first Minuend is 1·737, and because the right-hand figure 7 is in 3rd place of Decimals, First Quotient-figure will be in 3rd place of Decimals; for, *when Divisor is an Integer every Quotient-figure falls in same place as right-hand-figure of its Minuend.* (See page 115·).

Ex. completed. *Ciphers expressed.* Divr. Dividend. Quotient.	*Ciphers suppressed.* Divr. Dividend. Quotient.	
462·)1·737813(·0037615	462·)1·737813(·0037615	
1·386	1·386	462· Div. as Multr.
·3518	3518	·007523
·3234	3234	·22569
·02841	2841	1·5046
·02772	2272	1·737813 *Proof.*
·000693	693	
·000462	462	
·0002310	2310	
·0002310	2310	

The ciphers being omitted, it becomes impossible to retain the Decimal Point in the work.

THIRD EXAMPLE WORKED OUT AND PROVED. Divide 73148629· by 6025·.

Example completed; *Ciphers omitted.*

Divisor Dividend.	Quotient.
6025·)73148629·(	12140$\frac{5129}{6025}$
6025	6025·
12898	65829·
12050	242800·
8486	72840000·
6025	73148629· *Proof.*
24612	
24100	
5129 = $\frac{5129}{6025}$ of 6025·.	

In proving this Example, the first step is to multiply the Vulgar Fraction $\frac{5129}{6025}$ by its Denominator.

But a Fraction multiplied by its Denominator yields for Product its Numerator. (PRINCIPLE XIX., Cor. I.)

Therefore, we have only to carry and *add in* the Numerator 5129· of the Vulgar Fraction.

EXERCISE LXI.

EXAMPLES IN DIVISION OF INTEGERS AND DECIMALS BY INTEGERS GREATER THAN 12·.*

When Divisor is an Integer with one or more ciphers next to Decimal Point, the process is abbreviated by working under Principle XXI., which governs Division by Decimals.

1· Divide 843615·435 by 75·
2· Divide 6438·4286 by 604·
3· Divide 7169843·1946 by 1916·
4· Divide ·0182392 by 2048·
5· Divide ·008645375 by 4096·
6· Divide 8643·625 by 15625·
7· Divide 16· by 9375·
8· Divide 1· by 1024·
9· Divide ·001 by 128·
10· Divide 3·1416 by 640·

Find the Quotients

11· Of 36· ÷ 45· & 54· ÷ 96·
12· Of 714· ÷ 1214· & ·5 ÷ 64·
13· Of ·0014 ÷ 61843·
14· Of ·01486 ÷ 2408·
15· Of 5· ÷ 512· and 4· ÷ 25·
16· Of 7· ÷ 16· and 11· ÷ 75·

17· What Factor of 1804599·297 corresponds to 23·?
18· By what No. must 34· be multiplied to produce 275357·6581?
19· How many times 46· are contained in 31624·816?
20· What Multiple of 914749· is 53055442·?
21· Sixty-nine is one Factor of 48964·815, what is the Corresponding Factor?
22· Of what No. is 3701·2312 the 47th Multiple?
23· Find the $\frac{1}{86874}$th part of 59944·85049.
24· What part of 68118176511· is 6945873·?
25· Find that No. which must be multiplied by 879146· to produce 326034·810684.
26· What part of 22748·8025 is $\frac{1}{7}$ of 43038275·?
27· Of what No. is $\frac{1}{8934}$ of 7425028227636· the 29th Multiple?
28· What Multiple of 5· is equal to $\frac{1}{7895}$ of 448349273425·?
29· How many times can 4839786· be obtained from 29062914930·?
30· What Factor of 58543382524158· corresponds to 7146874·?

DIVISION OF INTEGERS AND DECIMALS BY COMPOSITE NUMBERS.

When the Divisor is a Composite Number, we may divide by the Corresponding Factors, one after another, and when each of the Factors is less than 13·, we thus avoid Long Division.

How many twenty-eights can 319· be distributed into?

The Factors of 28· are 7· and 4·; for 28· = 7· × 4·

Dividing by 28· is, then, the same thing as dividing first by 7· and then the Quotient by 4·, or dividing first by 4· and then the Quotient by 7·

---

* Note to Teacher. As Quotients will now occur containing more than twelve places of figures, the pupil should at once study the Third Numeration Table. (See Table of Contents.)

To do this experimentally, we should first arrange 319· in groups of *seven* each. There would be 45· such groups, and 4· units remaining, shewing that 319· = $45\frac{4}{7}$ *sevens.*

Now, 4· groups of *seven* must be put together to make a group of *twenty-eight.* There will be 11· such groups, and 1 *seven* over, which, with the 4· units remaining from the first division, will make altogether a Remainder of 11· units.

∴ No. of twenty-eights in 319· = $11\frac{11}{28}$ twenty-eights.

Every unit of First Quotient being equal to the First Divisor, (Principle XIV.) every Unit *over* from Second Divisor is equal to the First Divisor. Hence, to reduce Second Remainder into Simple Units, Multiply it by First Divisor.

If we Divide by a third Factor, and have a Third Remainder, each of its Units is (like each Unit of Second Quotient, from which it remains) equal to Product of First and Second Divisors. Hence, to reduce Third Remainder to Simple Units, Multiply it continually by First and Second Divisors.

In the same way, if we Divide successively by any number of Factors, each Remainder must (in order to Reduce it to Simple Units,) be Multiplied continually by all the preceding Divisors, except the last.

To Divide Integers or Decimals by any Composite Integer.

First. Divide Dividend by any Factor of Divisor.

Second. Divide Quotient by a Corresponding Factor of Divisor.

Third. Divide Second Quotient by Third Corresponding Factor, (if any,) and so on, until the whole set of Factors have been employed as Divisors.

Fourth. To find the True Remainders, (when Dividend has no Decimals,)

Multiply 2nd Remainder by 1st Divisor, and add in 1st Remainder.

Multiply 3rd Remainder continually, by 1st and 2nd Divisors, and add in 2nd True Remainder.

Multiply 4th Remainder, continually, by 1st, 2nd, and 3rd Divisors, and add in 3rd True Remainder.

And so on, always multiplying New Remainder continually by all preceding Divisors (except the last), and *adding* in last-found True Remainder.

FIRST EXAMPLE WORKED OUT. DIVISOR HAVING TWO FACTORS, EACH LESS THAN 13·. Divide 56497663. by 72·.

The Divisor having two sets of Factors, this Example may be worked in *four* ways.

First, Dividing by 12· and 6·

$$72\cdot\left\{\begin{array}{l}12\cdot)\underline{56497663\cdot} \\ \quad 6\cdot)\underline{4708138\tfrac{7}{12}}\ \textit{twelves in } 56497663\cdot\end{array}\right.$$
$$\qquad\qquad\underline{\underline{784689\tfrac{55}{72}}}\ \textit{seventy-twos in } 56497663\cdot$$

By first method: Second Remainder 4· = 4· twelves, because each Digit of First Quotient is a twelve. (PRINCIPLE XIV.) And 7· units remain from first Division.

∴ *True Remainder* = 4· × 12· + 7· = 55· = $\frac{55}{72}$ of *Seventy-two.*

Second, Dividing by 6· and 12·

$$72\cdot\left\{\begin{array}{l}6\cdot)\underline{56497663\cdot} \\ 12\cdot)\underline{9416277\tfrac{1}{6}}\ \textit{sixes in } 56497663\cdot\end{array}\right.$$
$$\qquad\qquad\underline{\underline{784689\tfrac{55}{72}}}\ \textit{seventy-twos in } 56497663\cdot$$

By second method: First Remainder = 1· *Unit.*
Second Remainder = 9· *Sixes.* (Why?)

∴ *True Remainder* = 9· × 6· + 1· = 55· = $\frac{55}{72}$ of *Seventy-two.*

Third, Dividing by 8· and 9·

$$72\cdot\left\{\begin{array}{l}8\cdot)\underline{56497663\cdot} \\ \quad 9\cdot)\underline{7062207\tfrac{7}{8}}\ \textit{eights in Dividend.}\end{array}\right.$$
$$\qquad\qquad\underline{\underline{784689\tfrac{55}{72}}}\ \textit{seventy-twos in Dividend.}$$

By third method: First Remainder = 7· *Units.*
Second Remainder = 6· *Eights.* (Why?)

∴ *True Remainder* = 6· × 8· + 7· = 55· = $\frac{55}{72}$ of 72·

Fourth, Dividing by 9· and 8·

$$72\cdot\left\{\begin{array}{l}9\cdot)\underline{56497663\cdot} \\ \quad 8\cdot)\underline{6277518\tfrac{1}{9}}\ \textit{nines in Dividend.}\end{array}\right.$$
$$\qquad\qquad\underline{\underline{784689\tfrac{55}{72}}}\ \textit{seventy-twos in Dividend.}$$

By fourth method: First Remainder = 1· *Unit.*
Second Remainder = 6· *Nines.* (Why?)

∴ *True Remainder* = 6· × 9· + 1· = 55· = $\frac{55}{72}$ of 72·

EXERCISE LXII.

EXAMPLES IN DIVISION OF INTEGERS AND DECIMALS BY COMPOSITE INTEGERS, HAVING TWO FACTORS LESS THAN 13·.

1· Divide 37169564· by 25·, and 38·16 by 45·
2· Divide 43684256· by 36·, and 6154·3 by 32·
3· How many times 42· in 916427·?

4· Find $\frac{1}{81}$ of 64321864·, and $\frac{1}{66}$ of 6·49
5· Find $\frac{1}{96}$ of 31454986·, and $\frac{1}{84}$ of 932·15
6· What is $\frac{1}{108}$ of 614325684·?
7· Divide 7145438953· by 121·
8· Find Quotients of 614382· ÷ 81·
9· Of 34200006·48 ÷ 132· and of 7146832· ÷ 55·
10· Of 74965394· ÷ 35· and of 396435· ÷ 144·

Second Example worked out. Divisor having three Factors, each less than 13·. Divide 99805354· by 315·.

The Divisor having three Factors, namely, 9·, 5·, and 7·, this Example may be worked in six ways. For we may Divide either

By 9·, 5·, and 7·,	By 5·, 9·, and 7·,	By 7·, 5·, and 9·, or,
By 9·, 7·, and 5·,	By 5·, 7·, and 9·,	By 7·, 9·, and 5·.

First, Dividing by 9·, 5·, and 7·.

$$315\cdot\left\{\begin{array}{l} 9\cdot)\underline{99805354\cdot} \\ 5\cdot)\underline{11089483\frac{7}{9}}\ \textit{nines in Dividend.} \\ \quad 7\cdot)\underline{2217896\frac{34}{45}}\ \textit{forty-fives in Dividend.} \end{array}\right.$$
$$\qquad\qquad 316842\tfrac{124}{315}\ \textit{three-hundred-and-fifteens in Dividend.}$$

1st Remainder = 7 *Units.*
2nd Remainder = 3· *Nines.*
2nd *True Remr.* = 3· × 9· + 7· = 34· = $\frac{34}{45}$ of 45·
3rd Remainder = 2· *Forty-fives.*
3rd *True Remr.* = 2· × 45· + 34· = 124· = $\frac{124}{315}$ of 315·

Second, Dividing by 5·, 7·, and 9·.

$$315\cdot\left\{\begin{array}{l} 5\cdot)\underline{99805354\cdot} \\ 7\cdot)\underline{19961070\frac{4}{5}}\ \textit{fives in Dividend.} \\ \quad 9\cdot)\underline{2851581\frac{19}{35}}\ \textit{thirty-fives in Dividend.} \end{array}\right.$$
$$\qquad\qquad 316842\tfrac{124}{315}\ \textit{three-hundred-and-fifteens in Dividend.}$$

1st Remainder = 4· *Units.*
2nd Remainder = 3· *Fives.*
2nd *True Remr.* = 3· × 5· + 4· = 19· = $\frac{19}{35}$ of 35·
3rd Remainder = 3· *Thirty-fives.*
3rd *True Remr.* = 3· × 35· + 19· = 124· = $\frac{124}{315}$ of 315·

EXERCISE LXII. (*continued.*)

11· Divide 18936975· by 308·
12· What is $\frac{1}{168}$ of 12069660423·?
13· Find the Quotient of 22743145705· ÷ 396·
14· Find the Quotient of 83357016675· ÷ 264·
15· Find the Quotient of 23104448971· ÷ 252·
16· Divide 905490·50634 by 126·
17· Divide 24804028·6362 by 726·
18· Divide 849260137·6116 by 1188·
19· Divide 150478967·016 by 504·
20· Divide 618376·08 by 588·

DIVISION OF VULGAR FRACTIONS.

When Divisor or Dividend is a Vulgar Fraction or Mixed Number, there are three ways of arriving at the Quotient.

First. When the Divisor and Dividend are Simple Fractions with a Common Denominator, they may be considered as like units (of the magnitude described by their Common Denominator), and their *Quotient is at once expressed, fractionally, by writing Numerator of Divisor under Numerator of Dividend.* (PRIN. XV. p. 99·.)

$$\therefore\ \tfrac{95}{113} \div \tfrac{142}{113} = \tfrac{95}{142}\ (\text{of } \tfrac{142}{113}).$$

$$\text{And, } \tfrac{314}{500} \div \tfrac{65}{500} = \tfrac{314}{65}\ (\text{of } \tfrac{65}{500}) = 4\tfrac{54}{65}\ (\text{of } \tfrac{65}{500}).$$

$$\text{Also, } 19\tfrac{5}{8} \div 4\tfrac{7}{8} = \tfrac{157}{8} \div \tfrac{39}{8} = \tfrac{157}{39}\ (\text{of } \tfrac{39}{8}) = 4\tfrac{1}{39}\ (\text{of } \tfrac{39}{8}).$$

Any two Vulgar Fractions may, of course, be treated thus, if previously reduced to a Common Denominator. And there is no other way by which the Division of one Fraction by another can be experimentally effected.

The second means of obtaining the Quotient is to Multiply Dividend by Reciprocal of Divisor. (PRINCIPLE XVII. p. 106·.) Taking the same three examples as above:

$$\tfrac{95}{113} \div \tfrac{142}{113}\ (\doteq \tfrac{95}{113} \times \tfrac{113}{142} \text{ or } \tfrac{95}{142} \times \tfrac{113}{113}) = \tfrac{95}{142}, \text{ as above.}$$

$$\text{Also, } \tfrac{314}{500} \div \tfrac{65}{500}\ (\doteq \tfrac{314}{500} \times \tfrac{500}{65} \text{ or } \tfrac{314}{65} \times \tfrac{500}{500}) = \tfrac{314}{65} = 4\tfrac{54}{65}.$$

$$\text{And } 19\tfrac{5}{8} \div 4\tfrac{7}{8} = \tfrac{157}{8} \div \tfrac{39}{8}\ (\doteq \tfrac{157}{8} \times \tfrac{8}{39} \text{ or } \tfrac{157}{39} \times \tfrac{8}{8}) = \tfrac{157}{39} = 4\tfrac{1}{39}.$$

$$\text{Again, } \tfrac{7}{11} \div \tfrac{5}{9}\ (\doteq \tfrac{7}{11} \times \tfrac{9}{5}\ \&\text{c.}) = \tfrac{63}{55}\ (\text{of } \tfrac{5}{9}) = 1\tfrac{8}{55}\ (\text{of } \tfrac{5}{9})$$

Now, comparing this process with the former one, we shall find that the two are, in reality, the same. For, in reducing the two Fractions to a Common Denominator, and then writing New Numerator of Divisor under New Numerator of Dividend, we do really and only Multiply Dividend by Reciprocal of Divisor. This is shewn by working last Example by first method.

$$\text{For, } \tfrac{7}{11} \div \tfrac{5}{9} = \tfrac{63}{99} \div \tfrac{55}{99} = \frac{7 \times 9}{11 \times 9} \div \frac{5 \times 11}{9 \times 11} = \frac{7 \times 9}{11 \times 5} \doteq \tfrac{7}{11} \times \tfrac{9}{5}.$$

Here, *Num. of Quo. = Num. of Dividend* × *Den. of Divisor.*
Den. of Quo. = Den. of Dividend × *Num. of Divisor.*
∴ *Quotient* ≐ *Dividend* × *Reciprocal of Divisor.*

The third method of Dividing one Vulgar Fraction by another is derived from PRINCIPLE XIX.

For, since Dividend is Product of Divisor and Quotient, and any Product divided by one of its Factors yields the Corresponding Factor;

$$\therefore \tfrac{39}{72} \text{ being } = \tfrac{3}{8} \times \tfrac{13}{9} \text{ or } = \tfrac{3}{9} \times \tfrac{13}{8},$$

$$\tfrac{39}{72} \div \tfrac{3}{8} = \tfrac{13}{9} = \frac{39 \div 3}{72 \div 8} \qquad \tfrac{39}{72} \div \tfrac{3}{9} = \tfrac{13}{8} = \frac{39 \div 3}{72 \div 9}$$

$$\tfrac{39}{72} \div \tfrac{13}{9} = \tfrac{3}{8} = \frac{39 \div 13}{72 \div 9} \qquad \tfrac{39}{72} \div \tfrac{13}{8} = \tfrac{3}{9} = \frac{39 \div 13}{72 \div 8}$$

In all four of those instances, to obtain the Quotient we merely Divide Numerator by Numerator, and Denominator by Denominator.[*]

To Divide a Vulgar Fraction by an Integer, we either Divide Numerator, or Multiply Denominator by the Integer.

For, $\tfrac{63}{11} \div \tfrac{3}{1} \doteq \frac{63 \div 3}{11} = \tfrac{21}{11}$

or, $\tfrac{63}{11} \div \tfrac{3}{1} \doteq \tfrac{63}{11} \times \tfrac{1}{3} = \tfrac{63}{33}$

These results agree, for $\tfrac{21}{11} = \tfrac{63}{33}$ by PRIN. XIII., p. 79·.

To Divide Integer, Vulgar Fraction, or Mixed Number by Vulgar Fraction or Mixed Number.

First. Reduce Integers and Mixed Numbers to Improper Fractions.

Second. Multiply Dividend by Reciprocal of Divisor.[†]

The Product thus found only *coincides* with the true Quotient, that is to say, is expressed by the same figures similarly situated. (See pages 101· to 106·, and PRINCIPLE XVII.)[‡]

FIRST EXAMPLE WORKED OUT. AN INTEGER ÷ A VULGAR FRACTION.

$43\cdot \div \tfrac{7}{11} = \tfrac{43}{1} \div \tfrac{7}{11}$ ($\doteq \tfrac{43}{1} \times \tfrac{11}{7}$ and) $= 67\tfrac{4}{7}$ (of $\tfrac{7}{11}$).

SECOND EXAMPLE WORKED OUT. VULGAR FRACTION ÷ VULGAR FRACTION. What part or multiple are $\tfrac{9}{11}$ of $\tfrac{4}{5}$? Or, How many times are $\tfrac{4}{5}$ contained in $\tfrac{9}{11}$?

$\tfrac{9}{11} \div \tfrac{4}{5}$ ($\doteq \tfrac{9}{11} \times \tfrac{5}{4}$ and) $= \tfrac{45}{44}$ (of $\tfrac{4}{5}$) $= 1\tfrac{1}{44}$ (of $\tfrac{4}{5}$),

THIRD EXAMPLE WORKED OUT. MIXED NUMBER

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[*] NOTES TO TEACHER. This curious variety of Division of Fractions is rarely or never alluded to. Although only advantageously available when the terms of Dividend are respectively Multiples of those of Divisor, it holds true in all cases: as the following Example will shew.

$\tfrac{7}{11} \div \tfrac{5}{9} \doteq \frac{7 \div 5}{11 \div 9} \doteq \tfrac{7}{5} \div \tfrac{11}{9} \doteq \tfrac{7}{5} \times \tfrac{9}{11} \doteq \tfrac{7}{11} \times \tfrac{9}{5}$ as by usual method.

[†] To avoid the trouble of previously explaining the word "Reciprocal," most Arithmetical "Guide Books" direct the pupil to "*Invert the Divisor and Multiply*," a remarkably vague injunction, for the inversion of $\tfrac{9}{7}$ (for example,) might be as fairly expected to produce $\tfrac{1}{9}$ or $\tfrac{9}{2}$ as $\tfrac{7}{9}$.

[‡] *The* fact that we are writing for pupils in Elementary Schools must *excuse tautology.*

÷ VULGAR FRACTION. How many parcels, each containing $\frac{6}{13}$ of 1·lb., can be procured from $54\frac{5}{7}$ lb.?

$$54\tfrac{5}{7} \div \tfrac{6}{13} \doteq \tfrac{383}{7} \times \tfrac{13}{6} \text{ and} = \tfrac{4979}{42} \text{ (of } \tfrac{6}{13}) = 118\tfrac{23}{42} \text{ (of } \tfrac{6}{13})$$

∴ Out of $54\frac{5}{7}$ lb. $118\frac{23}{42}$ parcels could be made, each containing $\frac{6}{13}$ of 1·lb.

FOURTH EXAMPLE WORKED OUT. INTEGER ÷ MIXED NUMBER. Divide 98· by $11\frac{1}{5}$

$$98\cdot \div 11\tfrac{1}{5} = \tfrac{98}{1} \div \tfrac{56}{5} \doteq \tfrac{98}{1} \times \tfrac{5}{56} \text{ and} = \tfrac{490}{56} \text{ (of } 11\tfrac{1}{5}) = 8\tfrac{42}{56} \text{ (of } 11\tfrac{1}{5})$$

FIFTH EXAMPLE WORKED OUT. VULGAR FRACTION ÷ MIXED NUMBER. Divide $\frac{9}{16}$ by $4\frac{1}{13}$

$$\tfrac{9}{16} \div 4\tfrac{1}{13} = \tfrac{9}{16} \div \tfrac{53}{13} \doteq \tfrac{9}{16} \times \tfrac{13}{53} \text{ and} = \tfrac{117}{848} \text{ (of } 4\tfrac{1}{13})$$

SIXTH EXAMPLE WORKED OUT. MIXED NUMBER ÷ MIXED NUMBER. How many lb. of beef at $4\frac{1}{4}$ pence per lb. can be purchased with $20\frac{1}{2}$ pence?

To ascertain this, find how many times $4\frac{1}{4}$ pence are contained in $20\frac{1}{2}$ pence.

$$20\tfrac{1}{2} \div 4\tfrac{1}{4} = \tfrac{41}{2} \div \tfrac{17}{4} \doteq \tfrac{41}{2} \times \tfrac{4}{17} \text{ and} = \tfrac{164}{34} \text{ (of } 4\tfrac{1}{4}) = 4\tfrac{28}{34} \text{ (of } 4\tfrac{1}{4})$$

∴ $20\frac{1}{2}$ pence would pay for $4\frac{28}{34}$ lb. of beef, at $4\frac{1}{4}$ pence per lb.

To Divide a Vulgar Fraction by an Integer.

Either Divide Numerator, or Multiply Denominator, by Integer.

SEVENTH EXAMPLE WORKED OUT. NUMERATOR ÷ INTEGER. What part of 2· are $\frac{416}{595}$ of a Unit?

$$\tfrac{416}{595} \div 2\cdot = \tfrac{208}{595} \text{ (of } 2\cdot)$$

EIGHTH EXAMPLE WORKED OUT. DENOMINATOR × INTEGER.

$$\tfrac{416}{595} \div 2\cdot = \tfrac{416}{1190} \text{ (of } 2\cdot\text{), and } \tfrac{416}{1190} = \tfrac{208}{595} \text{ by PRIN. XIII. p. 171}\cdot$$

NINTH EXAMPLE WORKED OUT. MIXED NUMBER ÷ INTEGER. How many stone, of 8·lb. each, are contained in $94\frac{5}{7}$ lb.?

To discover this, Divide $94\frac{5}{7}$ by 8·

$$94\tfrac{5}{7} \div 8\cdot = \tfrac{663}{7} \div 8\cdot \doteq \tfrac{663}{7} \times \tfrac{1}{8} \text{ and} = \tfrac{663}{56} \text{ (of } 8\cdot) = 11\tfrac{47}{56} \text{ (\textit{eights.})}$$

∴ $94\frac{5}{7}$ lb. are equal to $11\frac{47}{56}$ stone of 8·lb. each.

TENTH EXAMPLE WORKED OUT. COMPOUND FRACTION ÷ COMPOUND FRACTION. Divide $\frac{4}{5}$ of $9\frac{2}{3}$ by $\frac{3}{8}$ of 19·

$$(\tfrac{4}{5} \text{ of } 9\tfrac{2}{3}) \div (\tfrac{3}{8} \text{ of } 19\cdot) = (\tfrac{4}{5} \text{ of } \tfrac{29}{3}) \div (\tfrac{3}{8} \text{ of } \tfrac{19}{1}) \doteq \tfrac{4}{5} \times \tfrac{29}{3} \times \tfrac{8}{3} \times \tfrac{1}{19} \text{ and} = \tfrac{928}{855}$$

EXERCISE LXIII.

Find the Quotients

1· Of $\frac{7}{15} \div \frac{9}{11}$
2· Of $\frac{16}{19} \div \frac{3}{11}$
3· Of $\frac{4}{65} \div \frac{9}{2}$
4· Of $7\frac{1}{4} \div \frac{6}{7}$
5· Of $13\frac{4}{11} \div \frac{19}{25}$
6· Of $64· \div \frac{17}{95}$
7· Of $\frac{17}{95} \div 64·$
8· Of $\frac{3}{4} \div 9\frac{1}{5}$
9· Of $113\frac{4}{5} \div \frac{6}{13}$
10· Of $18\frac{7}{12} \div 18\frac{7}{12}$
11· Of $\frac{39}{45} \div \frac{13}{45}$
12· Of $\frac{65}{11} \div \frac{5}{114}$
13· Of $\frac{28}{91} \div \frac{7}{13}$
14· Of $\frac{64}{77} \div \frac{4}{7}$
15· Of $\frac{32}{45} \div \frac{8}{9}$
16· Of $\frac{374}{29} \div 11·$
17· Of $\frac{8624}{451} \div 11·$
18· Of $7\frac{5}{9} \div 12·$
19· Of $39\frac{4}{13} \div 16·$
20· Of $17\frac{5}{8} \div 7·$

21· Of $(\frac{21}{37}$ of $\frac{6}{14}) \div (9\frac{1}{4} \times \frac{6}{17}$ of $4\frac{3}{5})$
22· Of $(\frac{15}{18}$ of 64· times $3\frac{1}{2}) \div (\frac{2}{3}$ of $\frac{5}{8}$ of 9·)
23· Of $(·84 \times \frac{13}{5}) \div (7·5 \times 7\frac{1}{2})$
24· Of $(\frac{11}{119}$ of 63·75$) \div (4\frac{1}{2}$ of $6\frac{3}{4}$ of $\frac{7}{12})$
25· Of $\frac{112}{64} \div 16·$; and of $\frac{112}{137} \div 95·$
26· What part of $\frac{11}{12}$ is $\frac{13}{29}$?
27· Express $\frac{7}{18}$ of $14\frac{3}{5}$ "*in terms of*" $64\frac{1}{7}$
28· What must $19\frac{4}{7}$ be multiplied by to produce 1·?
29· How many times $\frac{3}{4}$ will make 29·?
30· What Fraction of 29· is $\frac{3}{4}$?
31· Express $\frac{7}{8}$ as a Fraction of 21·; of $6\frac{1}{4}$; and of 17·
32· Reduce $\frac{48}{5}$ to a Fraction of 429·

This is only another way of saying, "*Find what Fraction* $\frac{48}{5}$ *are of* 429·," or, "*Express* $\frac{48}{5}$ *in terms of* 429·."

33· If $6\frac{1}{4}$ articles cost $16\frac{1}{4}$ shillings, what will 1· cost?

To pay for each article, *separately*, $16\frac{1}{4}$ shillings must be distributed into $6\frac{1}{4}$ equal amounts.

The No. of s. in each of those amounts is the same as the No. of lots of $6\frac{1}{4}$s. each that can be obtained from $16\frac{1}{4}$s. (PRIN. III.)

$\therefore$ *Cost of* 1· *article in s.* $\doteq 16\frac{1}{4}$*s.* $\div\ 6\frac{1}{4}$*s.*

34· If $7\frac{5}{8}$ yards of cloth cost $4\frac{2}{3}$s., what will 1· yard cost?
35· If $11\frac{4}{5}$ gallons of wine cost £$23\frac{3}{5}$, what will 1· gallon cost?
36· If $9\frac{4}{7}$ cheeses weigh $19\frac{1}{7}$ lb., what is the weight of 2· cheeses

Find weight of 1· *cheese first.*

37· Seven boys together weighed $595\frac{7}{8}$ lb., what was their average weight?
38· A steamer went, for 4· hours at the rate of $6\frac{4}{5}$ miles per hour, and for five hours, at 11· miles per hour, what was her average speed per hour?
39· A and B are travelling in the same direction; A has the start of B by $15\frac{3}{4}$ miles, but B travels $\frac{7}{8}$ of a mile per hour faster than A. In how many hours will A be overtaken by B?
40· C starts to walk from Portsmouth to London (72· miles) at the same time that D leaves London to walk to Portsmouth. C walks $3\frac{1}{4}$ miles an hour; D $2\frac{7}{8}$ miles an hour. When will they meet, and at what distance from London?

The end of each hour finds them $(3\frac{1}{4} + 2\frac{7}{8})$ *miles nearer together.*
$\therefore$ *No. of hours to meet* $\doteq$ 72· miles $\div\ (3\frac{1}{4} + 2\frac{7}{8}) =$ &c. *miles.*

41· What was the circumference of a coach-wheel which made $392\frac{5}{8}$ revolutions in 4000· feet?
42· How many revolutions will a coach-wheel whose circumference is $11\frac{2}{3}$ feet make in $4\frac{3}{7}$ mile? A mile = 5280· feet.

DIVISION OF DECIMALS.

When the Divisor contains one or more Decimals, the *work* of Division is carried on precisely in the same manner as with an Integral Divisor, and the only new point to be attended to, in placing the Quotient-figures, is the distance of the right-hand Significant Figure of Divisor from Units' place, as we now proceed to explain.

Because Dividing by any Number produces the same figures (similarly situated) as Multiplying by the Reciprocal of Divisor (PRINCIPLE XVII., p. 106·); therefore,

Quotient of 6184·3219 ÷ ·1 ≐ Product of 6184·3219 × 10·
Quotient of 6184·3219 ÷ ·01 ≐ Product of 6184·3219 × 100·
&c. &c. &c. &c.
Quotient of 6184·3219 ÷ 10· ≐ Product of 6184·3219 × ·1
Quotient of 6184·3219 ÷ 100· ≐ Product of 6184·3219 × ·01
&c. &c. &c. &c.

Consequently,

6184·3219 ÷ 100000· = ·061843219
6184·3219 ÷ 10000· = ·61843219
6184·3219 ÷ 1000· = 6·1843219
6184·3219 ÷ 100· = 61·843219
6184·3219 ÷ 10· = 618·43219
*6184·3219 ÷ 1· = 6184·3219
6184·3219 ÷ ·1 = 61843·219
6184·3219 ÷ ·01 = 618432·19
6184·3219 ÷ ·001 = 6184321·9
6184·3219 ÷ ·0001 = 61843219·
6184·3219 ÷ ·00001 = 618432190·

* Begin at the line where the Divisor is 1·, and read upwards and downwards.

And so on, extending the table either upwards or downwards.

∴ *Dividing by Digit* 1, *standing in any place, merely removes every Digit of Dividend as many places*, BUT IN THE CONTRARY DIRECTION, *as Dividing Digit is from Units' place.*

Compare this with PRINCIPLE X., p. 60·

Again, because 20· = 2· × 10·, Dividing by 20· is the same thing as first Dividing by 2· and then by 10· (EXERCISE LXII.)

But, *Dividing by* 10· *merely removes Digits one place to the right.* Therefore, to Divide by 20·, we Divide by 2·, and set each Quotient-figure one place to the right.

Similarly, to Divide by 30·, we Divide by 3·, and set each Quotient-figure one place to the right. And so on.

Also, because ·2 = $\frac{1}{10}$ of 2· (PRINCIPLE IV.), = 2· × ·1· (PRINCIPLE IX.); therefore, Dividing by ·2 is the same thing as Dividing by 2·, and then Dividing the Quotient by ·1·

But *Dividing by ·1 merely removes Digits one place to the left.* Hence, to Divide by ·2, we Divide by 2·, and set each Quotient-figure one place to the left.

To Divide by ·3, Divide by 3·, and set each Quotient-figure one place to the left. And so on.

∵ 163·28 ÷ 2· = 81·64
∴ 163·28 ÷ ·002 = 81640·
and 163·28 ÷ 20· = 8·164
&c. &c.

∵ 163·28 ÷ 8· = 20·41
∴ 163·28 ÷ ·08 = 2041·
and 163·28 ÷ 8000· = ·02041
&c. &c.

PRINCIPLE XXI. *Dividing by a Single Digit, standing in any place of Integers or Decimals, removes every Quotient-figure as many places,* BUT IN THE CONTRARY DIRECTION, *as Dividing Digit is from Units' place.*

Compare with PRINCIPLE XII., p. 66·

The *Natural Place* of any Quotient-figure is that which it would occupy if Divisor were Units. (See page 115·)

Hence, wherever Dividing Digit may stand, we consider it, for the moment, as being in Units' place, and then, *Natural Place of First Quotient-figure is the same as right-hand place of First Minuend.*

We then remove the First Quotient-figure the necessary number of places to the right or left, in obedience to Principle XXI., and the other Quotient-figures will fall into their proper places without further trouble.

To Divide Integers, or Decimals, by a Single Digit standing in any place of Integers or Decimals.

First. Decide the Natural Place of First Quotient-figure.

Second. Divide as usual: but set every Quotient-figure as many places to the LEFT or RIGHT of its Natural Place as Dividing Digit is to the RIGHT or LEFT of Units' place. (PRINCIPLE XXI.)

Observe: 11 and 12 are to be considered as Single Digits in their right-hand place. (See page 67·)

FIRST EXAMPLE WORKED OUT. Divide 9543·21 by 400·

Ex. completed.
400·)9543·21
23·858025

Natural Place of 1st Quotient-figure is (like that of the 9) 4th place of Integers. But Dividing Digit is *two places to the left of Units.*

Therefore, 1st Quotient-figure is removed *two places to the right of its Natural Place.*

SECOND EXAMPLE WORKED OUT. Divide 76543·2 by ·00012

Ex. completed.
·00012)76543·2
637860000·

Natural Place of 1st Quotient-figure is (like that of the 6) 4th place of Integers. But Dividing Digit is *five places to the right of Units.*

Therefore, 1st Quotient-figure is removed *five places to the left of its Natural Place.*

EXERCISE LXIV.

EXAMPLES IN DIVISION OF INTEGERS AND DECIMALS BY A SINGLE DIGIT (OR BY 11 OR 12) IN ANY PLACE OF INTEGERS OR DECIMALS. ABSTRACT NUMBERS.

Find the Quotients

1· Of 743981·252 ÷ 700·
2· Of 95763256· ÷ ·04
3· Of 8654·3218 ÷ ·008
4· Of 129·8643 ÷ 1200·
5· Of 64·9548 ÷ 30000·
6· Of 1· ÷ 80000·
7· Of 7·875 ÷ ·00005
8· Of 11· ÷ 8· and 11· ÷ ·08
9· Of ·00004 ÷ ·00004
10· Of 40000· ÷ ·0004
11· Divide 843216473· by 11000·
12· Divide 342186·423 by ·008
13· Divide 76543·2168 by ·0012
14· Divide 84· by ·005 and by 500·
15· Div^e· 1· by 2· by 20· by 200· & by ·02
16· Divide 4· by ·08 by 80· and by ·008
17· Divide 3· by 120· and by 4000·
18· Divide 7· by ·08 by 800· and by ·0008
19· Divide 6· by ·0012 and by ·00008
20· Divide 5· by ·02 by ·04 and by 800·

DIVISION OF ANY INTEGER OR DECIMAL BY ANY INTEGER OR DECIMAL.

Any Number whatever would, if we had Digits enough, be expressed by a single Digit; and that single Digit would occupy the right-hand place of the Digits which now express the number. (See page 115·)

Therefore, any combination of Significant figures may be considered as equivalent to a possible single Digit in their right-hand place; just as 11 and 12 are always treated.

It follows, then, that Principle XXI. is true, not only of Numbers expressed by single Digits, but of any Number whatever.

EXTENSION OF PRINCIPLE XXI. *Dividing by any Integer or Decimal removes every Quotient-figure as many places,* BUT IN THE CONTRARY DIRECTION, *as right-hand Significant figure of Divisor is from Units' place.*

Now, 295460· ÷ 395· = 748·, therefore the Natural Place of first Quotient-figure 7 is the third place of Integers.

∴ 295460· ÷ 39·5 = 7480· Each Quo. Fig. being moved *one place*
295460· ÷ 3·95 = 74800· Each Quo. Fig. being moved *two places*
295460· ÷ ·395 = 748000· Each Quo. Fig. being moved *three places*
295460· ÷ ·0395 = 7480000· Each Quo. Fig. being moved *four places*
to the left. Why?
&c. &c. &c. &c.

Also 295460· + 3950· = 74·8 Each Quo. Fig. being moved *one place*
295460· ÷ 39500· = 7·48 Each Quo. Fig. being moved *two places*
295460· ÷ 395000· = ·748 Each Quo. Fig. being moved *three places*
295460· ÷ 3950000· = ·0748 Each Quo. Fig. being moved *four places*
to the right. Why?

Again, 29546· ÷ 395· = 74·8, where Natural Place of first Quotient-figure 7 is the second place of Integers.

∴ 29546· ÷ 39·5 = 748· Quo. Figs. moved *one place to left. Why?*

Also, 2954·6 ÷ 3·95 = 748· 295·46 ÷ ·395 = 748·
&c. &c. &c.

To Divide any Integer or Decimal by any Integer or Decimal.

First. Decide the Natural Place of First Quotient-figure.

Second. Divide as usual; but set every Quotient-figure as many places to the LEFT or RIGHT of its Natural Place, as right-hand Significant Figure of Divisor is to the RIGHT or LEFT of Units' place.

Observe. If Divisor be an Integer with one or more ciphers adjoining Decimal Point, consider these as no part of the Divisor. Their only effect will be to remove the Quotient-figures so many places to the right.

EXERCISE LXV.

EXAMPLES IN DIVISION OF ANY INTEGER OR DECIMAL BY ANY INTEGER OR DECIMAL. ABSTRACT NUMBERS.

Find the Quotients

1· Of 1468·038 ÷ 43·5
2. Of 275·843378 ÷ ·353
3· Of 5751·306 ÷ 9·3
4· Of 642882· ÷ ·0654
5· Of 3·21715 ÷ 684·5
6· Of ·36921335 ÷ 738·4267
7· Of 20261330·46 ÷ 643·21684
8· Of 458441780· ÷ 58·43
9· Of 6108·15 ÷ ·0645
10· Of 4·03828014 ÷ 743·698
11· Of ·0856515 ÷ 98·45
12· Of 566319·6 ÷ 3·96
13· Divide 69338·51 by 97· by ·97 and by ·0097
14· Divide 1· by 512· by 5120· by 5·12 by 512000· and by ·512
15· Divide 7· by 40·96 by ·04096 by ·512 by ·000512 and by 1024000·
16· Divide 16·5 by ·256, and 29·875 by ·064
17· By what Number must I multiply 3·125 to produce 2281250·?

GREATEST COMMON MEASURE.

A Measure of any Number is an exact Divisor of it; that is to say, one which leaves no Remainder.

12· ÷ 6· = 2·, 18· ÷ 6· = 3·; no Remainder in either instance.

Therefore, 6· is a Measure of 12·, and of 18·.

15· ÷ 6· = $2\frac{3}{6}$, 20· ÷ 6· = $3\frac{2}{6}$; Remainders 3· and 2·.

Therefore, 6· is not a Measure of 16·, or of 20·.

Two things are, then, evident.

First. "Measure" is only another name for "Factor."

Second. Any Number is a Measure of its own Multiples *only.*

A Common Measure of any two or more Numbers is a Factor of each of them.

The Greatest Common Measure of two or more Numbers is the greatest Number which is an exact Divisor (or Factor) of each of them. For the words, "Greatest Common Measure," we write G.C.M.

3· is a Common Measure of 12·, 18·, and 42·.
But 6· is G.C.M of 12·, 18·, and 42·.

EXERCISE LXVI.

Find by Inspection,

1· A Common Measure of 16·, 24·, 18·, 32·, and 100·.
2· Two Common Measures of 72·, 18·, 24·, 108·, and 30·.
3· Three Common Measures of 96·, 144·, 18·, 48·, 30·, and 36·.
4· Four Common Measures of 30·, 120·, 450·, and 900·.
5· The G.C.M. of 28·, 42·, and 56·; and G.C.M. of 30·, 24·, 18·, and 12·.
6· The G.C.M. of 36·, 45·, 72·, and 27·; and G.C.M. of 22·, 143·, and 187·.
7· The G.C.M. of 24·, 16·, 56·, 112·, and 64·; and G.C.M. of 45·, 78·, and 91·.
8· How many Common Measures have 16·, 24·, 36·, and 48·?
9· What Numbers less than 100· are measured both by 4· and by 6·?
10· Name the first ten Nos. of which 6·, 4·, and 5· are Common Measures.

GREATEST COMMON MEASURE OF ANY TWO NUMBERS.

In ordinary Arithmetic we never have occasion to find the G.C.M. of more than two Numbers at a time.

The method of finding the G.C.M. of any two Numbers is derived from four simple facts.

I. *Any Number is a Measure of its own Multiples only* (See page 130·).

II. *The Sum of two Multiples of a Number is also a Multiple of that Number.*

For, considering, for example, 8· as a Complex Unit, it is plain that,

∵ 5· + 7· = 12· ∴ 5· (*eights*) + 7· (*eights*) = 12· (*eights.*)
∴ 5th Multiple of 8· + 7th Multiple of 8· = 12th Multiple of 8·, and so on with any two Multiples of any other Number.

III. THE DIFFERENCE *of two Multiples of a Number is also a Multiple of that Number.*

For, ∵ 7· — 5· = 2· ∴ 7· (*eights*) — 5· (*eights*) = 2· (*eights*).
∴ 7th Multiple of 8· — 5th Multiple of 8· = 2nd Mult^{e} of 8·, and so on with any two Multiples of any other Number.

IV. ANY MULTIPLE *of a Multiple of a Number is also a Multiple of that Number.*

For, ∵ 7· × 2· = 14· ∴ 7· (*eights*) × 2· = 14· (*eights.*)
∴ 2nd Multiple of 7th Multiple of 8· = 14th Multiple of 8·.
The same is true of other Numbers.

These four facts, combined, constitute

PRINCIPLE XXII. *Every Measure of any two Numbers measures also*

a. THEIR SUM. *b.* THEIR DIFFERENCE.
c. ANY MULTIPLE OF EITHER OF THEM.

The process for finding the G.C.M. of two Numbers is exhibited in the following Example worked out.

Find the G.C.M. of 1387· and 2508·

```
1387·)2508·(1·
      1387·
      1121.)1387·(1·
            1121·
             266·)1121·(4·
                  1064·
                    57·)266·(4·
                        228·
                         38·)57·(1·
                             38·
Greatest Common Measure }
  of 1387· and 2508·    } = 19·)38·(2·
                                38·
```

It is here seen, that, to find G.C.M. of two Numbers, we Divide the Greater by the Less; then Divide 1st Divisor by Remainder; then, 2nd Divisor by 2nd Remainder; and so on, until we get no Remainder. *The Last Divisor is the Greatest Common Measure.*

To prove that this method is the true one, we have to shew two things.

First. That 19· *must* be a Common Measure of 1387· and 2508·.

Second. That there is no Common Measure of those two Numbers, greater than 19·.

For the first part of the proof, we trace the work upwards from 19·.

19· is a Measure of 38·	For 38· ÷ 19· leaves no Remainder.	
∴ 19· is a Measure of 57·	For 57· = 38· + 19·	PRINCIPLE XXII. *a.*
∴ 19· is a Measure of 228·	For 228· = 57· × 4·	PRINCIPLE XXII. *c.*
∴ 19· is a Measure of 266·	For 266· = 228· + 38·	PRINCIPLE XXII. *a.*
∴ 19· is a Measure of 1064·	For 1064· = 266· × 4·	PRINCIPLE XXII. *c.*
∴ 19· is a Measure of 1121·	For 1121· = 1064· + 57·	PRINCIPLE XXII. *a.*
∴ 19· is a Measure of 1387·	For 1387· = 1121· + 266·	PRINCIPLE XXII. *a.*
∴ 19· is a Measure of 2508·	For 2508· = 1387· + 1121·	PRINCIPLE XXII. *a.*

19· is, then, a Common Measure of 2508· *and* 1387·

For the second part of the proof, we trace the work downwards to 19·.

Now, if 19· be not the G.C.M. of 1387· and 2508·, some Number greater than 19· is their G.C.M. Suppose that a greater Number than 19· measures 1387· and 2508·; then,

∵ A greater No. than 19· measures 1387· and 2508·
∴ A greater No. than 19· measures 1121· For 1121· = 2508· — 1387· Pr. XXII. *b.*
∴ A greater No. than 19· measures 266· For 266· = 1387· — 1121· Pr. XXII. *b.*
∴ A greater No. than 19· measures 1064· For 1064· = 266· × 4· Pr. XXII. *c.*
∴ A greater No. than 19· measures 57· For 57· = 1121· — 1064· Pr. XXII. *b.*
∴ A greater No. than 19· measures 228· For 228· = 57· × 4· Pr. XXII. *c.*
∴ A greater No. than 19· measures 38· For 38· = 266· — 228· Pr. XXII. *b.*
∴ *A greater No. than* 19· *measures* 19· For 19· = 57· — 38· Pr. XXII. *b.*

But this conclusion is absurd. Consequently, the supposition that led us to it is false. There is, then, no greater Common Measure of 1387· and 2508· than 19·. Hence, 19· is the Greatest Common Measure of 1387· and 2508·, and the process is proved to be the true one.

EXERCISE LXVII.

Find G.C.M.

1· Of 5191· and 597719·
2· Of 7967· and 1355227·
3· Of 21781· and 1657219·
4· Of 42247· and 6991007·
5· Of 336841· and 15245449·
6· Of 170587· and 10484567·
7· Of 16507· and 557617·
8· Of 18664043· and 310067·
9· Of 27060823· and 438871·
10· Of 80892719· and 1683529·

Numbers prime to each other. Numbers which have no Common Measure are prime to each other, although neither of them may be a really Prime Number. (See page 43·)

Thus, although 8·, 15·, 49·, and 187· are all Composite Numbers, they are prime to each other, for they have no Common Measure.

We shall call Numbers which are prime to each other, "*Primes.*"

THE LOWEST TERMS OF A FRACTION.

We have already seen that the same Fraction may be expressed by different Numbers; for, by Principle XIII.,

$\frac{1}{2} = \frac{2}{4}$ or $\frac{3}{6}$ or $\frac{4}{8}$ or $\frac{5}{10}$ or $\frac{6}{12}$ or $\frac{7}{14}$ or $\frac{21}{42}$ or $\frac{500}{1000}$, &c. &c.

When a Fraction has its lowest possible Numerator and Denominator, we say it is in its lowest terms.

Thus, $\frac{21}{42}$, in its lowest terms, is written $\frac{1}{2}$. See above.

Now, since Dividing any Number by 1· makes no alteration in it, and since $\frac{3}{3}$ is equal to 1·; therefore,

$\frac{24}{36} \div \frac{3}{3} = \frac{24}{36} \div 1\cdot = \frac{24}{36}$; but, $\frac{24}{36} \div \frac{3}{3} = \frac{8}{12}$. See third method of Dividing Fraction by Fraction, p. 123·

$$\therefore\ \frac{24}{36} = \frac{24}{36} \div \frac{3}{3}.$$

The same is also evident from Principle XX. For,

$$\because \tfrac{8}{12} = \tfrac{8}{12} \times \tfrac{3}{3} = \tfrac{24}{36} \quad \therefore \tfrac{24}{36} \div \tfrac{3}{3} = \tfrac{8}{12} = \tfrac{24}{36}.$$

Hence, Dividing Numerator and Denominator by the same Number does not alter the value of the Fraction.

Extension of Principle XIII. *Multiplying or Dividing the Numerator and Denominator of a Fraction by the same Number makes no alteration in the value of that Fraction.*

To Reduce a Fraction to its lowest terms.

First. Find the G.C.M. of Numerator and Denominator.

Second. Divide Numerator and Denominator by their G.C.M.

Another Method.

First. Divide Numerator and Denominator by any Common Measure. This will give a New Numerator and Denominator.

Second. Divide New Numerator and Denominator by any Common Measure.

Third. Proceed thus until you find a Numerator and Denominator which are prime to each other.

A Fraction is in its lowest terms when Numerator and Denominator are prime to each other.

Example worked out. Reduce $\frac{6440}{11480}$ to its lowest terms.

By First Method. G.C.M. of 6440· and 11480· = 280·

Lowest terms of $\frac{6440}{11480} = \frac{6440}{11480} \div \frac{280}{280} = \frac{23}{41}$

By Second Method. (The successive Divisors stand over the Fractions.)

$$\overset{(10\cdot)}{\tfrac{6440}{11480}} = \overset{(4\cdot)}{\tfrac{644}{1148}} = \overset{(7\cdot)}{\tfrac{161}{287}} = \tfrac{23}{41} \text{ lowest terms of } \tfrac{6440}{11480}.$$

The Second Method is most frequently employed. And it will be made still more convenient by bearing in mind the following useful facts.

A Number is divisible
By 2· when its right-hand Digit is so.
By 3· when the Sum of its Digits is so.
By 4· when the No. expressed by its two right-hand Digits is so.
By 5· when it ends in 0, or 5·
By 6· when it is divisible by 2· and by 3·
By 8· when the No. expressed by its three right-hand Digits is so.
By 9· when the Sum of its Digits is so.
By 10· when it ends in 0.

By 11· when the Sums of its alternate Digits are equal; or, when the Difference of those Sums is a Multiple of 11·.
By 12· when it is divisible by 4· and by 3·.

EXERCISE LXVIII.

Reduce to their lowest terms the following Fractions.

1· $\frac{176}{484}$, and $\frac{48}{180}$
2· $\frac{7845}{96780}$
3· $\frac{8664}{8986}$, and ·875
4· $\frac{740}{8675}$, and ·106275
5· $\frac{764}{5240}$, and ·75
6· $\frac{644}{1728}$, and ·005
7· $\frac{768}{1024}$, and ·5
8· $\frac{3360}{4032}$, and ·25
9· $\frac{76923}{999999}$, and ·625
10· $\frac{13032}{607896}$
11· $\frac{21588}{756924}$
12· $\frac{37140}{439260}$, and ·125
13· $\frac{5220}{379980}$
14· $\frac{39320}{380840}$
15· $\frac{102276}{495828}$

Whenever the result of any operation contains a Fraction, it should be reduced to its lowest terms.

LEAST COMMON MULTIPLE.

The Least Common Multiple of any set of Numbers is the least Number which each of them *measures*, or divides without a Remainder.

The letters L.C.M. stand for the words, "Least Common Multiple." The process for ascertaining the L.C.M. of any set of Numbers is deduced from the following two facts.

I. *The* L.C.M. *of any set of primes is their Continued Product.*

4· and 5· are prime to each other. Their L.C.M. is the lowest Multiple of 4· which can be exactly divided by 5·.

Now, $4 \div 5 = \frac{4}{5}$
$8 \div 5 = 1\frac{3}{5}$
$12 \div 5 = 2\frac{2}{5}$
$16 \div 5 = 3\frac{1}{5}$
$20 \div 5 = 4$

The 5th Multiple of 4· is, then, the lowest which, being divided by 5·, gives an Integral Quotient. ∴ L.C.M. of 4· and 5· = 20· = 4· × 5· The same reasoning applies to any two Numbers prime to each other.

∴ L.C.M. *of any two primes is their product.*[*]

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[*] NOTE TO TEACHER. This is a deduction from the following General Proposition, for demonstration of which see Hall's Algebra, and De Morgan's Elements of Arithmetic, under the head, "Properties of Numbers."

"If any two Numbers, a and b be prime to each other, and each term of the series

$$b,\ 2 \cdot b,\ 3 \cdot b,\ \&c.\ (a - 1 \cdot)\ b,$$

be divided by a, each will leave a different Remainder." Whence follow two Corollaries.

I. Since the Remainders are all different, and $(a-1)$ in number, they must include all the Numbers from 1· to $(a - 1 \cdot)$.

II. Hence, lowest Multiple of b which leaves no Remainder when divided by a, is ab; or, L.C.M. of a and b is $a \times b$ when a is prime to b; which is the conclusion stated in the text.

Next, take *three* Numbers prime to each other, as 4·, 5·, and 7·.

Now, L.C.M. of 4· and 5· = 20· = 4· × 5·, and 20· is prime to 7·;
∴ L.C.M. of 20· and 7· = 20· × 7·
∴ L.C.M. of 4·, 5·, and 7· = 4· × 5· × 7·

The L.C.M. *of any three primes is their Continued Product.*

Again, take any *four* Numbers, prime to each other, as 3·, 4·, 5·, and 7·. L.C.M. of 4· and 5· = 20·; L.C.M. of 7· and 3· = 21·; and 20· is prime to 21·.

∴ L.C.M. of 4·, 5·, 3·, & 7· = L.C.M. of 20· & 21· = 4· × 5· × 3· × 7·.

In the same way it may be shewn that the L.C.M. of *any set* of Numbers, prime to each other, is their Continued Product.

II. *The* L.C.M. *of any set of Numbers whatever is the Continued Product of all their prime Factors.*

Since each of the six Numbers, 21·, 15·, 12·, 35·, 28·, and 20· is a Product of two Factors out of the set 3·, 7·, 5·, 4·, it is plain that L.C.M. of 21·, 15·, 12·, 35·, and 28· cannot be *less* than L.C.M. of 3·, 7·, 5·, and 4·. Neither can it be *greater*, for every Common Multiple of 3· and 5· is also a Multiple of 15·; every Common Multiple of 3· and 7· is also a Multiple of 21·, and so on.

Hence, L.C.M. of 21·, 15·, 12·, 35·, 28·, and 20·, is neither greater nor less than L.C.M. of all their prime Factors 3·, 7·, 5·, 4·; and the L.C.M. of these last is their Continued Product.

∴ L.C.M. of 21·, 15·, 12·, 35·, 28·, and 20· = 3· × 7· × 5· × 4· = 420·

The same reasoning will shew that the L.C.M. of any set of Numbers whatever is equal to the Continued Product of all their prime Factors.

Two steps, then, are necessary, in order to discover the L.C.M. of any set of Numbers.

I. Resolve them into their prime Factors.

II. Multiply these prime Factors continually together.

If any one of the Numbers be a Factor of any other of them, it may be neglected, for any Multiple of the Greater will be also a Multiple of the Less which is its Factor. (PRINCIPLE XXII. *c.*)

If any of the set be Prime Numbers, they will also be disregarded in the first step, inasmuch as, being Prime Numbers, they have no Factors but themselves and Unity.

FIRST EXAMPLE WORKED OUT. Find L.C.M. of 5·, 6·, 7·, 8·, 15·, 18·, 21·, 32·, 13·, and 19·.

Of these ten Numbers, the 5· is a Factor of 15·, the 6· is a Factor of 18·, the 7· is a Factor of 21·, and the 8· is a Factor of 32·.

Therefore, 5·, 6·, 7·, and 8· may be neglected. The two Prime Numbers 13· and 19· will also be disregarded in the first step.

We have, then, in the first place, to find the prime Factors of 15·, *18·, 21·, and 32·* only.

Example completed.

Factors neglected, 5·, 6·, 7·, 8·.

Prime Numbers set aside during 1*st step*, 13·, 19·.

Finding prime Factors of remaining Numbers.

3·)15·, 18·, 21·, 32·,
2·)5·, 6·, 7·, 32·,
5·, 3·, 7·, 16·

These I arrange in a horizontal line. I then seek a *Common Factor of as many as possible* of them. 15·, 18·, and 21·, have the Common Factor 3·; I divide these by 3·, and re-write the 32· in the same line with the Quotients. I again seek *a Common Factor of as many as possible* of this new set of Numbers. 6· and 32· having the Common Factor 2·, I divide these by 2·, and, in the same line with the Quotients, re-write the 5· and the 7·. The third line of Numbers being all prime to one another, the first step is completed, and it is ascertained that the prime Factors of 15·, 18·, 21· and 32· are 3·, 2·, 5·, 3·, 7· and 16·.* Now, taking in again the two prime Numbers which were set aside. (13· and 19·), we have

L.C.M. of 5·, 6·, 7·, 8·, 15·, 18·, 21·, 32·, 13·, and 19· = 3· × 2· × 5· × 3· × 7· × 16· × 13· × 19· = 2489760·.

It is evident that this No. will still be the L.C.M., if, among the given set of Numbers, we include any others which are Factors of any of them, as 2·, 3·, 4·, 9·, and 16·.

To find L.C.M. *of any given set of Numbers.*

First. Select from the given set of Numbers all those which are neither prime Numbers nor evident Factors of any of the others: and arrange them in a horizontal line.

Second. Divide as many as possible of these by their Greatest Common Factor. Re-write, in the same line with Quotients, the Numbers which cannot be divided by that Common Factor.

Third. Proceed similarly with this new line of Numbers; and with the next line; and so on, until you obtain a line of Numbers all prime to each other.

Fourth. Multiply continually together:

The last line of Numbers,
The Divisors, and
All the prime Numbers omitted at starting.

This Product is the L.C.M. of the given set of Numbers.

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* NOTE TO TEACHER. Observant pupils will object that the 16· is not prime to 2·. Strictly, the 16· should, by three successive divisions, be resolved into 2·, 2·, 2·, and 2·; but as no other Number would be affected by these divisions, which would be immediately cancelled by Multiplication, useless labour is avoided by stopping at this point.

Contraction. When any Number in the line does not exactly contain the Divisor, but does exactly contain a Factor of the Divisor, Divide that Number by that Factor of the Divisor.

$$\begin{array}{r|l} 6\cdot & 15\cdot,\ 18\cdot,\ 21\cdot,\ 32\cdot, \\ \hline & 5\cdot,\ \ 3\cdot,\ \ 7\cdot,\ 16\cdot \end{array}$$

This would have saved us one line in the last Example. For, taking as our Divisor 6· instead of 3·, and noting that 15· is divisible by 3·, and that 18· is also divisible by 3·, and 32· by 2·, we arrive at the same result as before, the only difference being that we have now to multiply by the single Factor 6·, instead of by the two Factors 3· and 2·. 12· is a very convenient Divisor, because, having *four* Factors, it gives us *five* chances of dividing, namely, by 2·, by 3·, by 4·, by 6·, or by 12·.

EXERCISE LXIX.

Find the L.C.M.

1· Of 7·, 12·, 15·, and 22·; also of 3·, 9·, 6·, 18·, and 25·
2· Of 1·, 2·, 3·, 4·, 5·, 6·, 7·, 8·, and 9·
3· Of 2·, 4·, 8·, 6·, 12·, 32·, and 48·
4· Of 34·, 17·, 68·, 2·, and 4·; also of 61·, 19·, 36·, and 9·
5· Of 128·, 144·, 35·, 15·, 25·, and 19·; also of 11·, 145·, and 209·
6· Of 9·, 16·, 7·, 72·, 42·, 63·, 21·, 176·, 72·, and 14·
7· Of 7·, 12·, 40·, 495·, 15·, and 11·
8· Of 15·, 16·, 18·, 77·, 22·, 55·, and 198·
9· Of 718·, 613·, and 549·; also of 7164·, and 9358·
10· Of 14·, 19·, 100·, 150·, 180·, 200·, and 211·
11· Of 8·, 16·, and 20·; also of 16·, 14·, 9·, 12·, and 18·
12· Of 144·, 729·, 18·, 27·, and 900·

LEAST COMMON DENOMINATOR.

The Common Denominator of any set of Fractions, found by the ordinary method, (explained on page 79·,) is the Continued Product of all their Denominators.

But when these are not prime to each other, time and Figures are economized by employing for Common Denominator their L.C.M. An Example will shew this.

Reduce $\frac{3}{4}$, $\frac{11}{12}$, $\frac{7}{8}$, and $\frac{2}{3}$ to a Common Denominator, and also to their Least Common Denominator.

Common Denominator by usual Method.	Least Common Denominator.
$\frac{3}{4} = \frac{864}{1152}$	$\frac{3}{4} = \frac{18}{24}$
$\frac{11}{12} = \frac{1056}{1152}$	$\frac{11}{12} = \frac{22}{24}$
$\frac{7}{8} = \frac{1008}{1152}$	$\frac{7}{8} = \frac{21}{24}$
$\frac{2}{3} = \frac{768}{1152}$	$\frac{2}{3} = \frac{16}{24}$

To Reduce Fractions to their Least Common Denominator.

First. Find the L.C.M. of all the Denominators. This is the L.C.D.

Second. Divide this L.C.D. by each Denominator, and Multiply its Numerator by Quotient. This gives the New Numerators.

Third. Write the L.C.D. under these.

EXAMPLE WORKED OUT. Reduce to their L.C.D. $\frac{7}{9}$, $\frac{5}{6}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{5}{12}$ and $\frac{13}{18}$. L.C.D. = L.C.M. of 9·, 6·, 4·, 3·, 12·, 18·, = 36·.

36· ÷ 9· = 4·; $\frac{7}{9} = \frac{28}{36}$
36· ÷ 6· = 6·; $\frac{5}{6} = \frac{30}{36}$
36· ÷ 4· = 9·; $\frac{1}{4} = \frac{9}{36}$
36· ÷ 3· = 12·; $\frac{2}{3} = \frac{24}{36}$
36· ÷ 12· = 3·; $\frac{5}{12} = \frac{15}{36}$
36· ÷ 18· = 2·; $\frac{13}{18} = \frac{26}{36}$

This is, in fact, Multiplying Numerator and Denominator of each Fraction by the same Number; namely, by that Factor of L.C.D. which corresponds with Denominator of that Fraction.

EXERCISE LXX.

Reduce to their Least Common Denominators,

1· $\frac{3}{8}$, $\frac{7}{12}$, $\frac{8}{15}$, $\frac{11}{20}$
2· $\frac{5}{14}$, $\frac{17}{42}$, $\frac{12}{77}$, $\frac{9}{56}$
3· $\frac{23}{34}$, $\frac{1}{2}$, $\frac{12}{17}$, $\frac{15}{68}$
4· $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{12}$
5· $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{13}{49}$
6· $\frac{5}{6}$ of $\frac{7}{8}$, $\frac{2}{3}$ of $\frac{3}{16}$, $\frac{3}{5}$ of $\frac{5}{48}$
7· $\frac{11}{128}$, $\frac{15}{144}$, $\frac{224}{315}$, $\frac{7}{36}$
8· $\frac{17}{21}$, $\frac{11}{12}$, $\frac{9}{14}$, $\frac{5}{18}$, $\frac{23}{24}$, $\frac{41}{54}$
9· $\frac{19}{36}$, $\frac{11}{16}$, $\frac{29}{144}$, $\frac{23}{27}$, $\frac{39}{56}$
10· $\frac{3}{4}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{5}{7}$, $\frac{5}{9}$, $\frac{7}{16}$, $\frac{3}{10}$

REDUCTION OF VULGAR FRACTIONS TO EQUIVALENT DECIMALS.

A Decimal Fraction has for its Denominator some Power of 10·. Certain Vulgar Fractions are equal to Decimal Fractions.

Thus, $\frac{1}{2} = \frac{5}{10}$, $\frac{1}{4} = \frac{25}{100}$, $\frac{3}{4} = \frac{75}{100}$, $\frac{1}{8} = \frac{125}{1000}$, &c.

Now, seeing that the Denominator of the Decimal must be a Multiple of the Denominator of the Vulgar Fraction, it is evident that, if a Vulgar Fraction be equal to a Decimal, its Denominator will measure some Power of 10·.

And, since the only prime Factors of any Power of 10· are *twos* and *fives*,

For, 10· = 2· × 5·
100· = 2· × 2· × 5· × 5·
1000· = 2· × 2· × 2· × 5· × 5· × 5·
10000· = 2· × 2· × 2· × 2· × 5· × 5· × 5· × 5·,

and so on, it is plain that a Vulgar Fraction is not equal to a Decimal, unless that Vulgar Fraction, when in its lowest terms, has a Denominator composed of the prime Factors 2· and 5·, or of their Powers.

Hence, any Vulgar Fraction is equal to a Decimal, if its Denominator be any of the following; namely, 2·, 4·, 5·, 8·, 16·, 20·, 25·, 32·, 40·, 50·, 64·, &c. &c.

And no Vulgar Fraction is equal to a Decimal, if, *when in its lowest terms*, its Denominator be any of the following; namely, 3·, 6·, 7·, 9·, 11·, 12·, 13·, 14·, 15·, 17·, 18·, 19·, &c. &c.

Nevertheless, each of the following is equal to a Decimal: $\frac{3}{6}$, $\frac{7}{14}$, $\frac{3}{15}$, $\frac{6}{15}$, $\frac{9}{15}$, $\frac{12}{15}$, $\frac{3}{12}$, $\frac{6}{12}$, $\frac{9}{12}$, $\frac{9}{18}$, because, when reduced to their lowest terms, these Fractions become $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{2}$.

It is most useful to be able to decide, at a glance, whether a Vulgar Fraction has, or has not, any equivalent Decimal.

EXERCISE LXXI.

Select from the following Vulgar Fractions those which have equivalent Decimals.

1· $\frac{11}{35}$, $\frac{5}{35}$, $\frac{7}{35}$, $\frac{15}{35}$
2· $\frac{4}{57}$, $\frac{3}{57}$, $\frac{38}{57}$, $\frac{19}{57}$
3· $\frac{1}{14}$, $\frac{2}{14}$, $\frac{12}{14}$, $\frac{7}{14}$
4· $\frac{4}{44}$, $\frac{12}{44}$, $\frac{11}{44}$, $\frac{33}{44}$
5· $\frac{4}{64}$, $\frac{6}{64}$, $\frac{16}{64}$, $\frac{15}{64}$
6· $\frac{19}{95}$, $\frac{5}{95}$, $\frac{15}{95}$, $\frac{25}{95}$
7· $\frac{24}{192}$, $\frac{13}{192}$, $\frac{32}{192}$, $\frac{33}{192}$
8· $\frac{15}{85}$, $\frac{75}{85}$, $\frac{34}{85}$, $\frac{51}{85}$
9· $\frac{64}{256}$, $\frac{19}{521}$, $\frac{16}{1024}$, $\frac{29}{128}$
10· $\frac{39}{768}$, $\frac{16}{3584}$, $\frac{999}{9216}$, $\frac{209}{1408}$
11· $\frac{108}{765}$, $\frac{78}{510}$, $\frac{196}{595}$, $\frac{864}{1020}$
12· $\frac{106}{848}$, $\frac{742}{848}$, $\frac{16}{96}$, $\frac{13}{52}$

Since $\frac{3}{8} = 3\cdot \div 8\cdot$, PRIN. XV., and $3\cdot \div 8\cdot = \cdot375$
$\therefore \frac{3}{8} = \cdot375$.

And we say, that the Vulgar Fraction $\frac{3}{8}$ is here Reduced to its equivalent Decimal, which process consists merely in expressing the Quotient of Numerator and Denominator as a Decimal instead of as a Vulgar Fraction. (See last twenty-seven Examples of Exercise LVIII., page 111·)

To Reduce a Vulgar Fraction to its Equivalent Decimal.

Divide Numerator by Denominator, and carry out Quotient-figures in Decimal Places.

EXERCISE LXXII.

Reduce to Equivalent Decimals,

1· $\frac{5}{8}$, $\frac{9}{12}$, $\frac{3}{4}$, $\frac{7}{16}$, $\frac{1}{64}$, $\frac{1}{2}$
2· $\frac{34}{512}$, $\frac{95}{5000}$, $\frac{8}{20}$, $\frac{39}{600}$
3· $\frac{81}{225}$, $\frac{96}{2048}$, $\frac{77}{1792}$
4· $\frac{7}{15}$ of $\frac{9}{32}$, & $\frac{28}{35} \times \frac{49}{128}$
5· $\frac{26}{75} \times \frac{6}{104}$, & $\frac{208}{288} \times \frac{279}{1625}$
6· $6\cdot \div 48\cdot$, & $4\frac{1}{2} \div 52\frac{4}{11}$
7· $143\cdot \div 1300\cdot$, & $1\frac{12}{17} \div 26\frac{89}{136}$
8· $\frac{1}{64} \div \frac{5}{15}$, & $27\frac{2}{15} \div 77\frac{1}{12}$
9· $\frac{442}{25} \div 3\frac{1}{5}$, & $3\frac{5}{7} \div 1\frac{89}{119}$
10· $1\frac{1}{5} \div 2\frac{14}{35}$, & $\frac{7}{12} \times \frac{9}{10} \times \frac{3}{14} \times \frac{7}{8}$

INTERMINATE OR ENDLESS DECIMALS.

In attempting to Reduce to an Equivalent Decimal a Vulgar Fraction which has no such equivalent, we obtain an **Interminate Decimal**; that is, one in which the Division can never be completed, for one or more of the Quotient-figures constantly recur.

In a **Repeating Decimal** a single Digit is constantly repeated.

Thus, $\frac{1}{9} = \cdot 1111$ &c., a Repeating Decimal.

In a **Circulating Decimal** a set of two or more Digits is constantly repeated.

Thus, $\frac{1}{7} = \cdot 142857\ 142857$ &c., a Circulating Decimal.

The repeating figure or figures constitute a **Period, Repetend,** or **Circle.**

A Repetend is distinguished by a dot over its first, and another over its last figure.

$\therefore\ \frac{1}{9} = \cdot\dot{1}$, and $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$

A **Pure Repetend** commences at the Decimal Point.

A **Mixed Decimal** is partly terminate and partly interminate.

Thus, $\frac{219}{1750} = \cdot 125\dot{1}4285\dot{7}$, a Mixed Decimal, for the last six Digits, only, repeat.

REDUCTION OF INTERMINATE DECIMALS TO THEIR EQUIVALENT VULGAR FRACTIONS.

I. DENOMINATOR OF A PURE REPETEND.

Since $\frac{1}{9} = \cdot 111$ &c., $= \cdot\dot{1}$
" $\frac{2}{9} = \cdot 222$ &c., $= \cdot\dot{2}$
" $\frac{3}{9} = \cdot 333$ &c., $= \cdot\dot{3}$

It follows that any pure Repetend of a Single Digit has for its Denominator the Digit 9.

And, since

$\frac{1}{99} = \frac{1}{9} \div 11\cdot = \cdot 111$ &c. $\div\ 11\cdot = \cdot 0101$ &c. $= \cdot\dot{0}\dot{1}$

$\frac{5}{99} = \frac{5}{9} \div 11\cdot = \cdot 555$ &c. $\div\ 11\cdot = \cdot 0505$ &c. $= \cdot\dot{0}\dot{5}$

And, since

$$\frac{30}{99} = \cdot 303030 \text{ \&c.} = \cdot\dot{3}\dot{0}$$

$$\frac{4}{99} = \cdot 040404 \text{ \&c.} = \cdot\dot{0}\dot{4}$$

$$\therefore\ \frac{34}{99} = \cdot 343434 \text{ \&c.} = \cdot\dot{3}\dot{4}$$

A Repetend of two Figures has for its Denominator 99.

And, similarly, any Pure Repetend is the Numerator of a Vulgar Fraction whose Denominator is as many nines as there are Places of Figures in the Repetend.

$$\cdot\dot{6}8\dot{4} = \tfrac{684}{999},\quad \cdot\dot{0}158\dot{4} = \tfrac{1584}{99999},\quad \cdot\dot{0}05\dot{4} = \tfrac{54}{9999}$$

II. Denominator of a Mixed Decimal.

Again, $\frac{1}{90} = \frac{1}{9} \div 10\cdot = \cdot111$ &c. $\div 10\cdot = \cdot011$ &c. $= \cdot0\dot{1}$

Also, $\frac{1}{900} = \cdot00\dot{1}$; $\frac{1}{9000} = \cdot000\dot{1}$; $\frac{1}{90000} = \cdot0000\dot{1}$

Hence, $\cdot0\dot{1} = \frac{1}{90}$ | $\cdot00\dot{0}4\dot{6} = \frac{46}{99900}$ | $\cdot0000\dot{6} = \frac{6}{90000}$

$\cdot00\dot{2} = \frac{2}{900}$ | $\cdot00\dot{5}0\dot{4} = \frac{504}{99900}$ | $\cdot000\dot{8}64\dot{3} = \frac{8643}{9999000}$

If there be one or more Significant Figures between the Decimal Point and the Repetend, the Denominator will be the same as if these were ciphers, as in above examples.

For $\cdot8\dot{1} = \frac{8}{10} + \frac{1}{90} = \frac{72}{90} + \frac{1}{90} = \frac{73}{90}$

$\cdot4\dot{6}\dot{2} = \frac{4}{10} + \frac{62}{990} = \frac{396}{990} + \frac{62}{990} = \frac{458}{990}$

Therefore, a Repetend separated from the Decimal Point by any number of Figures must have the same number of ciphers annexed to the nines in its Denominator.

III. Numerator of a Mixed Decimal.

A Mixed Decimal is equal to two Fractions. First, a Terminate Decimal, which has a Power of 10· for its Denominator; and, second, an Interminate Decimal which has its Denominator expressed by nines, or by nines and ciphers.

The sum of these two is the Vulgar Fraction equivalent to the whole Mixed Decimal.

Their Common Denominator may always be the Denominator of the Repetend Decimal.

And to Reduce the first Fraction (the Terminate Decimal) to this Common Denominator, its Numerator and Denominator must merely be multiplied by the nines in Denominator of second Fraction (the Interminate Decimal).

∴ If Repetend be a Single Digit, the Multiplier will be 9·
If Repetend contain two Digits, the Multiplier will be 99· and so on.

Now, Subtracting any Number from its 10th Multiple gives its 9th Multiple,
And, Subtracting any Number from its 100th Multiple gives its 99th Multiple.
&c. &c. &c.

$$\begin{aligned} \therefore\quad 9\cdot \text{ times } 645\cdot &= 6450\cdot - 645\cdot = 5805\cdot \\ 99\cdot \text{ times } 645\cdot &= 64500\cdot - 645\cdot = 63855\cdot \\ 999\cdot \text{ times } 645\cdot &= 645000\cdot - 645\cdot = 644355\cdot \\ &\text{\&c.} \end{aligned}$$

$$\begin{aligned} 7\cdot + 9\cdot \text{ times } 43\cdot &= 7\cdot + 430\cdot - 43\cdot = 437\cdot - 43\cdot = 394\cdot \\ 69\cdot + 99\cdot \text{ times } 25\cdot &= 69\cdot + 2500\cdot - 25\cdot = 2569\cdot - 25\cdot = 2544\cdot \\ &\text{\&c.} \end{aligned}$$

$$\therefore\quad \cdot 74\dot{3} = \frac{74}{100} + \frac{8}{900} = \frac{8 + (74 \times 9)}{900} = \frac{8 + 740 - 74}{900} = \frac{743 - 74}{900}.$$

$$\text{Again, } \cdot 35\dot{6}\dot{2} = \frac{85}{100} + \frac{62}{9900} = \frac{62 + (85 \times 99)}{9900} = \frac{62 + 8500 - 85}{9900} = \frac{8562 - 85}{9900}.$$

Which Examples shew, that the Numerator of the Vulgar Fraction, equivalent to a Mixed Decimal, is found by subtracting the Terminate part from the whole Mixed Decimal.

To Reduce any Interminate Decimal to its Equivalent Vulgar Fraction.

First. Take for Numerator all the Figures from Decimal Point to the end of the First Period, Subtracting the Number expressed by the non-repeaters, if any.

Second. Write for Denominator as many *nines* as there are Repeating Figures, followed by as many *ciphers* as there are non-repeaters.

Third. Reduce Vulgar Fraction to its lowest terms.

EXERCISE LXXIII.

Reduce to Equivalent Vulgar Fractions,

1· $\cdot\dot{3}$, $\cdot\dot{4}\dot{5}$, $\cdot\dot{6}1\dot{3}$, $\cdot\dot{4}57\dot{8}$;
2· $\cdot\dot{0}6\dot{4}$, $\cdot\dot{0}051\dot{3}$, $\cdot\dot{0}001\dot{4}$;
3· $\cdot\dot{0}1\dot{8}$, $\cdot0\dot{6}$, $\cdot06\dot{4}\dot{0}$;
4· $\cdot0004\dot{3}$, $\cdot00\dot{0}7649$, $\cdot00\dot{5}$;
5· $\cdot74\dot{3}\dot{2}$, $\cdot3\dot{4}1\dot{2}$, $\cdot051\dot{6}\dot{9}$;
6· $\cdot\dot{8}431\dot{6}$, $\cdot085\dot{4}0\dot{3}$;
7· $\cdot26\dot{4}2857\dot{1}$, $\cdot759\dot{6}325\dot{8}$;
8· $\cdot0001\dot{6}8\dot{4}$, $\cdot09\dot{0}0\dot{5}$;
9· $\cdot\dot{1}\dot{7}$, $\cdot1\dot{7}$, $\cdot\dot{0}1\dot{7}$, $\cdot0\dot{1}\dot{7}$;
10· $\cdot01\dot{7}$, $\cdot\dot{0}01\dot{7}$, $\cdot0\dot{0}1\dot{7}$, $\cdot00\dot{1}\dot{7}$.

ADDITION, MULTIPLICATION, SUBTRACTION, AND DIVISION OF INTERMINATE DECIMALS.

First. Reduce the Interminate Decimals to their equivalent Vulgar Fractions.

Second. Add, Subtract, Multiply, or Divide, as may be required, and Reduce the Result to its Equivalent Decimal.

FIRST EXAMPLE WORKED OUT. Find the sum of $\cdot\dot{5}$, $\cdot 0\dot{1}\dot{6}$, and $\cdot 34\dot{2}\dot{5}$

$$\cdot\dot{5} + \cdot 0\dot{1}\dot{6} + \cdot 34\dot{2}\dot{5} = \frac{5}{9} + \frac{16}{990} + \frac{3391}{9900} = \frac{5500 + 160 + 3391}{9900} = \frac{9051}{9900} = \cdot 91\dot{4}\dot{2}$$

SECOND EXAMPLE WORKED OUT. What is the difference of $\cdot 063\dot{4}$ and $\cdot 4\dot{3}$?

$$\cdot 4\dot{3} - \cdot 063\dot{4} = \frac{39}{90} - \frac{571}{9000} = \frac{3900 - 571}{9000} = \frac{3329}{9000} = \cdot 369\dot{8}$$

THIRD EXAMPLE WORKED OUT. Multiply $\cdot\dot{3}\dot{6}$ by $\cdot 2\dot{5}$

$$\cdot\dot{3}\dot{6} \times \cdot 2\dot{5} = \frac{36}{99} \times \frac{23}{90} = \frac{4}{11} \times \frac{23}{90} = \frac{92}{990} = \cdot 0\dot{9}\dot{2}$$

FOURTH EXAMPLE WORKED OUT. Divide $\cdot\dot{6}$ by $2\cdot\dot{3}$

$$\cdot\dot{6} \div 2\cdot\dot{3} = \frac{6}{9} \div 2\frac{3}{9} = \frac{2}{3} \div \frac{7}{3} \left(\doteq \frac{2}{3} \times \frac{3}{7} \text{ and}\right) = \frac{2}{7} = \cdot\dot{2}8571\dot{4}$$

EXERCISE LXXIV.

1· Add together $\cdot 381\dot{3}$, $\cdot 42$, $\cdot 5\dot{2}1\dot{6}$, and $\cdot 0\dot{6}\dot{3}$
2· Find the sum of $15\cdot 875$, $7\cdot\dot{3}1\dot{6}$, $19\cdot 0\dot{0}\dot{4}$ and $\cdot 63\dot{2}\dot{5}$
3· From $\cdot\dot{4}2\dot{7}$ take $\cdot 0\dot{3}\dot{4}$; and from $24\cdot 3\dot{8}\dot{4}$ take $9\cdot 0\dot{7}\dot{2}$
4· What must be subtracted from $24\cdot 3\dot{8}\dot{4}$ to leave $15\cdot 3\dot{1}\dot{2}$?
5· By how much does $84\cdot 769\dot{7}$ fall short of $476\cdot 3\dot{2}$?
6· What must be added to $\cdot 0382$ to make up $3\cdot 85\dot{6}\dot{4}$?
7· Multiply $7\cdot\dot{7}\dot{2}$ by $\cdot\dot{2}9\dot{7}$; and $3\cdot\dot{9}7\dot{3}$ by $8\cdot$
8· Multiply $8574\cdot 3$ by $87\cdot\dot{5}$; and $421\cdot\dot{4}2\dot{1}$ by $\cdot 00\dot{2}$
9· Find the 4th Power of $\cdot\dot{3}\dot{4}$, and the Cube of $\cdot 1\dot{5}$
10· Divide $5\cdot 764\dot{8}$ by $\cdot 6\dot{8}$, and $234\cdot\dot{6}$ by $\cdot\dot{7}$
11· Divide $1327\cdot 871\dot{9}$ by $7\cdot 68\dot{4}$
12· Divide $41\cdot\dot{9}71\dot{7}$ by $34\cdot 1\dot{2}$

The above method is laborious, but there is no other so simple and accurate, especially for Multiplication and Division.

Interminate Decimals should be, as much as possible, avoided, as may be done by attending to the remarks on page 140·.

They will, nevertheless, sometimes occur in the course of ordinary Arithmetical operations, and then, if strict accuracy be not demanded, it will suffice to increase the first Figure of the Repetend by 1, and to consider the Decimal as finite, and ending with the Digit so increased.

So that $3\cdot 429\dot{5}\dot{8}$ would be called $3\cdot 4296$, and so on.

RATIO AND PROPORTION.

The **Ratio** of one Number to another is the Fraction which the first is of the second.

6· is $\frac{2}{3}$ of 9·; ∴ Ratio of 6· to 9· = $\frac{2}{3}$.
9· is $1\frac{1}{2}$ times 6·; ∴ Ratio of 9· to 6· = $\frac{3}{2}$.

The **Inverse Ratio** of any two Numbers, is the Ratio of their Reciprocals.

Thus, Ratio of $\frac{1}{8}$ to $\frac{1}{4}$ = Inverse Ratio of 8· to 4·

To express the Ratio of two Numbers.

First. Divide the first Number by the second, expressing the Quotient Fractionally. (PRINCIPLE XV.) The Fraction thus found is the Ratio.

Second. Reduce the Ratio to its lowest terms.

EXAMPLES WORKED OUT. What are the Ratios

1· Of 14· to 6·?
2· Of 6· to 14·?
3· Of 9· to $\frac{4}{5}$?
4· Of $\frac{4}{5}$ to 9·?
5· Of $11\frac{1}{4}$ to $3\frac{1}{2}$?
6· Of $3\frac{1}{2}$ to $11\frac{1}{4}$?
7· Of 3·75 to 7·5?
8· Of 7·5 to 3·75?

1· Ratio of 14· to 6· $= \frac{14}{6} = \frac{7}{3}$
2· Ratio of 6· to 14· $= \frac{6}{14} = \frac{3}{7}$
3· Ratio of 9· to $\frac{4}{5}$ $= \frac{9}{1} \div \frac{4}{5} = \frac{45}{4}$
4· Ratio of $\frac{4}{5}$ to 9· $= \frac{4}{5} \div \frac{9}{1} = \frac{4}{45}$
5· Ratio of $11\frac{1}{4}$ to $3\frac{1}{2}$ $= \frac{45}{4} \div \frac{7}{2} = \frac{90}{28} = \frac{45}{14}$
6· Ratio of $3\frac{1}{2}$ to $11\frac{1}{4}$ $= \frac{7}{2} \div \frac{45}{4} = \frac{28}{90} = \frac{14}{45}$
7· Ratio of 3·75 to 7·5 $= 3·75 \div 7·5 = ·5 = \frac{1}{2}$
8· Ratio of 7·5 to 3·75 $= 7·5 \div 3·75 = 2·$

EXERCISE LXXV.

Find the Ratio

1· Of 25· to 15· and of 15· to 25·
2· Of 42· to 39· and of 39· to 42·
3· Of 64· to 40· and of 40· to 64·
4· Of 24· to 60· to 108· and to 192·
5· Of 12· to 22· to 28· and to 64·
6· Of 78· to 112· and of 112· to 42·
7· Of 35· to $3\frac{1}{2}$ and of $2\frac{1}{2}$ to 30·
8· Of $7\frac{1}{2}$ to 60· and of $4\frac{1}{4}$ to 51·
9· Of $\frac{7}{8}$ to 14· and of 14· to $\frac{7}{8}$
10· Of $4\frac{1}{2}$ to 22· and of 6· to $1\frac{1}{4}$
11· Of $1\frac{1}{2}$ to 6· and of $2\frac{1}{2}$ to 10·
12· Of $7\frac{1}{2}$ to 30· and of $3\frac{3}{4}$ to 15·
13· Of 25· to 45· and of 72· to 32·
14· Of ·8125 to 3·125

The two Numbers of which the Ratio is sought or expressed are called the "Terms of the Ratio." The first Term is the **Antecedent**, and the second is the **Consequent**

It is evident that in Concrete Numbers the Antecedent and the Consequent must express the same kind of unit (PRINCIPLE V., page 35·) Any one Sum of Money is either a Part or Multiple of any other Sum of Money.

There exists, then, a Ratio between any two Sums of Money.

Ratio of 6· *shillings* to 9· *shillings* $= \frac{2}{3}$. For 6·*s.* $= \frac{2}{3}$ of 9·*s.* But there is no Ratio between 6· *shillings* and 9· *men*, for 6· *shillings* are neither Fraction nor Multiple of 9· *men.*

Similarly, it is ridiculous to speak of the Ratio of 14· *pence* to 2· *lb. of butter;* or of 24· *hours* to 48· *bushels.*

These expressions are absurd, because they really imply that *men* may become *shillings*, and that *pence* can literally be turned into *butter*, and *bushels* into *hours.* (See PRINCIPLE V. p. 35·)

COROLLARY TO PRINCIPLE V. *There is no Ratio between unlike Concrete Quantities.*

This is a truth of the utmost importance.

PROPORTION.

Proportion is the equality of Ratios. If one pair of Numbers have the same Ratio as another pair have, the four form a Proportion, and are called **Proportionals.**

And we say, "The First Term *is to* the Second *as* the Third *is to* the Fourth."

Thus, 14· $= \frac{2}{3}$ of 21·; and 8· $= \frac{2}{3}$ of 12·
∴ Ratio of 14· to 21· = Ratio of 8· to 12·

The four Numbers, 14·, 21·, 8·, and 12· are, therefore, Proportionals.

And 14· *is to* 21· *as* 8· *is to* 12· Also 8· *is to* 12· *as* 14· *is to* 21·

We may also say,

As 14· *is to* 21· *so is* 8· *to* 12· Or, *as* 8· *is to* 12· *so is* 14 *to* 21·

Arithmetical Symbols. The signs of Proportion are : and ::, the former stands for "*is to*" or "*to*," and the latter for "*as*" or "*so is.*" Hence, the above Proportions are expressed arithmetically thus:

14· : 21· :: 8· : 12·	As 14· : 21· :: 8· : 12·
8· : 12· :: 14· : 21·	As 8· : 12· :: 14· : 21·

When four Numbers are Proportionals, the first and fourth terms are called the **Extremes,** and the second and third are called the **Means.**

METHOD OF FINDING ANY TERM OF A PROPORTION.

In each of the following Proportions one of the four terms is wanting. Until it be found, we put a note of interrogation in its place.

FIRST EXAMPLE WORKED OUT. FIRST EXTREME UNKNOWN. What Number bears the same Ratio to 24· as 5· bears to 6·?

? : 24· :: 5· : 6· Now 5· = $\frac{5}{6}$ of 6·

∴ *Unknown Term* = $\frac{5}{6}$ of 24· = 24· × $\frac{5}{6}$ = 20· (PRIN. IX.)

SECOND EXAMPLE WORKED OUT. FIRST MEAN UNKNOWN. To what Number has 12· the same Ratio as 3· has to 7·?

12· : ? :: 3· : 7· Now Ratio of 3· to 7· = $\frac{3}{7}$

∴ 12· = $\frac{3}{7}$ *of Unkn. Term*, ∴ 4· = $\frac{1}{7}$ *of Unkn. Term.*

∴ *Unknown Term* = 4· × 7· = 28· = 12· × 7· ÷ 3·

THIRD EXAMPLE WORKED OUT. SECOND MEAN UNKNOWN. 7· has to 5· the same Ratio as *a Certain Number* has to 10· What is that Number?

7· : 5· :: ? : 10· Now, 7· = $\frac{7}{5}$ of 5·

∴ *Unkn. Term* = $\frac{7}{5}$ of 10· = 10· × $\frac{7}{5}$ = 14· (PRIN. IX.)

FOURTH EXAMPLE WORKED OUT. LAST EXTREME UNKNOWN. 12· has the same Ratio to 54· as 42· has to *a Certain Number*. What is that Number?

12· : 54· :: 42· : ? Now, 12· = $\frac{2}{9}$ of 54·

∴ 42· = $\frac{2}{9}$ *of Unkn. Term*, ∴ 21· = $\frac{1}{9}$ *of Unkn. Term.*

∴ *Unknown Term* = 189· = 42· × 54· ÷ 12·

The Unknown Term in each of the above instances has been ascertained by Inspection, that is to say, without the aid of any fixed Rule, such as we will now establish.

Take any four Proportionals, as 6·, 9·, 14·, and 21·

∴ 6· : 9· :: 14· : 21·

Now, by merely expressing each of the two Antecedents "in terms of" its Consequent, we have

$\frac{2}{3}$ of 9· : 9· :: $\frac{2}{3}$ of 21· : 21·

Hence, *The Product of the Extremes* = $\frac{2}{3}$ of 9· × 21· = $\frac{2}{3}$ × 9· × 21· (PRINCIPLE IX.)

And, *The Product of the Means* = 9· × $\frac{2}{3}$ of 21· = 9· × $\frac{2}{3}$ × 21· (PRINCIPLE IX.)

But, by PRINCIPLE VII., $\frac{2}{3}$ × 9· × 21· = 9· × $\frac{2}{3}$ × 21·

∴ *The Product of the Means = The Product of the Extremes.*

If we express each Consequent "in terms of" its Antecedent, the result must be the same. For, we then have

$$6\cdot : \tfrac{3}{2} \text{ of } 6\cdot :: 14\cdot : \tfrac{3}{2} \text{ of } 14\cdot$$

Where Product of Extremes $= 6\cdot \times \frac{3}{2} \times 14\cdot$; Product of Means $= \frac{3}{2} \times 6\cdot \times 14\cdot$

And, $6\cdot \times \frac{3}{2} \times 14\cdot = \frac{3}{2} \times 6\cdot \times 14\cdot$ By PRIN. VII., as before.

Precisely the same reasoning applies to Proportional Fractions. For example,

$\frac{1}{4} : \frac{5}{8} :: \frac{7}{11} : \frac{35}{22}$. Now $\frac{1}{4} = \frac{2}{5}$ of $\frac{5}{8}$, and $\frac{7}{11} = \frac{2}{5}$ of $\frac{35}{22}$; therefore the four Proportionals may be written thus,

$$\tfrac{2}{5} \text{ of } \tfrac{5}{8} : \tfrac{5}{8} :: \tfrac{2}{5} \text{ of } \tfrac{35}{22} : \tfrac{35}{22}.$$

Where Product of Extremes $= \frac{2}{5} \times \frac{5}{8} \times \frac{35}{22}$. And Product of Means $= \frac{5}{8} \times \frac{2}{5} \times \frac{35}{22}$.

And $\frac{2}{5} \times \frac{5}{8} \times \frac{35}{22} = \frac{5}{8} \times \frac{2}{5} \times \frac{35}{22}$. By PRINCIPLE VII.

PRINCIPLE XXIII. *When four Numbers are Proportionals, the Product of the Extremes is equal to the Product of the Means.*

This is the fundamental Principle of the celebrated "Rule of Three," in which three of the four Proportionals are given to find the Fourth.

Suppose we have the two Means and one Extreme given, to find the other Extreme.

Product of means = Product of Extremes.

Now, the known Extreme is one Factor of this Product, and the unknown Extreme is the Corresponding Factor of it.

But, by PRINCIPLE XX., The Product divided by one Factor yields the Corresponding Factor.

∴ *Either Extreme* = *Product of Means* ÷ *The other Extreme.*
Also *Either Mean* = *Product of Extremes* ÷ *The other Mean.*

So that 1st Term = 2nd × 3rd ÷ 4th | 3rd Term = 1st × 4th ÷ 2nd
2nd Term = 1st × 4th ÷ 3rd | 4th Term = 2nd × 3rd ÷ 1st

We can verify these conclusions by testing each Term of the following Proportion. 16· : 44· :: 12· : 33·. Here,

$16\cdot = 44\cdot \times 12\cdot \div 33\cdot$ | $12\cdot = 16\cdot \times 33\cdot \div 44\cdot$
$44\cdot = 16\cdot \times 33\cdot \div 12\cdot$ | $33\cdot = 44\cdot \times 12\cdot \div 16\cdot$

We always manage in the "Rule of Three" that the unknown Term shall be the fourth.

To find a Fourth Proportional to three Given Numbers.

Multiply Second and Third together and Divide by the First.

If the Numbers represent Concrete Quantities, take care that the First and Second be of the same kind. The Third must be of the same kind as the Fourth.

The Fourth is at once expressed fractionally by writing Second and Third as Factors of Numerator, and First as Denominator; Thus:

$$9\cdot\ :\ 11\cdot\ ::\ 31\cdot\ :\ \frac{11\times 31}{9}$$

It is sometimes convenient to leave the Fourth Term in this form.

FIRST EXAMPLE WORKED OUT. Find a Fourth Proportional to 16·*s*, ¾lb., and 5½lb.

Here, 16·*s.* must be Third. Either of the others may be First.

For ¾lb. : 5½lb. :: 16·*s.* : 16·*s.* × 5½ ÷ ¾; (or 117⅓*s.*)
And 5½lb. : ¾lb. :: 16·*s.* : 16·*s.* × ¾ ÷ 5½; (or $2\frac{2}{11}$*s.*)

Proof of First Operation. 117⅓ × ¾ = 88· = 16· × 5½

The Product of Means = Product of Extremes.

Or thus: $\frac{3}{4} = \frac{3}{22}$ of 5½, and 16· = $\frac{3}{22}$ of 117⅓.

Ratio of First to Second = Ratio of Third to Fourth.

The Four Numbers are Proportionals.

Observe. As soon as we proceed to employ the Second Term, 5½, as a Multiplier, it ceases to represent pounds; for it is impossible to Multiply anything by pounds. The Product of 16·*s.* × 5½ is 88· *shillings;* hence the First Term, ¾, also ceases to represent part of a pound weight, so soon as we employ it to divide 88· *shillings.* For, although 88· *lb.* can be divided by ¾ *lb.*, 88· *shillings* cannot.

EXERCISE LXXVI.

Complete the following Proportions:

1· As 9· : 42· :: £8· : ?
2· As 13· : 64· :: 14·*s.* : ?
3· As 8·oz : 11·oz :: 3· yds. : ?
4· As 16·*s.* : 15·*s.* :: 48· tons : ?
5· As 28· : 21· :: 1· : ?
6· As 33· : 57· :: 6· : ?
7· As 54· : 21· :: 36· : ?
8· As 112· : 36· :: 44· : ?

REDUCTION OF RATIO. CANCELLING.

When we Multiply by any Number, and then Divide the Product, the result of this double operation does not depend on the magnitude of Multiplier and Divisor, but on their Ratio to each other.

If Divisor be equal to Multiplier, Result will be equal to Multiplicand; and the Division will *cancel* the Multiplication. $34\cdot5 \times 19\cdot75 \div 19\cdot75 = 34\cdot5$

If Multiplier be double of Divisor, Result will be double of Multiplicand.

$$34\cdot5 \times 19\cdot75 \times 2\cdot \div 19\cdot75 = 34\cdot5 \times 2\cdot = 69\cdot$$

Where the Common Factor (namely 19·75) is **Cancelled** in Multiplier and Divisor.

In the same way any Common Factor may be Cancelled in Multiplier and Divisor; and, consequently, by PRINCIPLE VI., in Multiplicand and Divisor: but never, of course, in Multiplier and Multiplicand.

Hence, having arranged the three given terms of a Proportion so that the Unknown Term shall be fourth, we may shorten the work by cancelling any Common Factor in First and Second Terms, or in First and Third, but never in Second and Third.

$$\left.\begin{array}{l}\text{As } \cancel{9}\cancel{5}\cdot : \cancel{1}\cancel{1}\cancel{4}\cdot \\ \text{or as } 5\cdot : \quad 6\cdot\end{array}\right\} :: 7\cdot : \frac{7\times6}{5} \text{ or } 8\tfrac{2}{5}.$$

Here, the First and Second Terms have the Common Factor 19· Cancelling this Common Factor, the Fourth Proportional is found by Multiplying 7· by 6· and Dividing by 5·, instead of Multiplying by 114· and Dividing by 95·

Cancelling, as employed in working out Proportionals, may be accounted for in another way.

The Fourth Term being the same Fraction (Proper or Improper) of the Third as the Second is of the First, and that Fraction being the Ratio of the Second to the First, it is plain that

$$\textit{Fourth Term} = \textit{Third} \times \textit{Ratio of Second to First}$$
$$= \textit{Third} \times \frac{\textit{Second}}{\textit{First}} = \textit{Second} \times \frac{\textit{Third}}{\textit{First}}$$

And Cancelling in First and Second or First and Third is merely Reducing the Multiplying Fraction to its lowest terms by the application of PRINCIPLE XIII. (See p. 134·)

Cancelling is of great utility when we have to Divide the Continued Product of one set of Numbers by that of another set.

EXAMPLE WORKED OUT. How many times is the continued Product of 35·, 51·, 85·, 112·, and 42·, contained in that of 11·, 27·, 374·, 98·, and 56·?

$$\left.\begin{array}{c}\textit{Result}\\ \textit{expressed}\\ \textit{Fractionally}\end{array}\right\} = \frac{11\cdot\times\overset{3\cdot}{\overset{\cancel{9}\cdot}{\cancel{27}\cdot}}\times\overset{11\cdot}{\overset{\cancel{22}\cdot}{\cancel{374}\cdot}}\times\overset{\cancel{2}\cdot}{\overset{\cancel{14}\cdot}{\cancel{98}\cdot}}\times\cancel{56}\cdot}{\underset{5\cdot}{\cancel{35}\cdot}\times\underset{\cancel{3}\cdot}{\cancel{51}\cdot}\times 85\cdot\times\underset{\cancel{2}\cdot}{\cancel{112}\cdot}\times\underset{\underset{\cancel{3}\cdot}{\cancel{21}\cdot}}{\cancel{42}\cdot}} = \frac{11\cdot\times 3\cdot\times 11\cdot}{5\cdot\times 85\cdot} = \frac{363}{425}$$

The Numbers of the upper line are all to be considered as Multipliers, and those of the lower line as Divisors. PRINCIPLE XV.

Then, selecting one of each, having a Common Factor, we cancel

In 374· and 51· the Common Factor 17·; setting down 22· and 3·;
In 27· and 3· the Common Factor 3·; setting down 9·;
In 22· and 42· the Common Factor 2·; setting down 11· and 21·;
In 98· and 21· the Common Factor 7·; setting down 14· and 3·;
In 14· and 35· the Common Factor 7·; setting down 2· and 5·;
In 56· and 112· the Common Factor 56·; setting down 2·;
In 2· and 2· the Common Factor 2·;
In 9· and 3· the Common Factor 3·; setting down 3·

We do not set down 1, as it is useless either for Multiplication or Division.

The Multipliers being all Cancelled except 11·, 3·, 11·, the Continued Product of these is the Numerator of Fractional Result; while that of 5· and 85· (the only uncancelled Divisors,) becomes Denominator.

TRANSPOSITION OF MULTIPLICATION AND DIVISION.

It has been shewn, in PRINCIPLE XVIII., page 107·, that Multiplying the Dividend (before Dividing) has the same effect as Multiplying the Quotient; and that previously Dividing the Dividend also Divides the Quotient of the subsequent Division.

In other words: Division followed by Multiplication produces the same result as Multiplication followed by Division, supposing, of course, the Divisor and Multiplier to remain unchanged.

Hence, $\frac{4}{5}$ of $40\cdot \doteq 40\cdot \div 5\cdot \times 4\cdot$; or, $\doteq 40\cdot \times 4\cdot \div 5\cdot = 32\cdot$
Also, $\frac{5}{8}$ of $1\cdot = \frac{1}{8}$ of $5\cdot \doteq 1\cdot \div 8\cdot \times 5\cdot$; or, $1\cdot \times 5\cdot \div 8\cdot$

TO FIND ANY FRACTION OF A NUMBER.

Either, Divide by Denominator and Multiply Quotient by Numerator; or, Multiply by Numerator and Divide Product by Denominator.

The latter method is the more convenient, as it avoids Multiplication of Fractions. For example,

By 1st Method, $\frac{4}{7}$ of $32\cdot \doteq 32\cdot \div 7\cdot \times 4\cdot \doteq 4\frac{4}{7} \times 4\cdot = 18\frac{2}{7}$
By 2nd Method, $\frac{4}{7}$ of $32\cdot \doteq 32\cdot \times 4\cdot \div 7\cdot \doteq 128\cdot \div 7\cdot = 18\frac{2}{7}$

MULTIPLICATION OF AN INTEGER, DECIMAL, OR MIXED NUMBER BY A MIXED NUMBER.

To Multiply by a Mixed Number, it is not always most convenient to reduce the Multiplier to an Improper Fraction, as directed on page 77· Instead of doing so, we may Multiply first by the Fraction, and then by the Integer, and add the results together.

EXAMPLE WORKED OUT. Multiply 6148329· by $4\frac{7}{8}$

$$\begin{array}{rl}
 & 6148329\cdot \\
 & \underline{\qquad\quad 7\cdot} \\
8\cdot) & \underline{43038303\cdot} \\
 & 5379787\frac{7}{8} \doteq \frac{7}{8} \text{ of } 6148329\cdot \\
 & \underline{24593316\cdot} = 4\cdot \text{ times } 6148329\cdot \\
 & \underline{\underline{29973103\frac{7}{8}}} = 4\frac{7}{8} \text{ of } 6148329\cdot = 6148329\cdot \times 4\frac{7}{8}.
\end{array}$$

PRINCIPLE IX.

Working by the other method we should have a Multiplier 39· instead of 7· and 4·, and a Dividend 239784831· instead of 43038303·

For $6148329\cdot \times 4\frac{7}{8} \times \frac{6148329}{1} \times \frac{39}{8} = \frac{239784831}{8} = 29973103\frac{7}{8}$

EXERCISE LXXVII.

Find the Product

1· Of 6148354· × $\frac{7}{12}$
2· Of 7000645$\frac{1}{9}$ × $\frac{3}{8}$
3· Of 56432159$\frac{4}{7}$ × $\frac{11}{15}$
4· Of 41964328$\frac{6}{11}$ × $\frac{7}{16}$
5· Of 9463714·5 × 6$\frac{5}{8}$
6· Of 732142·14 × 342$\frac{7}{16}$
7· Of 61483·005 × 7487$\frac{7}{8}$
8· Of 541288·722 × 6459$\frac{11}{13}$

DIVISION OF INTEGER, DECIMAL OR MIXED NUMBER BY A FRACTION OR MIXED NUMBER.

To Divide by a Fraction or Mixed Number, we adopt the first method explained on page 123·; namely, Reducing Divisor and Dividend to a Common Denominator, and then Dividing New Numerator of Dividend by New Numerator of Divisor.

First Example worked out. Divide 431682· by $4\frac{2}{7}$.

$431682\cdot \div 4\frac{2}{7} = \frac{3021774}{7} \div \frac{30}{7} = 3021774\cdot \div 30\cdot = 100725\frac{4}{5}$

And the work would appear thus:

$$\begin{array}{rl} 4\frac{2}{7} &)431682\cdot \\ 7 & 7\cdot \\ \hline 30\cdot &)3021774\cdot \\ \hline & 100725\frac{4}{5} \\ \hline\hline \end{array}$$

Here we Multiply Dividend and Divisor by Denominator of the Dividend-Fraction.

The New Divisor and Dividend are so many *sevenths.*

Second Example worked out. Divide $31684\frac{5}{8}$ by $11\frac{6}{7}$.

$$\begin{array}{rl} 11\frac{6}{7} &)31684\frac{5}{8}(\\ 7\cdot & 8\cdot \\ \hline 83\cdot & 253477\cdot \quad \text{PRIN. XIX.} \quad \text{COR. I.} \\ 8\cdot & 7\cdot \\ \hline 664\cdot &)1774339\cdot(2672\frac{131}{664} \end{array}$$

In Examples like the present, Multiply Divisor and Dividend, respectively, *first* by Denominator of its *own* Fraction, and Product by Denominator of the other Fraction.

Here, New Divisor and Dividend are so many *fifty-sixths.*

EXERCISE LXXVIII.

1· Divide 2874126· by $7\frac{3}{4}$
2· Divide 34015·404 by $23\frac{3}{7}$
3· Divide 67893284· by $29\frac{5}{6}$
4· Divide 85794361· by $11\frac{3}{8}$
5· Divide 87548679· by $42\frac{3}{8}$
6· Divide 72386748· by $55\frac{4}{11}$
7· Divide 87654869· by $24\frac{11}{12}$
8· Divide 87289654· by $38\frac{10}{17}$
9· Divide 897538· by $365\frac{1}{4}$
10· Divide 829642· by $23\frac{3}{7}$

EXERCISE LXXIX.

1· Divide $718432\frac{9}{11}$ by $14\frac{3}{5}$
2· Divide $910008\frac{4}{5}$ by $23\frac{6}{7}$
3· Divide $8421684\frac{5}{9}$ by $47\frac{4}{5}$
4· Divide $732186\frac{9}{13}$ by $104\frac{6}{7}$
5· Divide $318462\frac{5}{8}$ by $29\frac{7}{12}$
6· Divide $8461235\frac{6}{17}$ by $95\frac{3}{18}$
7· Divide $9342186\frac{4}{31}$ by $632\frac{4}{15}$
8· Divide $3416215\frac{8}{17}$ by $193\frac{8}{19}$
9· Divide $7004061\frac{13}{19}$ by $394\frac{9}{22}$
10· Divide $9432184\frac{17}{25}$ by $346\frac{17}{26}$

COMPLEX FRACTIONS.

A Complex Fraction is one whose Numerator or Denominator is a Simple or Compound Fraction, or a Mixed Number.

Thus, $\frac{11\frac{1}{2}}{5}$, $\frac{4}{7\frac{2}{3}}$, $\frac{5\frac{4}{17}}{43\frac{1}{8}}$, $\frac{\frac{3}{4}}{\frac{5}{8}\text{ of }\frac{4}{13}}$, $\frac{5\frac{4}{11}\text{ of }\frac{6}{18}\times\frac{7}{8}}{6\frac{1}{4}+\frac{3}{4}\text{ of }7\cdot}$ are Complex Fractions.

Complex Fractions are produced by expressing fractionally the Quotient of a Division in which Divisor or Dividend is not an Integer. $64\frac{1}{3} \div \frac{5}{8} = \frac{64\frac{1}{3}}{\frac{5}{8}}$

And Complex Fractions are easily simplified by the application of PRINCIPLE XV. (page 99·) which shews that the Numerator of any Fraction is a Dividend, of which the Denominator is the Divisor.

To Simplify a Complex Fraction.

Take Numerator for Dividend, and Denominator for Divisor, and Divide.

EXAMPLES WORKED OUT. Find the Simple Fractions, Integers, or Mixed Numbers respectively equal to the following Complex Fractions:

$$\frac{8\frac{1}{16}}{2\frac{11}{32}};\quad \frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{6}};\quad \text{and}\quad \frac{\frac{7}{8}\text{ of }2\frac{30}{49}\times 2\frac{7}{29}}{1\frac{56}{87}\times\frac{57}{112}\times 8\frac{56}{209}}.$$

Ex. 1· $\frac{8\frac{1}{16}}{2\frac{11}{32}}=8\frac{1}{16}\div 2\frac{11}{32}=\frac{129}{16}\div\frac{75}{32}\,(\doteq\frac{129}{16}\times\frac{32}{75})^{*}=\frac{86}{25}=3\frac{11}{25}$

Ex. 2· $\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{6}}=(\frac{1}{2}+\frac{1}{3}+\frac{1}{4})\div(\frac{1}{3}+\frac{1}{4}+\frac{1}{6})=(\frac{6}{12}+\frac{4}{12}+\frac{3}{12})$
$\div(\frac{4}{12}+\frac{3}{12}+\frac{2}{12})=\frac{13}{12}\div\frac{9}{12}\doteq\frac{13}{12}\times\frac{12}{9}=\frac{13}{9}=1\frac{4}{9}$

Ex. 3· $\frac{\frac{7}{8}\text{ of }2\frac{30}{49}\times 2\frac{7}{29}}{1\frac{56}{87}\times\frac{57}{112}\times 8\frac{56}{209}}=(\frac{7}{8}\times\frac{128}{49}\times\frac{65}{29})\div(\frac{143}{87}\times\frac{57}{112}$
$\times\frac{1728}{209})\doteq\frac{7}{8}\times\frac{128}{49}\times\frac{65}{29}\times\frac{87}{143}\times\frac{112}{57}\times\frac{209}{1728}=\frac{20}{27}$

Cancelling Common Factors materially abbreviates the work of this example.

EXERCISE LXXX.

Reduce the following Complex Fractions to their equivalent Simple Fractions, or Integers, or Mixed Numbers.

1· $\frac{9\frac{7}{9}}{2\frac{1}{27}}$
2· $\frac{5\frac{3}{11}}{2\frac{7}{11}}$
3· $\frac{8\frac{3}{4}}{5\frac{5}{8}}$
4· $\frac{15\frac{3}{5}}{7\frac{4}{5}}$
5· $\frac{23}{2\frac{2}{3}+\frac{2}{5}}$
6· $\frac{3\frac{1}{3}\text{ of }1\frac{1}{5}}{1\frac{1}{3}\text{ of }1\frac{1}{4}}$
7· $\frac{2\frac{1}{2}+1\frac{2}{3}}{3\frac{2}{3}-2\frac{1}{2}}$
8· $\frac{4\frac{4}{15}\text{ of }2\frac{5}{8}}{5\frac{1}{5}-4\frac{1}{2}}$
9· $\frac{3\frac{3}{7}\text{ of }2\frac{11}{12}}{\frac{1}{33}\text{ of }8\frac{9}{14}}$

10· Find the value of $\frac{5\frac{4}{5}-2\frac{1}{8}}{3\frac{3}{4}+\frac{9}{20}}$ of $\frac{4\frac{1}{2}+5\frac{19}{25}}{4\frac{1}{20}}$ of $\frac{2\frac{2}{5}+1\frac{2}{3}}{7\frac{19}{24}-2\frac{1}{4}}$.

In order to shew the propriety of calling such quantities as the foregoing by the name of "Fraction," we will now explain how these Complex Fractions may be produced (as Simple and Compound Fractions are) by the cutting up of a Unit.

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* See use of Vinculum, page 156·

Draw a Unit-square, and Divide it by seven vertical lines into eight equal parts, exactly as is done in Diagram 2· on page 14·

Now, commencing at the left, rub out the 1st, 2nd, 4th, and 5th of the seven vertical lines.

Then, supposing your Unit-square to be of the same size as Diagram 2·, it is divided into $2\frac{2}{3}$ parts of the size of Au. $\therefore \mathrm{A}u = \frac{1}{2\frac{2}{3}}$ (of the Unit,) a Complex Fraction; where Denominator, as usual, expresses the number of parts into which the Unit is cut; and the Numerator shews how many of those parts are contained in the piece spoken of, that is to say, in Au.

Similarly, $\mathrm{A}x = \frac{2}{2\frac{2}{3}}$ of the Unit. And, restoring the effaced lines,

$$\begin{array}{l|l|l}
\mathrm{A}s = \dfrac{\frac{1}{3}}{2\frac{2}{3}}; & \mathrm{A}v = \dfrac{1\frac{1}{3}}{2\frac{2}{3}}; & \mathrm{A}y = \dfrac{2\frac{1}{3}}{2\frac{2}{3}} \\
\mathrm{A}t = \dfrac{\frac{2}{3}}{2\frac{2}{3}}; & \mathrm{A}w = \dfrac{1\frac{2}{3}}{2\frac{2}{3}}; & \mathrm{AC} = \dfrac{2\frac{2}{3}}{2\frac{2}{3}} = 1\cdot
\end{array}$$

And, lastly, completing Diagram 2· by drawing two horizontal lines to divide the Unit-square into three equal parts, we have

$$\mathrm{A}b = \frac{\frac{1}{3}\text{ of }\frac{1}{3}}{2\frac{2}{3}};\ \mathrm{A}k = \frac{\frac{2}{3}\text{ of }\frac{1}{3}}{2\frac{2}{3}};\ \mathrm{A}l = \frac{\frac{2}{3}\text{ of }\frac{2}{3}}{2\frac{2}{3}}.$$

&c. &c. &c.

EXTENSION OF NOTATION AND NUMERATION TABLES.

The Notation and Numeration Tables already given shew how to write or read any Number requiring not more than twelve places on either side of Decimal Point.

To go further than this, it must be remembered that

A Million Millions = *One Billion*,
A Million Billions = *One Trillion*,
A Million Trillions = *One Quadrillion*,
A Million Quadrillions = *One Quintillion.*

And so on, each period of six places from Decimal Point to the left, having a new name; *Sextillions* coming next to Quintillions, and then *Septillions*, *Octillions*, and *Nonillions*.

And the Third Notation and Numeration Tables for Integers, may be constructed from inspection of the following Example.

Billions.						**Millions.**						**Units.**						
Hundred-thousands of	*Ten-thousands of*	*Thousands of*	*Hundreds of*	*Tens of*	*Units of (or single)*	*Hundred-thousands of*	*Ten-thousands of*	*Thousands of*	*Hundreds of*	*Tens of*	*Units of (or single)*	*Hundred-thousands of*	*Ten-thousands of*	*Thousands of*	*Hundreds of*	*Tens of*	*Single*	**Decimal Point**
4	6	8,	7	3	4;	5	6	8,	0	4	7;	5	2	3,	6	0	4	.

The extended Tables for Notation and Numeration of *Decimals* are easily derived from the same Example, by remembering that "The number of ciphers in the Denominator of a Decimal Fraction indicates the number of Decimal places for its Numerator." (See p. 18·) Hence,

∵ One Billion requires *twelve ciphers* between Digit 1 and Dl. Pt.
∴ Billionth parts require *twelve places of Decimals.*

The above is the English system; the following shews

THE FRENCH METHOD.

Trillions. H.T.S.	Billions. H.T.S.	Millions. H.T.S.	Thousands. H.T.S.	Units. H.T.S.	
2 1 4;	5 8 4;	8 6 4;	4 6 7;	3 1 4	.

And so on, each period containing only *three* places, instead of *six* as with us.

THE VINCULUM, BRACKET, AND BAR.

Arithmetical Symbols. The character which, in ordinary writing or printing, is termed an ellipsis or parenthesis ( ), is called in Arithmetic a **Vinculum,** which is the Latin word for "a chain."

Quantities enclosed in a Vinculum are to be considered as forming a Complex Unit. (See page 62·)

The expression $8\cdot \times (6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot)$ means that the sum of the three quantities within the Vinculum is to be multiplied by 8·

$\therefore 8\cdot \times (6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot)$ or $(6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot) \times 8\cdot = 12\cdot \times 8\cdot = 96\cdot$

Similarly, $72\cdot \div (6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot) = 72\cdot \div 12\cdot = 6\cdot$

Also, $(\frac{7}{8} \text{ of } 56\cdot - \frac{3}{4} \text{ of } 20\cdot) + (6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot) = 34\cdot + 12\cdot = 46\cdot$
And, $(\frac{7}{8} \text{ of } 56\cdot - \frac{3}{4} \text{ of } 20\cdot) - (6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot) = 34\cdot - 12\cdot = 22\cdot$

The **Bracket** { }, or [ ], or a horizontal **Bar** (such as separates the Numerator of a Fraction from its Denominator,) is sometimes employed as a Vinculum.

Hence, $\overline{6\cdot \times \frac{5}{8} \text{ of } \frac{3}{4} \div 9\cdot}$, and $\{6\cdot \times \frac{5}{8} \text{ of } \frac{3}{4} \div 9\cdot\}$ are respectively the same as $(6\cdot \times \frac{5}{8} \text{ of } \frac{3}{4} \div 9\cdot)$.

Whatever, then, is said of the Vinculum is true also of the Bracket and the Bar.

RULES FOR THE REMOVAL OF THE VINCULUM.

I. A Vinculum preceded by + may be at once effaced.

For $19\cdot + (\frac{3}{7} \text{ of } 42\cdot - \frac{5}{8} \text{ of } 64\cdot) = 19\cdot + \frac{3}{7} \text{ of } 42\cdot - \frac{5}{8} \text{ of } 64\cdot$

II. On removing a Vinculum preceded by ×, the sign and Common Factor connected with it must be repeated before *each* of the previously enclosed quantities. Thus,

$8\cdot \times \{6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot\} = 6\cdot \times 8\cdot + 3\frac{1}{2} \times 8\cdot + \frac{5}{8} \text{ of } 4\cdot \times 8\cdot = 96\cdot$

This operation depends on the self-evident Principle, that any Multiple of a Quantity is equal to the Sum of such Multiples of all the parts of that quantity.

III. A Vinculum whose contents are to be Divided by any Number may be removed by making *each* of the enclosed quantities a separate Dividend.

Thus, $\{6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot\} \div 3\cdot = 6\cdot \div 3\cdot + 3\frac{1}{2} \div 3\cdot + \frac{5}{8} \text{ of } 4\cdot \div 3\cdot = 4\cdot$

That is to say, $\dfrac{6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot}{3} = \dfrac{6}{3} + \dfrac{3\frac{1}{2}}{3} + \dfrac{\frac{5}{8} \text{ of } 4\cdot}{3} = 2\cdot + 1\frac{1}{6} + \frac{5}{6} = 4\cdot$

NOTE. When a Vinculum is *preceded* by ÷, shewing that the enclosed quantities are to be *combined* (according to their respective signs,) *to produce a Divisor*, there is no way of evading that Vinculum, but by working out all the operations indicated within it, and making the *Result* the Divisor.

For Example: $72\cdot \div \{6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot)$ or $\dfrac{72\cdot}{6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot}$ can only be simplified by collecting the quantities enclosed by the Vinculum or Bar, and making their Result (which is 12·,) the Divisor.

$$\therefore\ 72\cdot \div \{6\cdot + 3\tfrac{1}{2} + \tfrac{5}{8} \text{ of } 4\cdot\} \text{ or } \frac{72\cdot}{6\cdot + 3\frac{1}{2} + \frac{5}{8} \text{ of } 4\cdot} = \frac{72}{12} = 6\cdot$$

IV. A Vinculum preceded by the sign of Subtraction may be removed, if all the signs of the enclosed quantities be changed.

The expression 26· — (12· + 4·) signifies that the sum of 12· and 4· (that is 16·) is to be subtracted from 26· If we subtract 12· without first adding 4· to it, we must afterwards subtract 4·

$$\therefore 26\cdot - (12\cdot + 4\cdot) = 26\cdot - 12\cdot - 4\cdot$$

Where the removal of the Vinculum is accompanied by the change of the sign of the 4· from + to —.

The sign of the 12· is also changed. For *within the Vinculum* there is no minus sign belonging exclusively to the 12· The minus sign before the Vinculum will refer to the sum of 12· and 4· when that sum shall have been found. And *When any quantity has no sign expressed before it, + is always understood.* Hence, the 12· has + understood before it, and by removing the Vinculum, and allowing its minus sign to remain connected with the 12· we have changed the sign of this quantity as we did that of the 4·

Again, 26· — (12· — 4·) means that the difference of 12· and 4· (that is 8·), is to be subtracted from 26· If, then, we subtract 12· from 26·, we shall subtract 4· too many; and to correct our work we must afterwards add 4·

$$\therefore 26\cdot - (12\cdot - 4\cdot) = 26\cdot - 12\cdot + 4\cdot$$

Where the signs of the quantities which were enclosed are both changed, as before.

DIVISION OF A NUMBER INTO PROPORTIONAL PARTS.

It is frequently necessary to distribute a Number into a set of parts which shall have to each other the same ratios as a given set of Numbers (either Integers or Fractions) have.

First Example worked out. Distribute 72· into four portions, having the ratios of 3·, 4·, 5·, and 6·

These portions will be

First, $\frac{3}{18}$ of 72·, or 12·
Second, $\frac{4}{18}$ of 72·, or 16·
Third, $\frac{5}{18}$ of 72·, or 20·
Fourth, $\frac{6}{18}$ of 72·, or 24·

Second Example worked out. Three men find a purse containing £100·; A claims $\frac{1}{2}$ of the amount, B, $\frac{1}{3}$, and C, $\frac{1}{4}$. Now, as it is not possible to give to each so much as he demands, how shall the sum of money be divided into three parts, having the ratios of these Fractions?

$$\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} = \tfrac{6}{12} + \tfrac{4}{12} + \tfrac{3}{12}$$

Now, the Common Denominator 12· may be neglected. For, $\frac{6}{12}$, $\frac{4}{12}$, and $\frac{3}{12}$ have to each other the same ratios as any 6·, 4·, and 3· like units have.

This question, then, is precisely similar to the former one, and comes to this: "How shall £100· be distributed into three portions, having to each other the same ratios as 6·, 4·, and 3· have?"

A's portion will be $\frac{6}{13}$ *of* £100·, *or* £46$\frac{2}{13}$
B's portion will be $\frac{4}{13}$ *of* £100·, *or* £30$\frac{10}{13}$
C's portion will be $\frac{3}{13}$ *of* £100·, *or* £23$\frac{1}{13}$

From examination of these Examples, the following method is established.

To distribute a Number into any set of proportional parts, which shall be in the ratios of any given set of Numbers.

First. Make each of the given Numbers, if Integers, the Numerator of a Fraction, of which the Sum of those Numbers is the Denominator.

Second. Find those Fractions of the Number which is to be distributed.

If the given Numbers, or any of them, be Fractions, Reduce them to a Common Denominator. Consider the New Numerators as Integers, and proceed as directed above.

EQUATIONS.

When two equal quantities are written with the sign of equality between them, an **Equation** is expressed; and the two equal quantities are called the two sides of the Equation.

Now, supposing an unknown quantity to form a part or a Factor of one of the two sides of an Equation, then we may gradually remove from that side everything but the unknown quantity, taking care in so doing to keep the two sides constantly equal, and then the other side of the Equation will at last express the value of the quantity which was at first unknown. This removal of all the known quantities to one side is called **Solving the Equation.** It is effected by applying the following self-evident truths, or **Axioms.**

I. *If Equals be added to Equals the Wholes will be Equal.*

II. *If Equals be taken from Equals the Remainders will be Equal.*

III. *If Equals be multiplied by Equals the Products will be Equal.*

IV. *If Equals be divided by Equals the Quotients will be Equal.*

GENERAL PRINCIPLE FOR THE SOLUTION OF EQUATIONS.

PRINCIPLE XXIV. *If the same operation be performed upon equal quantities the results will be equal.*

Every Arithmetical Problem might be solved by the method of Equations, and very many can be more clearly and elegantly worked out by this than by any other contrivance. We proceed to give a few very simple Examples.

FIRST EXAMPLE WORKED OUT. What Number is 12· the 4th Multiple of?

Required No. × 4· = 12·

Divide both sides by 4· Then, by PRINCIPLE XX.

Required No. = 3· Proof. 3· × 4· = 12·

SECOND EXAMPLE WORKED OUT. To what No. must 14· be added to make 37·?

Required No. + 14· = 37·

Subtract 14· from each side.

∴ *Required No.* = 37· − 14· = 23· Proof. 23· + 14· = 37·

THIRD EXAMPLE WORKED OUT. What No. is less than its 4th Multiple by 44·?

4· *times Required No.* = *Required No.* + 44·

Subtract Required No. from each side.

∴ 3· *times Required No.* = 44·

Divide each side by 3· Then, by PRINCIPLE XX.

Required No. $= \frac{44}{3} = 14\frac{2}{3}$

Proof. $14\frac{2}{3} \times 4\cdot = 58\frac{2}{3} = 14\frac{2}{3} + 44\cdot$

FOURTH EXAMPLE WORKED OUT. Of what No. is 15· five times the half?

$\frac{1}{2}$ *of Required No.* × 5· = 15·

Divide both sides by 5·

∴ $\frac{1}{2}$ *of Required No.* = 3· PRIN. XX.

Multiply both sides by 2·

∴ *Required No.* = 6· Proof. $6\cdot \times \frac{5}{2} = 15\cdot$

FIFTH EXAMPLE WORKED OUT. A post is $\frac{1}{3}$ in the mud, $\frac{1}{4}$ in the water, and 10· feet above the water. What is its whole length?

Whole length, in feet, = $\frac{1}{3}$ *of length,* + $\frac{1}{4}$ *of length* + 10· *feet.*

Add the Fractions on the right-hand side.

∴ *Whole length, in feet* = $\frac{7}{12}$ *of length* + 10· *feet.*

Subtract $\frac{7}{12}$ of length from each side.

∴ $\frac{5}{12}$ *of whole length, in feet* = 10· *feet.*

Divide both sides by $\frac{5}{12}$. PRINCIPLES IX. and XX.

∴ *Whole length of post* = (10· ÷ $\frac{5}{12}$) *feet* = 24· *feet.*

Proof. $\frac{1}{3}$ of 24· + $\frac{1}{4}$ of 24· + 10· = 24·

SIXTH EXAMPLE WORKED OUT. The wages of a servant were £40· yearly, and a suit of livery. Quitting his situation at the end of 5· months, he was found to be entitled to his livery and £6$\frac{1}{6}$. What was the livery worth?

Having served $\frac{5}{12}$ of a year, he had earned $\frac{5}{12}$ of £40· + $\frac{5}{12}$ of livery.

But he was equitably paid with £6$\frac{1}{6}$ + the whole of the livery.

∴ *Value of Livery* + £6$\frac{1}{6}$ = $\frac{5}{12}$ *Value of Livery* + $\frac{5}{12}$ *of* £40·

Subtract $\frac{5}{12}$ of Value of Livery from each side.

∴ $\frac{7}{12}$ *of Value of Livery* + £6$\frac{1}{6}$ = $\frac{5}{12}$ *of* £40·

Subtract £6$\frac{1}{6}$ from each side.

∴ $\frac{7}{12}$ *of Value of Livery* = £10$\frac{1}{2}$

Divide both sides by $\frac{7}{12}$. PRINCIPLES IX. and XX.

∴ *Value of Livery* = £18·

Proof. 18· + 6$\frac{1}{6}$ = 24$\frac{1}{6}$ = $\frac{5}{12}$ of 18· + $\frac{5}{12}$ of 40·

SEVENTH EXAMPLE WORKED OUT. A traveller walks 24· miles a day, and 2· days after he has started another follows and overtakes him in 6· days. At what rate does the second traveller walk?

Distance travelled by each = (24· × 8·) *miles* = 192· *miles.*

∴ *Required rate* × 6· = 192· *miles.*

Divide both sides by 6·

∴ *Required rate* = $\frac{192}{6}$ *miles* = 32· *miles.*

Proof. 32· × 6· = 192· = 24· × 8·

∴ Second traveller goes as far in 6· days as first traveller does in 8· days, and consequently overtakes him.

Another Method.

Second traveller gains on first in 6· days 48· miles.

∴ *Gain of 2nd traveller per day* = $\frac{1}{6}$ of 48· *miles* = 8· *miles.*

Add first traveller's rate to each side.

∴ *Second traveller's rate* = (24· + 8·) *miles* = 32· *miles.*

EIGHTH EXAMPLE WORKED OUT. A gentleman, dying, bequeathed a legacy of £140· to three servants. A was to have twice as much as B, and B three times as much as C. What were their respective shares?

Call C's portion, (being the least,) *one* share.
Then, B's portion, = *three* shares.
And A's portion, = *six* shares.

∴ (1· + 3· + 6·) shares = 10· shares = £140·

Divide both sides by 10·

∴ One share = £$\frac{140}{10}$ = £14· = *C's portion.*
∴ Three shares = £14· × 3· = £42· = *B's portion.*
∴ Six shares = £42· × 2· = £84· = *A's portion.*

TRANSPOSITION IN EQUATIONS.

Any Quantity preceded by + or — may be **Transposed** from one side of an Equation to the other, if its sign be reversed. That is to say, in Transposing,

+ must be exchanged for —
— must be exchanged for +

This Transposition with exchange of signs is a necessary result of PRINCIPLE XXIV., as the following examples will shew.

17· + 6· = 23·
Subtract 6· from each side. PRIN. XXIV.
∴ 17· = 23· − 6·

The 6· is Transposed with its sign reversed.

Again 42· — 8· = 34·
Add 8· to each side. PRIN. XXIV.
∴ 42· = 34· + 8·

The 8· is Transposed with its sign reversed.

EXERCISE LXXXI.

RECAPITULATORY QUESTIONS.

1· What is Division? Describe its Arithmetical Symbol. What is the true meaning of the Quotient in the following expression, 42· ÷ 9· = 4$\frac{6}{9}$? 4· what? $\frac{6}{9}$ of what? Prove this by an experiment with 42· strips of paper. What is the absolute value of the $\frac{6}{9}$ in above Example? In the same Example, by how much does the Dividend 42· exceed in value the Quotient 4$\frac{6}{9}$?

2· Recite the Principle involving this latter fact.

3· How many methods have we expounded for ascertaining (for example,) how many eights are contained in 94·?
4· What is the nature of the first of these methods?
5· What is the nature of the second?
6· What processes are combined in the third?
7· Does Pure Division, or Distribution of a Number, increase or diminish it? Prove by experiment.
8· What is the difference between the Multiplication and Division Tables?
9· What is meant by expressing one Number "*in terms of*" another? How is this most briefly done?
10· How may the Quotient be at once expressed fractionally?
11· When can no other than a fractional Quotient be obtained?
12· What Ratio does the fractional Quotient express?
13· How may any one part of an Integer be at once expressed fractionally?
14· What part of Unity does any Fraction express?
15· What part of its Numerator is any Fraction?
16· When are two Fractions Reciprocals?
17· Shew by experiment that Dividing 45· by 9· will not exhibit $\frac{1}{9}$ of 45· How, then, is $\frac{1}{9}$ of 45· discovered from Dividing 45· by 9·?
18· Prove, by an Example, each of the four cases included under PRINCIPLE XVII.
    1st, An Integer ÷ an Integer; as 8· ÷ 11·
    2nd, A Fraction ÷ an Integer; as $\frac{1}{4}$ ÷ 9·
    3rd, An Integer ÷ a Fraction; as 6· ÷ $\frac{4}{7}$
    4th, A Fraction ÷ a Fraction; as $\frac{2}{11}$ ÷ $\frac{3}{8}$
19· In Dividing by Units where do Quotient-figures fall?
20· When Dividend contains Integers only, how many ways are there of treating Remainder?
21· What is the *Natural Place* of each Quotient-figure?
22· In Dividing successively by the Corresponding Factors of a Composite Divisor, how is Second True Remainder found? And why?
23· How is third True Remainder found? And why?
24· Describe each of the three methods of Dividing a Fraction by a Fraction.
25· Divide $\frac{4}{5}$ by $\frac{7}{11}$ after each of the three methods.
26· In what respect will the Quotients of the following differ, 314·83 ÷ by 1·; 314·83 ÷ ·0001; and 314·83 ÷ 100·?
27· How will Quotient of 314·83 ÷ 20· differ from Quotient of 314·83 ÷ 2·, and from Quotient of 314·83 ÷ ·002?
28· Dividing by ·02 includes two steps (one of which is concealed); what are they?
29· Divide 314·83 by ·02, in such a way as to exhibit the two steps separately.
30· Repeat Principle XXI., and shew wherein it differs from Principle XII.
31· How is the Greatest Common Measure of two Numbers found?
32· What is the difference between Prime Numbers and Numbers prime to each other? Give examples of each.

33· When is a Fraction in its lowest terms? How may it be reduced to that state?
34· How may a Multiple of 2· be easily recognized?
35· How may the Multiples of 3·, of 4·, of 5·, of 6·, of 8·, of 9· of 10·, of 11·, and of 12·, respectively, be known?
36· What two steps are necessary in order to ascertain the Least Common Multiple of any set of Numbers?
37· How are Fractions reduced to their Least Common Denominator?
38· What Vulgar Fractions only have equivalent Decimals?
39· When are Interminate Decimals produced?
40· What is a Repeating Decimal, and what a Circulating Decimal?
41· Give an example of a Pure Repetend of *one*, *two*, or *any number* of places?
42· What Denominator has any Mixed Decimal?
43· How is the Numerator of the Vulgar Fraction equivalent to any Mixed Decimal found?
44· What is *Ratio?* How are the "Terms of the Ratio" named?
45· What Quantities have no Ratio?
46· What is Proportion?
47· How are the "Terms of a Proportion" named, *collectively*, and *separately?*
48· What two Products are found equal in any four Proportionals?
49· Repeat the Principle which embodies this fact?
50· *Prove the truth of the Principle.*
51· What celebrated Rule is founded on this Principle?
52· What is Cancelling? Where can it be employed, and where not?
53· How may any Fraction of a Number be found?
54· Of the two methods, which is the more convenient, and why?
55· What is a Complex Fraction?
56· How are Complex Fractions produced?
57· Exhibit, by the partition of a Unit-square, the following Complex Fractions, $\frac{2\frac{7}{8}}{5}$, and $\frac{3\frac{1}{3}}{4\frac{2}{7}}$.
58· What is the characteristic difference between the English and French systems of Notation?
59· What is the meaning of the word "Vinculum?"
50· How are the entire contents of any Vinculum to be regarded?
61· Repeat the four rules for the removal of the Vinculum.
62· When can a Vinculum *not* be got rid of without working out?
63· Repeat the general Principle for the Solution of Equations.

EXERCISE LXXXII.

MISCELLANEOUS EXAMPLES.

1· Take half a dozen out of half a million.
2· Subtract 3· doz. from a million and a quarter.
3· How many eights are there in $\frac{1}{4}$ of 10000·?
4· How many nines in $\frac{1}{7}$ of 23· scores?
5· How many lb. in $\frac{3}{11}$ of 16· tons? A ton = 2240· lb.
6· What part of $5\frac{7}{8}$ are $4\frac{3}{5}$?

7· By what must $3\frac{5}{8}$ be multiplied to produce $1\frac{3}{7}$?

8· What Factor of $\frac{7}{11}$ corresponds to $\frac{9}{14}$?

9· Find the two Numbers whose *Sum* is 100·, and *Difference* 36·
(See Exercise lx. Examples 20·, 21·, and 22·)

10· What No. Subtracted from $14\frac{3}{8}$ will leave $1\frac{3}{4}$?

11· What No. Added to $(\frac{6}{11} + \frac{1}{2}$ of $\frac{4}{5})$ will give as *Sum* $\frac{1}{3}$ of $6\frac{3}{8}$?

12· Multiply $2\frac{2}{13}$ by $16\frac{5}{7}$, and Divide the result by $\frac{11}{12}$ of $\frac{7}{8}$ of 6·75.

13· Divide $6\frac{2}{5}$ by $3\frac{2}{3}$, and Add the Quotient to $\frac{7}{25}$.

14· What Number is that of which 14· is $\frac{7}{8}$?

$\frac{7}{8}$ *of Required Number* = 14·

Divide both sides by $\frac{7}{8}$.

∴ *Required Number* $\doteq$ 14· ÷ $\frac{7}{8}$ = &c. (See Prin. ix. and xx.)

15· Divide ·004 by ·00002; and find the value of $\frac{\cdot 002 \times \cdot 01}{\cdot 0005}$

16· Find the *Sum*, *Difference*, *Product*, and *Quotients* of $2\frac{7}{19}$ and $5\frac{9}{23}$.

17· Divide 1;472,005· by 9,200·; and prove the work.

18· Divide ·025 by ·000625·

19· Reduce $\frac{5}{7}$, $\frac{3}{10}$ and $\frac{9}{2}$ to a Common Denominator.

20· Find the L.C.M. of 36·, 42·, 24·, 18·, and 32·

21· Reduce $\frac{5}{12}$, $\frac{7}{8}$, and $\frac{3}{18}$ to their *Least Common Denominator*.

22· What No. is that whose 5th Multiple exceeds its 2nd Multiple by 27·?

Required No. × 5· = (*Required No.* × 2·) + 27·

23· The Difference of the 7th and 4th Multiples of a certain No. is 22· What is the No.?

24· One half of the 7th Multiple of a certain No. is 18· less than its 5th Multiple. What is the No.?

25· What Number is less by 5· than $\frac{1}{3}$ of its 7th Multiple?

Required No. × $\frac{1}{3}$ of 7· = *Required No.* + 5·

Divide both sides by $\frac{7}{3}$.

∴ *Reqd. No.* $\doteq$ (*Reqd. No.* + 5·) ÷ $\frac{7}{3}$ $\doteq$ $\frac{3}{7}$ *of Reqd. No.* + $\frac{3}{7}$ *of* 5·

Subtract $\frac{3}{7}$ of Required No. from each side.

∴ $\frac{4}{7}$ of *Required No.* = $\frac{3}{7}$ of 5· = $\frac{15}{7}$

Divide both sides by $\frac{4}{7}$.

26· If 15· geese cost 35· shillings more than ten geese, what is each worth, supposing them all to be of like value?

27· A boy spent half of his money in marbles, $\frac{1}{8}$ in fruit, and the remainder in the purchase of a half-crown book, which, being soiled, he obtained at $\frac{2}{3}$ of its price. What amount, in pence, had he at first? (See 5th Example worked out. Page 160·)

28· How many British miles are equivalent to 128· Irish; 11· Irish being equal to 14· British?

29· A hare, whose speed was $\frac{3}{11}$ of that of a greyhound, was overtaken by him after running 156· yards; how far had the greyhound run?

For every 3· *yards run by hare, the hound ran* 11·

30· Suppose the same two animals had been, at starting, 96· yards asunder, how far would the hare have run before being caught?

Hound gains 8· yards on every 3· yards run by hare.
Hound must gain, in all, 96· yards.
∴ *Hare will run as many times 3· yards as there are, &c. &c.*

31· A Shetland pony, which made 7· leaps of 5· feet long in the same time as an Exmoor pony made 5· leaps of 8· feet long, was overtaken by the latter, after making 210· leaps. How far was the Shetland in advance at starting?

Distance run by Shetland in 210· leaps = *feet.*
No. of leaps made by Exmoor = ;
∴ *Distance run by Exmoor* = *feet.*
Interval at starting = 150· *feet.*

32· How many apples at $3\frac{1}{2}$d. per dozen can be bought for $29\frac{3}{4}$d.?

Value of 1 *apple at* $3\frac{1}{2}$*d. per doz.* = $\dfrac{\overset{d.}{3\frac{1}{2}}}{12} = \frac{7}{24}$ *of a penny.*

No. of apples at $\frac{7}{24}$*d. for* $29\frac{3}{4}$*d.* ≑ $29\frac{3}{4}$*d.* ÷ $\frac{7}{24}$*d.* = 102·

Proof. 102· apples at $\frac{7}{24}$d. each, cost $\frac{7}{24}$d. × 102· = $29\frac{3}{4}$d.

33· How many apples at 4d. per dozen are worth 150· pears at 8d. per score?

Value of 1· *apple at* 4·*d. per doz.* = *of a penny.*
Value of 150· *apples* 8·*d. per score* = *pence.*
&c. *&c.* *&c.*

34· Express *nine* by the same Digit thrice written.

35· A could complete a piece of work in 4· days, and B could do the same in 5· days; in what time would they finish it both working together?

Call the piece of work 1·
Then A's day's work = $\frac{1}{4}$, *and B's day's work* = $\frac{1}{5}$;
∴ *A and B together do in* 1· *day* ($\frac{1}{4} + \frac{1}{5}$ *or*) $\frac{9}{20}$ *of the work.*
∴ *As many times as* $\frac{9}{20}$ *are contained in* 1· *so many days will the work occupy the two men.*
∴ *No. of days* = 1· ÷ $\frac{9}{20}$ ≑ 1· × $\frac{20}{9}$ = $2\frac{2}{9}$ *days.*

$$\text{Proof.}\ \left.\begin{array}{l}\text{In } 2\frac{2}{9} \text{ days, A will do } \frac{1}{4} \times 2\frac{2}{9} = \frac{5}{9}\\ \text{In } 2\frac{2}{9} \text{ days, B will do } \frac{1}{5} \times 2\frac{2}{9} = \frac{4}{9}\end{array}\right\} = 1\cdot, \left\{\begin{array}{c}\textit{or the entire}\\ \textit{work.}\end{array}\right.$$

36· Supposing that a peck loaf would suffice for a man 5· days, and for the man and his wife 3· days, how long would such a loaf last the latter alone?

37· A, working alone, could mow a field in $7\frac{1}{2}$ days, but with B's assistance in $4\frac{1}{4}$ days. In how many days would B do it alone?

38· How long must A work alone so that, on B's coming to help him, the job may be completed in 3· days from the arrival of the latter?

39· If A can paper a saloon in 7 hours, B in $6\frac{1}{4}$, and C in $6\frac{4}{5}$ hours; in how many hours would they complete it all working together?

40· E, F, and G working together can plaster a house in $6\frac{3}{4}$ days, which E alone can do in 27· days, and F alone in 21· days. In what time can G do it by himself?

41· Supposing E's earnings to be 5·s. per day, what will F's and G's be at the same rate?

42· A person possessing ⅝ of a tin mine sold ⅗ of his share for £2580· What was the whole mine worth at that rate?

43· A lady's fortune, being £2100·, was ⅘ of ⅞ of her brother's. How much was his?

44· A farmer sold 96· loads of hay to two persons. The first bought one half of his rick, and the second one fourth of it. How many loads did that rick contain?

45· A father left £600· to be divided between his two sons, of whom the eldest was to have twice as much as the younger. How much had each?

46· A certain sum of money is divided into two portions which are in the ratio of 4· : 9·, and the greater portion exceeds the less by £15· What is the amount divided, and the amount of each share? (See "Division into Proportional Parts." Page 158·)

47· M and N together could lay a floor in 5· days; but, after working together 3· days, N was compelled by illness to abandon the task, which M then finished in 3½ days. How long would N have been finishing, supposing he had been left alone instead of M?

48· If 4½ lb. of beef be worth 5¼ lb. of mutton, how much beef will be worth 1 cwt. of mutton?

$$1\cdot \textit{lb. of beef worth } \frac{5\frac{1}{4}}{4\frac{1}{2}} \textit{ lb. of mutton, \&c.}$$

49· How many pieces weighing 11⅗ lb. each can be cut from 1⅕ cwt. of beef?

50· E's rate of walking is ⅜ of G's. Now, supposing they start at the same moment in the same direction, how far will E have advanced when G will be 15· miles ahead of him?

51· Now, suppose the same parties to start at the same moment in opposite directions, and to meet at 9¾ miles from E's starting point. How far were they asunder at first?

52· £1280· are distributed among three persons so that their portions are in the ratios of 5·, 3·, and 2·, respectively. How much does each receive?

53· How many stone of 14· lb. each in 9· cwt?

54· How many dozens in 3½ millions?

55· What Fraction of a cwt. are 21· lb.?

56· A man had to walk 80· miles in 4· days; but, after walking 3· days at a certain rate, he found it necessary, in order to complete his journey within the stipulated time, to walk 4· miles more on the 4th day than on the 3rd. How many miles did he walk on each day?

57· The minute-hand of a clock moves 12· times as fast as the hour-hand. Now, at 12· o'clock the hands start from the same point; how many minute-spaces will they be asunder at 16· minutes past 12·?

58· How much does the minute-hand gain per minute on the hour-hand?

59· The whole circle on the clock-face is divided into 60· *minute-spaces*, each of which is traversed by the long-hand in a *minute of time*. When, between 12· o'clock and 1· o'clock, will the hands be first in the same straight line?

This will occur when M.H. has gained 30· *minutes on H.H.*

60· When will the hands be 15· minute-spaces asunder, between 12· and 1·?

61· At what time, between 3· o'clock and 4· o'clock, is the minute-hand 5· minute-spaces behind the hour-hand?

They were 15· *minute-spaces asunder at starting.*
∴ *M.H. has gained, &c. &c.*

62· Where is the hour-hand at 20· minutes past one?

63· Between 1· and 2· the hands were noticed to be exactly 23· minute-spaces asunder. What was the time?

They were 5· *minutes asunder at starting.*
∴ *M.H. has gained on H.H.* (5·+23·) *minutes, &c. &c.*

64· Peter Drainpurse paid away $\frac{1}{3}$ of his money, lost $\frac{1}{2}$ of the rest, lent $\frac{1}{4}$ of what then remained, was robbed of $\frac{5}{8}$ of the residue, and then had 9· shillings left. How much did he pay, lose, and lend respectively; and of how much was he robbed?

65· Add together $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{4}{11}$. Subtract their sum from $9\frac{1}{4}$ times $3\frac{3}{13}$. Multiply the result by $\frac{5}{6}$ of $\dfrac{19\frac{1}{2}}{7\frac{3}{4}}$. Find what Fraction the Product is of $84\frac{5}{7}$.

66· A bankrupt's effects realize £7848·, which will only pay £3· out of every £16· which he owes. What is the amount of his debts?

67· In a Division *Sum* (!), the Dividend was 879538·, and the Quotient $2457\frac{475}{1461}$. What was the Divisor?

68· In a Multiplication *Sum* (!!), the Multiplier was $19\frac{3}{8}$, and the Product 18445· What was the Multiplicand?

69· In a Subtraction *Sum* (!!!), the Difference was $\frac{4}{5}$ of the Subtrahend, and the Minuend was 641·55. Find the other two. (See page 87·)

70· If P can reap $1\frac{1}{2}$ acre in $2\frac{1}{4}$ days, and Q $1\frac{1}{4}$ acre in $2\frac{1}{2}$ days, how long will the two be occupied in reaping $10\frac{1}{2}$ acres?

71· How much should each receive, at 3·s. per acre?

72· L, M, and N travel from the same place, in the same direction, at the rates of 3·, 5·, and 6· miles an hour respectively. M sets out 12· hours after L; how many hours after M must N set out, so that they may overtake L at the same moment?

73· If $3\frac{1}{2}$ lb. of cheese be worth $2\frac{4}{5}$ lb. of butter, and $8\frac{1}{2}$ lb. of butter worth, $11\frac{1}{3}$ lb. of bacon; how many lb. of bacon are worth 20· lb. of cheese. (See Example 48·)

74· Supposing A to have got ahead of B $3\frac{1}{15}$ miles in 5· hours, how much per hour must B increase his speed to overtake A in $4\frac{3}{5}$ hours?

75· Swift and Sure started together in the same direction, at 2· o'clock, Swift walking at the uniform rate of $3\frac{5}{8}$ miles per hour. At $\frac{1}{4}$ past 6·, being then $2\frac{1}{8}$ miles in advance of Sure, (who also proceeded at a uniform speed,) he sat down and slept. On awaking, he found he had to walk 5· hours at the same rate as before to overtake Sure. How long had Swift slept?

76· If $1\frac{1}{2}$lb. of tea be worth $3\frac{1}{2}$lb. of coffee, $6\frac{1}{2}$lb. of coffee worth 24·lb. of sugar, and $7\frac{1}{2}$ lb. of sugar worth 4·lb. of soap, how many lb. of soap are 3·lb. of tea worth?

77· A Coachman whose annual wages were £35· and a suit of livery, left his place at the end of three months, when he was found to be entitled only to the livery, without its gold lace, which was worth £1·; what was the entire amount of his year's wages?

78· How many gallons of water in 1· cubic foot? (EXERCISE LIV. EXAMPLE 10·)

79· A cistern containing 4658· cubic feet of water is fitted with two valves, A and B, which, being both open together, would discharge it in $4\frac{1}{2}$ hours. Now the valve B alone would discharge the cistern in 6·6 hours. How many gallons of water per minute are let off by valve A?

Fraction discharged per hour by both valves =
Fraction discharged per hour by B alone =
Fraction discharged per hour by A alone =
Fraction discharged per min. by A alone = , = *galls.*

80· Connected with the same cistern is a pump which will fill it in $3\frac{5}{8}$ hours, when both valves are closed. In what time will the pump fill the cistern?
First. If valve A, alone, remain all the time open?
Second. If valve B, alone, remain all the time open?
Third. If both valves remain all the time open?

81· Now, suppose the cistern full, and the valve B to be opened. How long afterwards must the pump commence working to refill the cistern in $2\frac{1}{2}$ hours, valve B remaining still open?

Find how much the pump gains on valve B in $2\frac{1}{2}$ hours.
Pump must commence as soon as that quantity has run off.

82· The cistern being full, both valves are opened, and after $1\frac{1}{4}$ hour the pump commences, and continues working $\frac{3}{8}$ of an hour, both valves remaining still open. The pump and valves being then all stopped, how much water remains in the cistern?

83· A Farmer's daughter, being asked the number of her father's poultry, said that $\frac{1}{2}+\frac{1}{4}+\frac{1}{5}+\frac{1}{8}+\frac{1}{10}+\frac{1}{20}+\frac{1}{40}$ of the number of ducks was the 5th Multiple of the number of fowls; and that $\frac{1}{2}+\frac{1}{4}+\frac{1}{5}+\frac{1}{10}+\frac{1}{20}$ of the fowls amounted to 110· How many were there of each?

84· A, working alone, can do a piece of work in 8· days, but with B's assistance in $4\frac{4}{5}$ days; how long must A work alone, so that, with B's subsequent aid, the task may be completed in 6· days from its commencement?

85· Shew how to distribute £$64\frac{7}{12}$ between A and B; giving the latter $\frac{5}{8}$ of the amount given to the former.

86· Sixty pounds sterling may be distributed into three sums, such that $\frac{1}{3}$ of the first, $\frac{1}{4}$ of the second, and $\frac{1}{5}$ of the third shall be equal amounts. What are the three sums?

87· A gig and two horses cost together £75·; the first horse and the gig together cost £55·; the second horse and the gig together cost £50·; what was the cost of each?

88· A man engages to do a piece of work in 20· days for £5·; but after doing $\frac{2}{3}$ of it, he finds that, in order to complete the work in the time, he must call in the assistance of another workman of skill and activity equal to his own. How long must this assistant work, and what amount will be due to each?

89· A quantity of corn, which would suffice for a horse, a mule, and an ox 8· days, would be consumed by the horse and mule together in 12· days, but would last the mule and ox 16· days. How long would such a quantity serve each animal separately?

90· A, B, and C, sharing the profits of an adventure, find that the Sum of A's and B's gains is £13$\frac{1}{2}$, that of B's and C's £12$\frac{3}{5}$, whilst A's and C's shares, together, amount to £11$\frac{23}{40}$. What was the gain of each?

◆

SUMMARY OF PRINCIPLES.

PRINCIPLE I. The same Digit is made to represent different Numbers by changing its place, counting from the Decimal Point. Page 3·

PRINCIPLE II. The Cipher is employed to fill places where Significant Figures are not required; and the Cipher is only of service when it stands between a Significant Figure and the Decimal Point. Page 3·

PRINCIPLE III. There are as many Units in the Half of a Number as there are twos in the Whole of it; as many Units in the Third as threes in the Whole, &c. &c. Pages 23· and 24·

$$\text{There are as many times}\left\{\begin{array}{l}\frac{1}{2}\text{ in one Half of any Integer}\\ \frac{1}{3}\text{ in one Third ,, ,,}\\ \frac{1}{4}\text{ in one Fourth ,, ,,}\end{array}\right\}\text{as there are Units in the whole of that Integer. (P. 101·)}$$

&c. &c. &c.

COROLLARY. Any one part of an Integer is at once expressed by writing the Denominator of that part under the Integer. Page 100·

PRINCIPLE IV. In the Decimal System of Arithmetical Notation, each Digit has one tenth of the value it would have in the next place to the left, and tenfold the value it would have in the next place to the right. Page 26·

PRINCIPLE V. Unlike Units cannot be compared in *any* way. Page 35·

COROLLARY. There is no Ratio between unlike Concrete Quantities. Page 146·

PRINCIPLE VI. The Multiplicand and the Multiplier may be exchanged the one for the other, and no alteration will be made thereby in the Product.

PRINCIPLE VII. In whatever order Numbers are multiplied together, their Continued Product is always the same. Page 53·

PRINCIPLE VIII. Multiplying by any power of 10· merely removes every Digit of the Multiplicand as many places to the left as there are Ciphers in the Multiplier. Page 55·

PRINCIPLE IX. The word "of" between Numbers has the same meaning as the sign of Multiplication. Page 56·

PRINCIPLE X. Multiplying by Digit 1, standing in any place, merely removes every Digit of the Multiplicand to the right or left, as many places as the Multiplying Digit is from the Units' place. Page 60·

PRINCIPLE XI. The Product of any Number and a Composite Multiplier is equal to the Continued Product of that Number and any set of Corresponding Factors of the Composite Multiplier. Page 64·

PRINCIPLE XII. Multiplying by a Single Digit, standing in any place, removes each Figure of the Product, to the right or left, as many places as the Multiplying Digit is from the Units' place. Page 66·

PRINCIPLE XIII. Multiplying or Dividing the Numerator and Denominator of a Fraction by the same Number makes no alteration in the value of that Fraction. Pages 79· and 134·

PRINCIPLE XIV. Each Unit of the Quotient is a Complex Unit, and is equivalent to the whole Divisor. Page 99·

PRINCIPLE XV. The Quotient is at once expressed as a Vulgar Fraction, by placing the Divisor under the Dividend. Page 99·

PRINCIPLE XVI. Any Fraction is the same part of its Numerator that Unity is of its Denominator. Page 101·

PRINCIPLE XVII. Dividing by any Number produces the same figures as Multiplying by the Reciprocal of the Divisor. Page 106·

PRINCIPLE XVIII. The Dividend and the Quotient increase and decrease together. Multiplying the Dividend (before Dividing,) also multiplies the Quotient: and previously Dividing the Dividend also divides the Quotient of the subsequent Division. Page 107·

Or, Division followed by Multiplication produces the same result as Multiplication followed by Division; supposing, of course, the Divisor and Multiplier to remain unchanged. Page 151·

PRINCIPLE XIX. The Quotient and Divisor are Corresponding Factors of the Dividend. Page 112·

COROLLARY I. A Fraction multiplied by its Denominator gives its Numerator. Page 112·

COROLLARY II. The Divisor and Quotient may exchange places, if Dividend be retained unaltered. Page 113·

PRINCIPLE XX. Any Number divided by one of its Factors gives its Corresponding Factor. Page 113·

PRINCIPLE XXI. Dividing by a Single Digit, standing in any place of Integers or Decimals, removes every Quotient-figure as many places, BUT IN THE CONTRARY DIRECTION, as Dividing Digit is from Units' place. Page 128·

EXTENSION OF PRINCIPLE XXI. Dividing by any Integer or Decimal removes every Quotient-figure as many places, BUT IN THE CONTRARY DIRECTION, as right-hand Significant-figure of Divisor is from Units' place. Page 129·

PRINCIPLE XXII. Every Measure of any two Numbers measures also *a*. Their Sum. *b*. Their Difference. *c*. Any Multiple of either of them. Page 132·

PRINCIPLE XXIII. When four Numbers are Proportionals, the Product of the Extremes is equal to the Product of the Means. Page 148·

PRINCIPLE XXIV. If the same operation be performed upon equal quantities, the results will be equal. Page 160·

PRINCIPLE XXV. Any Multiple of a Quantity is equal to the Sum of such Multiples of all the parts of that Quantity. Page 157·

PRINCIPLE XXVI. The *Quotient of the Sum* of any set of Numbers by any Divisor is the same as the *Sum of the Quotients* of the separate Numbers by the same Divisor. Page 157· Rule III., for removal of Vinculum.

END OF PART I.

THE CONSTRUCTIVE ARITHMETIC.

PART II.

APPLICATION OF PRINCIPLES AND ELEMENTARY PROCESSES TO COMMERCIAL ARITHMETIC.

MEASUREMENT OF CONCRETE QUANTITIES.

THE various kinds of Concrete Quantity with which Arithmetic has to do, are: I. Time; II. Lineal Distance, consisting of Length only; III. Superficial Extent, or Area, made up of Length and Breadth only; IV. Solidity, having Length, Breadth, and Thickness; V. Capacity, or power of containing, depending on Length, Breadth, and Depth; VI. Weight; VII. Money-Value.

Every Concrete Quantity must, of necessity, be measured by a Unit of its own nature. Principle v.

We ascertain the magnitude of any given Length by trying how often it contains some settled Unit of Length. Duration can only be measured by a Unit of Time; Surface or Area, by a Unit of Surface, or a Superficial Unit. Solidity and Capacity are measured by Solid Units; Weight by Units of Weight; and Value by Units of Value.

The measuring units are called **Units of Comparison.**

Many of our Concrete Units of Comparison, having been selected in half-barbarous times and by illiterate persons, originated from very rude and clumsy contrivances. For example, our smallest Unit of Length was, until very recently, called *a barley-corn*, and is said to have originally been, really, the length of "a grain of ripe barley taken from the middle of the ear." The inch was the length of three such grains laid end to end.

The yard was the length of king Henry the First's arm; and, to the present day, a sailor roughly measures *a fathom* of rope by stretching it across his breast to the full extent of his arms.

The *foot*, the *hand*, the *span*, and the *pace*, were, as their names imply, similarly derived. So was also the weight called a *grain*.

The ancient Unit of Length, called a *cubit*, was originally the length of a man's forearm and hand, that is to say, from the elbow to the tip of the middle finger; and this was held to be the fourth part of a well-proportioned man's stature.

IMPERIAL STANDARD MEASURE.

It was enacted in 1824·, and again in 1855·, that all our legal Units of Length shall be tested by the **Imperial Standard Yard**; which is the distance, in a straight line, between the centres of the two gold plugs or pins in the bronze bar deposited in the office of the Exchequer. The foot is one third of this yard, and the inch is one thirty-sixth of it, or one twelfth of the foot.

METHOD OF RESTORING THE STANDARD.

As it is quite possible that the Imperial Standard Yard, and all certified copies* of it might be lost or defaced, it was necessary to provide a method by which the Standard of Length might be restored by reference to some natural unvarying unit.

The Act of 1824· provided that the natural Unit of Length to which we should refer, in order to restore our present system of Measures, should be a **Pendulum** † vibrating seconds of mean time *in vacuo*, at the level of the sea in the latitude of London.

By dividing such a Pendulum into 391393· equal parts, and then taking 10000· of those parts, we should obtain the length of our present inch, which is, consequently, $\frac{10000}{391393}$ of the length of the Pendulum in question. And the Pendulum is = 39·1393 inches.

But, as the said Pendulum must be of the length to vibrate *seconds*, it becomes first necessary to establish the duration of a second. Hence, the Unit of Time lies at the foundation of our whole system of Concrete Units of Comparison. And these are successively derived in the following order, which therefore indicates the natural arrangement of the Tables of Measures and Weights.

I. Time.
II. Lineal Measure.
III. Superficial Measure.
IV. Solid or Cubic Measure.
V. Capacity, or power of containing.
VI. Weight.
VII. Money-Value.

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* See note to Page 180·

† A Pendulum is a rod swinging freely on a fixed point, and it has been proved that every Pendulum vibrating in any certain time, (a second, for example,) will be of the same length, provided

I. That it meet with the same amount of resistance from the air.
II. That it be situated at the same height above the level of the sea.
III. That it remain in the same latitude.

The last *two* conditions amount to this *one*, namely, That it be at the same *distance from* the earth's centre.

TABLES OF MEASURES AND WEIGHTS.

I. TIME.

The earth, by revolving on its axis, turns, in succession, every point on its surface towards the sun. The time elapsing between two such successive presentations of the same point to the sun is called *a Solar Day*.

The twenty-fourth part of a mean Solar Day is an *Hour*. The sixtieth part of an Hour is a *Minute*: and the sixtieth part of a Minute is a *Second*.

60· seconds (marked *sec.*)	= 1· minute;	marked	*min.*
60· minutes	= 1· hour;	„	*ho.*
24· hours	= 1· day;	„	*da.*
7· days	= 1· week;	„	*we.*
4· weeks	= 1· common month;	„	*co. mo.*
13· mo. and 1· da., or 52· we. and 1· da., or 365· da.	= 1· common year;	„	*c. yr.*
365¼ days	= 1· Julian year;	„	*Ju. yr.*
366· days	= 1· bissextile or leap year.		

The Common Month of 4· weeks is sometimes called a Lunar Month; but the true Lunar Month, (that is, the time from new moon to new moon,) is 29· days, 12· hours, 44· minutes, 2·87 seconds. The true Tropical Year, on which the return of the seasons depends, is 365· days, 5· hours, 48· minutes, 49·7 seconds.

The year is also divided into twelve Calendar Months; namely,

January	31· days.	May	31· days.	September	30· days.
February	28· or 29· „	June	30· „	October	31· „
March	31· „	July	31· „	November	30· „
April	30· „	August	31· „	December	31· „

The number of days in each month is easily remembered by the aid of the following rhyme:

Thirty days hath September,
Like April, June, and November.
Each of the rest has thirty-one,
Excepting February alone;
Which numbers twenty-eight days clear,
And twenty-nine in each leap year.

Every fourth year is a Bissextile or Leap year. If the Number expressing the name of the year be a Multiple of 4·, that year is a Leap year. Thus 1836· was leap year, because 1836· ÷ 4· = 459· without a remainder; and 1843· was 3· years after leap year, for 1843· ÷ 4· leaves a remainder 3·. Again, if the year in question be the last of a Century, it will be excepted from this rule, and will not be leap year, unless the *number of hundreds* be exactly divisible by 4· Thus, neither 1900·, 2100·, nor 2200· will be leap year, because 19·, 21·, and 22· are not exact multiples of 4·. See Appendix, note on Old and New Styles.

THE FOUR QUARTER DAYS.

England and Ireland.

Lady-day, March 25th.	Michaelmas-day, September 29th.
Midsummer-day, June 24th.	Christmas-day, December 25th.

Scotland.

Candlemas-day, February 2nd.
Whitsun-day, May 15th.
Lammas-day, August 1st.
Martinmas, November 11th.

Christian nations reckon their chronology from two epochs: I. The Creation; II. The birth of Christ, or commencement of the Christian Era. A year counted from the Creation is distinguished by the letters A.M. standing for "Anno Mundi," "*In the year of the world.*" A year reckoned forward from the birth of Christ is marked by the letters A.D. an abbreviation of the words "Anno Domini," "*In the year of the Lord.*" When reckoned backward from the same event, the year is marked either by the letters A.C. "Ante Christum," or by B.C. "*Before Christ.*" For example,

The Deluge occurred A.M. 1656·; or A.C. or B.C. 2348·
America was discovered A.M. 5496·; or A.D. 1492·

The hours before and after noon are frequently distinguished from each other by the letters A.M. and P.M.; A.M. standing for the words "Ante Meridiem," "*Before Mid-day,*" P.M. standing for the words "Post Meridiem," "*After Mid-day.*"

II. LINEAL MEASURES.

Lineal Units must be considered as having neither Breadth nor Thickness, but Length only.*

I. ARTIFICERS' LINEAL MEASURE.

12· lines, or seconds (marked ″) = 1· inch; marked *in.*, or ′.
12· inches = 1· foot; „ *ft.*
3· feet = 1· yard; „ *yd.*
6· feet, or 2· yards, = 1· fathom; „ *fm.*

II. LAND-SURVEYORS' LINEAL MEASURE.

7·92 inches, or $7\frac{23}{25}$ inches = 1· link of Gunter's chain.
25· links, or 5½ yds., or 16½ ft. = 1· rod, pole, or perch *r.* or *p.*
100· lin., or 22· yds., or 66· ft., or 4· po. = 1· chain. *ch.*
10· ch., or 220· yds., or 660· ft., or 40· po. = 1· furlong. *fur.*
80· chains, or 8· fur., or 1760· yds., or 5280· ft. = 1· mile. *mi.*
3· miles = 1· leag. *leag.*

Links are usually written as Decimals of a Chain; but Chains are sometimes represented as hundreds of Links. Hence, 94· Chains and 35· Links may be expressed either thus; 94·35 Chains, (and this is the better method,) or thus: 9435· Links. The latter plan is employed to escape the trouble of learning to multiply Decimals.

III. CLOTH (LINEAL) MEASURE.

2¼ inches = 1· nail. *na.*
4· nails, or 9· inches = 1· quarter. *qr.*
4· quarters, or 16· na., or 3· ft., or 36· in. = 1· yard. *yd.*
3· quarters, or ¾ yd. = 1· Flemish Ell. *Fl. E.*
5· quarters, or 1¼ yd. = 1· English Ell. *E. E.*
6· quarters, or 1½ yd. = 1· French Ell. *Fr. E.*
37· inches, or $1\frac{1}{36}$ yd. = 1· Scotch Ell. *Sc. E.*

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* See note to Solid or Cubic Measure.

IV. LINEAL MEASURE OF THE CIRCLE, OR ANGULAR MEASURE.

One Three-hundred-and-sixtieth part of the Circumference of any Circle, great or small, is called a Degree.

A Degree is, then, *a variable Unit of Length*, being longer or shorter in proportion to the size of the Circle from which it is derived.

60· seconds (marked ″) = 1· minute. marked ′
60· minutes = 1· degree. „ °
360· degrees = 1· Circle.

To measure any plane Angle, we take the Angular point for a centre and describe so much of a Circle as will reach from line to line of the Angle. Then, the Fraction of the circumference included between the two lines shews the magnitude of the Angle. If *one fourth* of the Circle be so included, the Angle will contain 90·°, and be called a *Right Angle*. An Angle of 60·° includes between its two lines *one sixth* of any Circle which has the Angular Point for its centre.

Latitude and Longitude are measured in degrees, minutes, and seconds.

V. GEOGRAPHICAL OR NAUTICAL (LINEAL) MEASURE.

This measure is derived from the Earth's circumference at the Equator.

6075·8 feet (nearly) = 1· nautical mile. *n. mi.*
3· nautical miles = 1· nautical league. *n. leag.*
60· nautical miles, or 20· n. leag. = 1· degree of the Equator.
360· such degrees = the Earth's circumference at the Equator = 24855·5 common miles, nearly.

The degree of Latitude is nearly the same in any part of the Earth's surface, and may be reckoned as equivalent to 1·° of the Equator. The degree of Longitude is less and less as we recede from the equator.

VI. MISCELLANEOUS LINEAL MEASURES.

4· inches = 1· hand.*
3· inches = 1· palm, or hand-breadth.
3· palms, or 9· inches = 1· span.
18· inches, or 21· inches = 1· cubit.
3· feet = 1· pace.
120· fathoms = 1· cable's length.
7· yards = 1· Irish perch.
280· yards = 1· Irish furlong.
2240· yards = 1· Irish mile.
11· Irish miles are = 14· English miles.

III. SUPERFICIAL OR SQUARE MEASURES.

Superficial Units are to be considered as without Thickness. **A superficial inch** is equal to a square surface, one lineal inch in length, and the same in width.

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* Including the width of the thumb.

A superficial prime is equal to *an oblong surface*, one foot long, and one inch wide.

A superficial or square foot is equal to a square, measuring one lineal foot in length, and the same in width. And so on with other superficial Units.

I. ARTIFICERS' SUPERFICIAL MEASURE.

12· sup. or sq. in. or sec. marked	= 1· sup. prime.	′
12· sup. primes, or 144· sup. in.	= 1· sup. foot.	*s.ft.*
9· sup. feet	= 1· sup. yd.	*s.yd.*
100· sup. feet	= 1· square of flooring.	

II. LAND SURVEYORS' SUPERFICIAL MEASURE.

10,000· superficial links	= 1 supl. chain.	
10· superficial chains, or 100,000· sup. links	= 1 statute acre.	
30¼ sup. yds., or 625· sup. links	= 1· sup. rod, pole, or pch.	*s. p.*
40· sup. poles, or 2·5 sup. chains	= 1· rood.	*s. rd.*
4· roods, or 160· sup. poles, or 4840· sup. yds.	= 1· statute acre.	*ac.*
640· statute acres	= 1· square mile.	*s.mi.*
49· sup. yds.	= 1· Irish sup. perch.	*Ir. su.p.*
7840· sup. yds.	= 1· Irish acre.	*Ir. ac.*

Down to the perch, the Irish acre is divided similarly to the statute acre.

The rod, pole, or perch, (both lineal and superficial) is very often improperly termed a "*yard*," or "*landyard*." In many parts of England, the old or "customary" acre is 5760· sup. yds., being composed of 160· sup. rods of 36· sq. yds. each, instead of 30¼ sq. yds.

IV. SOLID OR CUBIC MEASURE.

Solid bodies have three dimensions, Length, Breadth, and Thickness.

A *Cube* is a solid having six equal sides, each of them a square.

A solid or cubic inch, or third,* is equal to *the bulk of a cube*, a lin. inch long, and the same wide, and thick, so that each of its six sides or faces is a superficial inch.

A solid second* is equal to *the bulk of an oblong column or prism*, one ft. long, one inch wide, and the same thick, so that each of its four long sides has a surface of 12· sup. inches; and each of its ends is a sup. inch.

A solid prime is equal to *the bulk of a square slab*, one the same wide, and one inch thick. Hence, its top and bottom have each a surface of 1· sup. foot; and each of its four sides, a surface of 1· sup. prime.*

A solid or cubic foot is equal to the bulk of a cube, having six superficial feet for its sides.

12· solid or cubic inches, or thirds, marked ‴	= 1· solid second.	″
12· solid seconds, or 144· cu. ins.	= 1· solid prime.	′
12· solid primes, or 1728· cu. ins.	= 1· cubic foot.	*c. ft.*
27· cubic feet	= 1· cubic yard.	*c. yd.*
40· cubic feet of rough timber, or 50· cubic feet of hewn timber	= 1· load.	*lo.*
42· cubic feet	= 1· ton of shipping.	*T. sh.*
128· cubic feet	= 1· cord of wood.	

V. MEASURES OF CAPACITY.

I. IMPERIAL MEASURE: DRY AND LIQUID.

"Capacity" means power of containing; hence, the Capacity of a vessel is only another name for the bulk or solidity which would exactly fill it. Consequently, Capacity is measured by solid units.

The *Standard of Capacity* of Vessels, both for liquids and dry goods, is *The Imperial Gallon*, which contains 277·2738 (or rather more than 277¼) cubic inches. One thirty-second part of this gallon is a *gill.*

				cubic inches.
4· gills, *gi.*	= 1· pint.	*pi.*	=	34·6592.
2· pints	= 1· quart.	*qt.*	=	69·3184.
4· quarts	= 1· gallon.	*gall.*	=	277·2738.
2· gallons	= 1· peck.	*pk.*	=	554·5477.
4· pecks	= 1· bushel.	*bu.*	=	2218·1908.
8· bushels	= 1· quarter.	*qr.*	=	17745·5266.
5· quarters	= 1· load.	*lo.*	=	88727·63.
3· bushels	= 1· sack	of coals.		*sa.*
12· sacks	= 1· chaldron			*ch.*

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* NOTE TO TEACHER. The permanent furniture of every elementary school should include the following articles, the frequent examination of which, *by sight and touch*, will secure the pupil's reception of the precise ideas conveyed by the names of those Units of Comparison.

I. A complete set of Imperial Measures and Weights.

II. A series of diagrams, (on the wall, or on boards,) shewing the exact size of the superficial inch, prime, foot, and yard. The sides of these diagrams will exhibit the lineal measures bearing the same names.

III. A solid foot, prime, and second, (all of wood,) with their several surfaces scored in superficial inches.

IV. A set of, at least, 350· separate cubic inches, also of wood. With these, cubes can be built, up to the cube of seven.

V. A wooden cube, 10· ins., and another 8· ins. in the edge. By grouping a sufficient number of the cubic inches around these two, respectively, the cubes of 9· and of 11· may be constructed; completing the series of the first 12· cubes.

The cubic inches may occasionally be taken to represent cubic feet or cubic yards, and thus the contents of solids of various dimensions may be experimentally ascertained or exhibited. Various devices for turning these things to profitable account will suggest themselves.

36· gallons of beer	=	1· barrel.	*bar.*
63· gallons of wine	=	1· hogshead.	*hhd.*
2· hhds. or 126· gals.	=	1· pipe.	*pi.*
2· pipes. or 252· gals.	=	1· tun.	*T.*

100· gallons of this Imperial measure are equal to 120· old wine gallons, or to 98·3 old ale gallons, or to 103·15 gallons, old dry measure.

The old "Winchester" bushel is equal to 7·7558 Imperial gallons, or to ·9694 Imperial bushels.

II. APOTHECARIES' FLUID MEASURE.

	1· fluid minim.	marked	m	=	·0045	cubic inches.
60· fluid minims	= 1· dram,	"	ʒ	=	·2708	" "
8· drams	= 1· ounce,	"	℥	=	2·1662	" "
16· ounces	= 1· pint.	"	O	=	34·6592	" "
8· pints	= 1· gallon	"	gal.	=	277·2738	" "

The words "*dram*" and "*ounce*" have, here, no reference to weight.

VI. WEIGHTS.

The Act of Parliament (18· and 19· Vict. cap. 72·) which re-establishes the Imperial Yard and Gallon, further declares the weight of the mass of platinum, marked "P. S. 1844·, 1· lb.," deposited in the office of the Exchequer, to be the legal and genuine Standard Measure of weight, and the Imperial Standard pound Avoirdupois, from which all our Units of Weight are to be derived.

The Act of 1824· provided that in case of the loss or defacement of this Standard it should be restored by reference to the weight of a cubic inch of distilled water, at a temperature of 62·° of Fahrenheit's thermometer, and exposed to a pressure of air sufficient to raise the mercury in the barometer to 30· inches.*

·001 of a cubic foot of distilled water, weighed under these conditions,† weighs 1· ounce Avoirdupois, which is equal to 437½ *grains*. A cubic foot of water (as above) weighs 62·5 lb. Avoirdupois or 1000· ounces.

1· lb. Avoirdupois = 7000· *grains*.
1· lb. Troy = 5760· *grains*.

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* If the water were colder or the pressure greater, the same bulk, being compressed, would contain a greater number of particles, and would, in consequence, weigh heavier.

† NOTE TO TEACHER. Beside the Standard Imperial Yard and Pound, which are kept in the Exchequer Office, there are four authorized copies of each, deposited, one of each respectively,

I. At the Royal Mint.
II. With the Royal Society of London.
III. At the Royal Observatory, Greenwich.
IV. In the New Palace at Westminster.

The Act, 18· and 19· Vict. cap. 72·, provides that the Standard shall be re-

I. TROY WEIGHT.

This weight is said to have received its name from the city of Troyes, in France.

It is employed for weighing gold, silver, and precious stones, and in philosophical experiments.

24· grains (*gr.*) = 1· pennyweight. *dwt.*
20· pennyweights = 1· ounce. *oz.*
12· ounces = 1· pound. *lb.*
·8 of a Troy grain = 1· diamond grain. *di. gr.*
4· diamond grains = 1· diamond carat.

Gold, being too soft to be worked up alone, is combined, for manufacturing or coining, with alloy, composed of silver or copper, or both.

A Carat of Gold is *one twenty-fourth part* of any mass containing that metal. When the whole is pure it is said to be "24· carats fine;" when $\frac{23}{24}$ are pure, and $\frac{1}{24}$ is alloy, the mass is "23· carats fine," and so on.

Manufactured articles partly composed of gold are tested at Goldsmiths' Hall, London, and, if found to contain not less than $\frac{18}{24}$ of their weight of gold, are stamped with their number of carats fine.

II. APOTHECARIES' WEIGHT.

This weight is employed only in dispensing drugs. The pound, ounce, and grain are the same as in Troy Weight.

20· grains = 1· scruple. ℈
3· scruples = 1· dram. ʒ
8· drams = 1· ounce. ℥
12· ounces = 1· pound. lb.

III. AVOIRDUPOIS WEIGHT.

The name of this weight is derived from three French words, "Avoir du poids," signifying "*To have weight.*" Avoirdupois weight is used for all articles sold by weight, except those mentioned under Troy and Apothecaries' Weights. It is sometimes called Grocers' Weight.

16· drams (*dr.*) = 1· ounce. *oz.*
16· ounces = 1· pound. *lb.*
14· pounds = 1· stone. *st.*
28· pounds = 1· quarter. *qr.*
4· quarters, or 112· lb. = 1· hundred-weight. *cwt.*
20· hundred-weights = 1· ton. *T.*

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stored when necessary, by reference to any one of these, at certain specified temperatures, instead of to the Pendulum: "Because," (as the preamble of this Act states,) "by the researches of scientific men doubts were thrown on the accuracy of the methods provided by the" previous Act "for the restoration of the Standard." The latter Act, however, prescribes *no other method* for the restoration of the Standard, in the event of *all* authorized copies of it being lost.

The common stone of 14· lb. is called a stone "horseman's weight." 8· lb. = 1· stone of butcher's meat.

∵ 1· *lb.* Troy = 5760· *grs.* while 1· *lb.* Avoir. = 7000· *grs.*
∴ 1· *lb.* Avoir. = 1· *lb.* Troy + 1240· *grs.*
And 144· *lb.* Avoir. = 175· *lb.* Troy.
And 192· *oz.* Avoir. = 175· *oz.* Troy.

VII. MONEY-VALUE.

The natural or intrinsic value of a coin depends on the *weight* and fineness of the metals composing it.

There are twelve sorts of sterling coins at present current in the British Islands; *three* of copper, namely, the *farthing*, *halfpenny*, and *penny*; *seven* of silver, namely, the *three-penny-piece*, *groat*, *sixpence*, *shilling*, *florin*, *half-crown*, and *crown*; and *two* of gold, that is to say, the *half-sovereign*, and the *sovereign* or *pound*.

These are all our legal *Coins of Exchange*. The "*Coins of Account*" are five: the *pound*, *shilling*, *penny*, *halfpenny* and *farthing*.

I. COINS OF ACCOUNT AND EXCHANGE.

					dwts.	grs.
2· farthings	= 1· halfpenny.					
4· farthings	= 1· penny.	*d.*				
12· pence	= 1· shilling.	*s.*	weighing		3·	$15\frac{4}{11}$.
20· shillings	= 1· pound.	£	,,		5·	3·2744.

II. COINS OF EXCHANGE ONLY.

3·d. = A threepenny-piece.
4·d. = A groat.
6·d. = A sixpenny-piece.
2·s. = 1· florin.

2·s. 6·d. or 30·d. = 1· half-crown.
5·s. or 60·d. = 1· crown.
10·s. or 120·d. = 1· half-sovereign.

III. NOMINAL AND OBSOLETE COINS.

21·s. = 1· guinea.
27·s. = 1· moidore.

36·s. = 1· Joannes.
13·s. 4·d. = 1· merk.

TRIANGULAR TABLE OF CURRENT COINS.

farth.	halfp.	d.	threeps.	grts.	sixps.	s.	flors.	hf-crns.	crns.	hf-sovs.	£.
2·=	1·										
4·=	2·=	1·									
12·=	6·=	3·=	1·								
16·=	8·=	4·=	$1\frac{1}{3}$=	1·							
24·=	12·=	6·=	2·=	$1\frac{1}{2}$=	1·						
48·=	24·=	12·=	4·=	3·=	2·=	1·					
96·=	48·=	24·=	8·=	6·=	4·=	2·=	1·				
120·=	60·=	30·=	10·=	$7\frac{1}{2}$=	5·=	$2\frac{1}{2}$=	$1\frac{1}{4}$=	1·			
240·=	120·=	60·=	20·=	15·=	10·=	5·=	$2\frac{1}{2}$=	2·=	1·		
480·=	240·=	120·=	40·=	30·=	20·=	10·=	5·=	4·=	2·=	1·	
960·=	480·=	240·=	80·=	60·=	40·=	20·=	10·=	8·=	4·=	2·=	1·

The *d.* naming *pence* stands for the Latin word denarii, of which the singular is denarius, a *penny*. £. represents the word Libra, a *pound*, or Libræ *pounds*.

The letters lb. also represent "libra" or "libræ." It is improper to write lbs., because the plural does not end in s., but in æ.

Farthings are always written as Fractions of a penny. Thus fourpence three farthings are written 4¾d. Fivepence halfpenny, thus; 5½d.; but fractions of a farthing have the letter *q*. placed over them, representing the Latin word "*quadrans*," *the fourth part*.

VIII. ALIQUOT PARTS.

An **Aliquot part** of any Quantity is a Fraction of it which has 1 for its Numerator.

Thus, 3·s. 4·d., being £$\frac{1}{6}$, is an Aliquot part of £1·.
But, 3·s., being £$\frac{3}{20}$, is not an Aliquot part of £1·.

TABLES OF ALIQUOT PARTS.

I. Of £1·

s.	*d.*	
10·	0·	= $\frac{1}{2}$
6·	8·	= $\frac{1}{3}$
5·	0·	= $\frac{1}{4}$
4·	0·	= $\frac{1}{5}$
3·	4·	= $\frac{1}{6}$
2·	6·	= $\frac{1}{8}$
2·	0·	= $\frac{1}{10}$
1·	8·	= $\frac{1}{12}$
1·	4·	= $\frac{1}{15}$
1·	3·	= $\frac{1}{16}$
1·	0·	= $\frac{1}{20}$
	10·	= $\frac{1}{24}$
	8·	= $\frac{1}{30}$
	7½	= $\frac{1}{32}$
	6·	= $\frac{1}{40}$
	5·	= $\frac{1}{48}$
	4·	= $\frac{1}{60}$
	3¾	= $\frac{1}{64}$
	3·	= $\frac{1}{80}$
	2½	= $\frac{1}{96}$
	2·	= $\frac{1}{120}$
	1½	= $\frac{1}{160}$
	1¼	= $\frac{1}{192}$
	1·	= $\frac{1}{240}$
	¾	= $\frac{1}{320}$
	½	= $\frac{1}{480}$
	¼	= $\frac{1}{960}$

II. Of a crown.

s.	*d.*	
2·	6·	= $\frac{1}{2}$
1·	8·	= $\frac{1}{3}$
1·	3·	= $\frac{1}{4}$
1·	0·	= $\frac{1}{5}$
	10·	= $\frac{1}{6}$
	7½	= $\frac{1}{8}$
	6·	= $\frac{1}{10}$
	5·	= $\frac{1}{12}$
	4·	= $\frac{1}{15}$
	3¾	= $\frac{1}{16}$
	3·	= $\frac{1}{20}$
	2½	= $\frac{1}{24}$
	2·	= $\frac{1}{30}$
	1½	= $\frac{1}{40}$
	1¼	= $\frac{1}{48}$
	1·	= $\frac{1}{60}$
	¾	= $\frac{1}{80}$
	½	= $\frac{1}{120}$
	¼	= $\frac{1}{240}$

III. Of a shilng.

d.	
6·	= $\frac{1}{2}$
4·	= $\frac{1}{3}$
3·	= $\frac{1}{4}$
2·	= $\frac{1}{6}$
1½	= $\frac{1}{8}$
1·	= $\frac{1}{12}$
¾	= $\frac{1}{16}$
½	= $\frac{1}{24}$
¼	= $\frac{1}{48}$

IV. Of a ton.

cwt.	*qr.*	
10·	0·	= $\frac{1}{2}$
5·	0·	= $\frac{1}{4}$
4·	0·	= $\frac{1}{5}$
2·	2·	= $\frac{1}{8}$
2·	0·	= $\frac{1}{10}$
1·	1·	= $\frac{1}{16}$
1·	0·	= $\frac{1}{20}$
	2·	= $\frac{1}{40}$
	1·	= $\frac{1}{80}$

V. Of a cwt.

lb.	
56·	= $\frac{1}{2}$
28·	= $\frac{1}{4}$
16·	= $\frac{1}{7}$
14·	= $\frac{1}{8}$
8·	= $\frac{1}{14}$
7·	= $\frac{1}{16}$
4·	= $\frac{1}{28}$
2·	= $\frac{1}{56}$

VI. Of a qr. Av.

lb.	
14·	= $\frac{1}{2}$
7·	= $\frac{1}{4}$
4·	= $\frac{1}{7}$
3½	= $\frac{1}{8}$
2·	= $\frac{1}{14}$
1¾	= $\frac{1}{16}$

VII. Of 1·lb. Av.

oz.	
8·	= $\frac{1}{2}$
4·	= $\frac{1}{4}$
2·	= $\frac{1}{8}$
1·	= $\frac{1}{16}$

VIII. Of a bush.

2·pe.	= $\frac{1}{2}$
1·pe.	= $\frac{1}{4}$
1·gal.	= $\frac{1}{8}$
2·qts.	= $\frac{1}{16}$
1·qt.	= $\frac{1}{32}$
1·pt.	= $\frac{1}{64}$

IX. Of a gallon.

2·qts.	= $\frac{1}{2}$
1·qt.	= $\frac{1}{4}$
1·pt.	= $\frac{1}{8}$
2·gi.	= $\frac{1}{16}$
1·gi.	= $\frac{1}{32}$

X. Of an acre.

2·r.	= $\frac{1}{2}$
1·r.	= $\frac{1}{4}$
20·p.	= $\frac{1}{8}$
16·p.	= $\frac{1}{10}$
10·p.	= $\frac{1}{16}$
8·p.	= $\frac{1}{20}$
5·p.	= $\frac{1}{32}$
4·p.	= $\frac{1}{40}$
2·p.	= $\frac{1}{80}$
1·p.	= $\frac{1}{160}$

Only the first three of the above tables are complete.

REDUCTION OF CONCRETE QUANTITIES.

Reduction, in Arithmetic, is the changing of the form and denomination of a quantity without altering its absolute value.

By means of Reduction any Concrete Quantity may be expressed in terms of either greater or smaller units of the same kind.

Descending Reduction expresses a quantity in a lower name. Ascending Reduction expresses a quantity in a higher name.

Descending Reduction is effected by Multiplication; Ascending Reduction, by Division.

DESCENDING REDUCTION.

To Reduce any Concrete Quantity to units of a lower denomination.

Multiply by the number of smaller units which make one of the greater, adding in with the Product any of the smaller units which may be given.

FIRST EXAMPLE WORKED OUT. INTEGERS OF MONEY-VALUE. Reduce £464· 3·s. 4$\frac{3}{4}$d. to farthings.

£.	s.	d.
464·	3·	4$\frac{3}{4}$
20·		

9283· *shillings in* £464· 3·s.
12·

111400· *pence in* £464· 3·s. 4·d.
4·

445603· *farthings in* £464· 3·s. 4$\frac{3}{4}$d.

Here, we multiply the £464· by 20·, (because 20·*s.* = £1·) and add in, as we proceed, the odd 3·*s.*

We next multiply the 9283·*s.* by 12·, (because 12·*d.* = 1·*s.*) and add in the odd 4·*d.*

Lastly, we multiply the 111400·*d.* by 4·, (because 4· farthings = 1·*d.*) and add in the odd $\frac{3}{4}$*d.*

THE IMPLICIT CHANGE OF UNIT.

This Example suggests an important remark, for it apparently contradicts what we have repeatedly affirmed of the nature of Multiplication, namely, that it is nothing more than repetition. Now, no amount of mere repetition can change the nature of a Concrete Unit. Hence, the Product must of necessity be of the same kind as the Multiplicand. Consequently, the repeating of £464· twenty times could only produce £9280· and not 9280· *shillings*, as stated above. The fact is, that one essential step in the foregoing process is concealed, being performed altogether mentally. This concealed step is the changing of the £464· into 464·*s.*, which 464·*s.* being repeated 20· times give 9280·*s.*, a result precisely the same as we should derive from multiplying 20·*s.* by 464 (PRINCIPLE VI.)

Similarly, there is an implied change of the 9283·*s.* into *pence*, previously to multiplying by 12·; and then a change of the 111400· *pence* into 111400· *farthings*, before multiplying by 4·.

If, in the first stage of the above Reduction, we consider the 20·*s*. as the Multiplicand, (by PRINCIPLE VI.) the £464· must still be changed into 464· units representing the number of repetitions that must be made of the 20·*s*., for it is impossible to multiply by pounds. (See p. 149·.)

The implicit or concealed change of Unit occurs in many processes involving Concrete Quantities, and we shall frequently have to call attention to it; for, if it be lost sight of, those processes become inexplicable.

It is from this "Implicit change of Unit" that the sign of Numerical Coincidence ($\doteq$) becomes necessary. (See Page 103·)

EXERCISE LXXXIII.

EXAMPLES IN DESCENDING REDUCTION OF INTEGERS.

Each Example marked with an asterisk may be worked in one line.

A.

1· In £346825· how many shillings, pence, and farthings?
2· In 8642· moidores how many guineas? (*Multiply by* $1\frac{2}{7}$.)
3· In 84356· guineas how many halfpence?
4· Reduce 7845· guineas to crowns. (*Multiply by* $4\frac{1}{5}$.)
5· Reduce 543625· half-crowns to shillings. (*Multiply by* $2\frac{1}{2}$.)
6· In 40000· merks how many shillings? (*Multiply by* $13\frac{1}{3}$.)
7· Express in crns., half-crns., flors., shillings, & sixpences, £4356·
8· Reduce £ 4638· 11·s. $4\frac{1}{4}$d. to farthings.
9· Reduce £ 8500· 4·s. 6·d. to sixpences.*
10· Reduce £ 1715· 8·s. 9·d. to threepences.*
11· Reduce £ 9452· 6·s. 10·d. to twopences.*
12· Reduce £ 4105· 14·s. $6\frac{1}{2}$d. to halfpence.
13· Reduce £ 6419· 18·s. 0·d. to florins.*
14· Reduce £ 9516· 15·s. 0·d. to crowns.*
15· Reduce £ 3765· 12·s. 6·d. to half-crowns.* (*Add in* 5·)
16· Reduce £ 1518· 10·s. 0·d. to half-sovereigns.*
17· Reduce £ 9542· 11·s. 0·d. to florins.*
18· Reduce £ 3654· 12·s. 6·d. to florins.*
19· Reduce £ 2148· 12·s. 0·d. to half-sovereigns.*
20· Reduce £ 3541· 12·s. 9·d. to half-crowns.* (*Add in* $5\frac{1}{10}$)
21· Reduce £ 4218· 13·s. 9·d. to half-crowns.* (*Add in* $5\frac{1}{2}$)
22· Reduce £ 7354· 11·s. 3·d. to half-crowns.*
23· Reduce £34186· 11·s. 4·d. to sixpences.* (*Add in* $22\frac{2}{3}$)
24· Reduce £53001· 8·s. 4·d. to threepences.* (*Add in* $33\frac{1}{3}$)
25· Reduce £27654· 12·s. 2·d. to threepences.*
26· Reduce £95438· 1·s. $4\frac{1}{4}$d. to halfpence.
27· Reduce £31427· 14·s. 6·d. to crowns.*
28· Reduce £94216· 6·s. 3·d. to florins.*
29· Reduce £21345· 10·s. $7\frac{1}{2}$d. to half-crowns.* (*Add in* $4\frac{1}{4}$)
30· Reduce £75642· 10·s. 8·d. to twopences.*
31· Reduce £54316· 19·s. $4\frac{1}{4}$d. to half-pence.
32· Reduce £63142· 5·s. $11\frac{3}{4}$d. to farthings.
33· Reduce £47654· 8·s. 4·d. to shillings.*
34· Reduce £35628· 11·s. $1\frac{1}{2}$d. to shillings.*
35· Reduce £43156· 4·s. 2·d. to florins.*
36· Reduce £61584· 13·s. 1·d. to sixpences.*
37· Reduce £73619· 15·s. 8·d. to shillings.*

B.

1· Reduce 3· tons to oz.
2· Reduce 14· tons, 17· cwts. to lb.
3· Reduce 18· tons, 3· qrs. to drs.
4· Reduce 15· cwts. 4· oz. to drs.
5· Reduce 13· cwts. 3· qrs. 17· lb., 6· oz. to oz.
6· Reduce 18⅙ tons to oz.
7· Reduce 5½ lb. to oz., and 17¼ oz. to drs.
8· Reduce 1⅐ qr. to lb., and 15¾ lb. to oz.
9· Reduce 4¾ tons to qr., and 11⅜ cwt. to lb.
10· Reduce 24· cwts. to stones, horseman's weight.
11· Reduce 14· tons, 3· qrs. 11· lb. 4· oz. to lb.
12· Reduce 16· tons, 2· qrs. 14· lb. to qrs.
13· Reduce 2· tons, 1· qr. 12· lb. to stones.
14· Reduce 8· tons, 7· lb. 12· oz. to lb.
15· Reduce 3· qrs., 14· lb. to qrs.
16· Reduce 2· qrs. 5· lb. 4· oz. 2· drs. to drams.
17· Reduce 11· cwts. 8· oz. to lb.
18· Reduce 16· cwts. 7· lb., 4· drs. to oz.
19· Reduce 364· tons, 14· cwts. 6· oz. to lb.
20· Reduce 45· tons, 8· cwts. to half-lb.

C.

1· Reduce 1· lb. Troy to dwts. and grs.
2· Reduce 4· lb. 3· oz., 14· dwts. to grs.
3· Reduce 4· lb. Apoth. to grs.
4· Reduce 7· lb. 5· ℥, 7· ʒ to ℈.
5· Reduce 8· lb. to dwts., and 14· lb. Apoth. to grs.
6· Reduce 7· miles, 3· fur. to inches.
7· Reduce 14· miles, 5· fur. 38· poles to yards.
8· Reduce 35· miles, 4· fur. to chains.
9· Reduce 7· leagues to links.
10· Reduce 1½ mile to yards and to links.
11· Reduce 6⅔ feet to hands.
12· Reduce Half a mile to lines.
13· Reduce 6439· links to inches.
14· Reduce 20⅝ miles to chains and feet.
15· Reduce 18· yards to nails.
16· Reduce 4⅞ yards to inches.
17· Reduce 3⅝ yards to inches.
18· Reduce 7· yards, 2· qrs. 1· nail to yards.
19· Reduce 13· yards, 1½ qrs. to yards.
20· Reduce 64· yards, 8· qrs., 3· n. to nails.
21· Reduce 14· Fl. E. to nails.
22· Reduce 33. E. E., 4· qrs. to inches.
23· Reduce 64· Fr. E. 1¾ yd. to qrs.
24· Reduce 18· Scotch Ells to inches.
25· Express in terms of 1· yard, "A yard and a half and a h
quarter."
26· Express 34· sup. yds. in inches.
27· Express 11· square poles in ft. and inches.
28· Express 95· acres in sq. yds. and inches.
29· Express 3· acres in sup. feet.

30· Express 8· sq. miles in sup. yds.

31· Express 14· sq. miles in sup. chains.

32· Express 17· ac. 3· r. 32· po. in sq. links.

33· Reduce 18· sq. miles to sup. poles.

34· Reduce 1· ac. to sup. ft. and inches.

35· Reduce 349· sup. yds. 3½ sup. ft. to seconds.

36· Reduce 429⅓ sq. yds. to sup. inches.

37· Reduce 9· cu. ft. 3·′ 9·″ 7·‴ to ins.

38· Reduce 14· galls. to cub. ins.

39· Reduce 87· cu. yds. 9‴ to thirds.

40· In 8· qrs. 5· bu. 3· pks. 1· gall. 3· qts. 1· pt. how many pints?

41· In 35· qrs. 7· bu. 3· pks. how many pecks?

42· Reduce 14· loads 3· qrs. 5· bu. to qrs.

43· In 95· galls. 3· pints, how many gills?

44· In 435· qrs. 7· bush. 1· gall. 2· pints, how many gallons?

45· Reduce 11· weeks to hours, minutes and seconds.

46· In 8· solar years, how many seconds?

47· Reduce to the triangular form each of the Tables of Concrete Units of Comparison, excepting the Money Table. (See Page 182·)

D.

EASY EXAMPLES, WHICH MAY BE WORKED MENTALLY.*

1· How many twopenny loaves may be bought for 1·s.; 1·s. 3·d.; 1·s. 9·d.; 2·s.; 7½d.; 2·s. 3·d.; 6·s.; 8·s. 6·d.; 10·s.; 10·s. 6·d.; 11·s. 3·d.; 12·s. 9·d.; 18·s. 4½d.?

2· How many threepenny, fourpenny, or sixpenny, loaves for each of those amounts?

3· How many groats are equivalent to 4·s.; 5·s. 8·d.; 6·s. 4·d.; 3·s. 2·d.; 6·s. 1·d.; 15·s. 9·d.; 8·s. 5·d.; 6·s. 7·d.; 2·s. 7·d.; 11·d.; 7·d.; 6·d.; 8½d.; £5·; £20·; £100·?

4· How many lb. beef at 4·d. will cost 10·s.; 7·s.; 5·s. 8·d.; 11·d.; 6·d.; 2·s. 6·d.; 4·s. 1·d.; 3·s. 4·d.; 11·s. 6·d.; 1·s. 8·d.; 2·s. 10·d.; 5·s. 7·d.; £1·; £2· 10·s.?

5· If a cap cost half-a-crown, how many such will cost £1·; £4·; 10·s.; 7·s. 6·d.; 12·s. 6·d.; 15·s.; 17·s. 6·d.; £3· 10·s.; £5· 12·s. 6·d.; £3· 2·s. 6·d.; £100·?

6· At a wage of a florin per day, how many days will a man be earning £10·; £4·; 10·s.; £3· 5·s.; £7· 10·s.; 1·s. 6·d.; 2·s. 6·d.; 8·s. 4·d.; £6· 6·s.; £22· 16·s.; £50·?

7· Supposing oranges to be making three-halfpence each, what number may be purchased for 1·s.; 2·s.; 3·s.; 4·s. 6·d.; 5·s. 3·d.; 6·s. 6·d.; 4½d.; 7½d.; 9·d.; 10½d.; 7·s. 7½d.; 10·s. 6·d.; £3·; £1· 5·s.; £4· 10·s.; 15·s.; 9·s. 6·d.?

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* Note to Teacher. *Examples like these, so simple as to be easily solved mentally, are the best calculated, at once, to elucidate the true nature, and to exhibit the practical utility of the processes which they involve.* Our limits restrict us to a few specimen questions of this kind, but their number may be augmented and their character varied in proportion to the intellectual exigencies of the learner. A directly beneficial result of such examples is the *familiarity with the tables* which they produce. No harm will be done by occasionally *anticipating* a little, in these mental exercises, the necessarily systematic arrangement of this work. The necessity thus imposed on the pupil to strike out his own path, by the light of Principles already inculcated, will tend materially to develope mental vigour and enterprise.

8· Eggs being sold at the rate of two for threehalfpence, how many are worth 6·d.; 9·d.; 1·s.; 1·s. 6·d.; 2·s. 6·d.; 3·s. 3·d.; 8·s. 7½d.; 10·s. 6·d.; £1·?

9· How many 2· oz. packets of tobacco can be weighed out from 1· lb.; 4· lb.; 3· lb. 6· oz.; 7¼ lb.; 5¼ lb.; 11¾ lb.; 16· lb.; 4⅜ lb.; 7⅝ lb.; 13⅞ lb.; 1· qr.; 5· lb. 13· oz.?

10· At the rate of 7· for 6·d., how many pears will cost 5·s.; 4·s.; 8·s. 6·d.; £1·; £2· 10·s.; £5· 5·s.; 14·s.; 11·s. 3·d.; 4·s. 9·d.; 30·s.; 30·d.; 1· crown?

11· How many "half quarters (of a lb.) of tea" may be obtained from 8· oz.; 12· oz.; 3· lb.; 4½ lb.; 4· lb. 4· oz.; ¼ lb.; 2· oz.; 10· oz.; 1· lb. 6· oz.; 4⅞ lb.?

12· Herrings at 5· for 2·d., how many may be bought for 4·d.; 6·d.; 8·d.; 10·d.; 1·s.; 2·s.; 3·s. 4·d.; 100·d.; half-a-crown; a florin; £1·?

13· How many lb. of 'five-shilling tea' may be purchased for £1·; £4·; £7· 10·s.; £3· 15·s.; 2·s. 6·d.; 7·s. 6·d.; 1·s.; 2·s.; 3·s.; 4·s.; 6·s.; 1·s. 3·d.; 3·s. 9·d.; £20·; £50·; 15·s.; 12·s. 6·d.; 7½d.; 3¾d.; 6·s. 3·d.; 17·s. 6·d.?

14· How many pint bottles of wine can be filled from 1· gal.; 3· gal.; 1· qt.; 14· gal.; 12½ gal.; 1· hhd.; 1· pipe; 1· tun; 7· qts. 3· gi.; 19⅝ qts.?

15· How many pints of peas in 1· peck; 4· p.; 7· p.; 4· gal.; 2· gal. 1· qt.?

16· How many half-pints of beer can be drawn from 1· bar.; 14· gal.; 7½ gal.; ½ bar.; 1½ bar.; 2· bar.; ¾ bar.; 15· gal.; 8½ gal.; 13· qts. 1· pi.?

17· Out of a bushel and a half of beans, how many horses can have a feed of 2· qts. each?

18· How many hands high is a horse whose stature is 5· ft. 4· in.?

19· In 3· furlongs how many paces? How many inches in a fathom?

20· How many cubic inches of water in 100· gal.? (PRINCIPLE VIII.)

DESCENDING REDUCTION OF FRACTIONS.

Concrete Fractions are reduced to lower denominations precisely as Concrete Integers are.

SECOND EXAMPLE WORKED OUT. Express $£\frac{7}{160}$ in terms of a shilling, and also in terms of a penny.

That is to say, "Reduce $£\frac{7}{160}$ to shillings and pence."

$$£\tfrac{7}{160} = \tfrac{7}{160} \text{ of } \overset{s.}{20\cdot} = \tfrac{7}{160} \times \overset{s.}{20\cdot} = \tfrac{7}{8} \textit{ of a shilling.}$$

$$\text{But, } \overset{s.}{\tfrac{7}{8}} = \tfrac{7}{8} \text{ of } \overset{d.}{12\cdot} = \tfrac{7}{8} \times \overset{d.}{12\cdot} = \overset{d.}{\tfrac{21}{2}} = \overset{d.}{10\tfrac{1}{2}},$$

$$\therefore \quad £\tfrac{7}{160} = \tfrac{7}{8} \text{ of a } \textit{shilling}, \text{ or} = \overset{d.}{10\tfrac{1}{2}}.$$

In the work of this Example there are two changes of Unit. First, $£\frac{7}{160}$ are changed into $\frac{7}{160}$ of a shilling; and, second, $\frac{7}{8}$ of a shilling are changed into $\frac{7}{8}$ of a penny.

THIRD EXAMPLE WORKED OUT. Find the value of $\frac{357}{4480}$ of a Ton.

That is to say, "Reduce $\frac{357}{4480}$ of a ton to cwts. qrs. lb., &c."

$$\overset{Ton.}{\tfrac{357}{4480}} = \tfrac{357}{4480} \text{ of } \overset{cwts.}{20\cdot} = \overset{cwt.}{\tfrac{357}{4480}} \times 20\cdot = \overset{cwt.}{\tfrac{51}{32}} = 1\tfrac{19}{32} \textit{ cwt.}$$

$$\text{But, } \overset{cwt.}{\tfrac{19}{32}} = \tfrac{19}{32} \text{ of } \overset{qrs.}{4\cdot} = \overset{qr.}{\tfrac{19}{32}} \times 4\cdot = \overset{qr.}{\tfrac{19}{8}} = \overset{qrs.}{2\tfrac{3}{8}}$$

$$\text{Again, } \overset{qr.}{\tfrac{3}{8}} = \tfrac{3}{8} \text{ of } \overset{lb.}{28\cdot} = \overset{lb.}{\tfrac{3}{8}} \times 28\cdot = \overset{lb.}{\tfrac{21}{2}} = \overset{lb.}{10\tfrac{1}{2}} = \overset{lb.}{10\cdot}\ \overset{oz.}{8\cdot}$$

$$\therefore \quad \overset{Ton.}{\tfrac{357}{4480}} = \overset{cwt.}{1\cdot}\ \overset{qrs.}{2\cdot}\ \overset{lb.}{10\cdot}\ \overset{oz.}{8\cdot}$$

FOURTH EXAMPLE WORKED OUT. To what fraction of a groat is £$\frac{1}{160}$ equivalent?

To ascertain this, "Reduce £$\frac{1}{160}$ to groats."

$$60\cdot \textit{ groats} = £1\cdot$$

$$\therefore \quad £\tfrac{1}{160} = \tfrac{1}{160} \text{ of } 60\cdot \textit{ groats} = \overset{grt.}{\tfrac{1}{160}} \times 60\cdot = \tfrac{3}{8} \text{ of a } \textit{groat.}$$

EXERCISE LXXXIV.

EXAMPLES IN DESCENDING REDUCTION OF VULGAR FRACTIONS.

1· Express £$\frac{17}{320}$ in terms of a shilling.
2· Reduce $\frac{11}{252}$ of a guinea to the fraction of a shilling.
3· Reduce £$\frac{5}{24}$ to the fr. of a crown; and £$\frac{11}{16}$ to the fr. of half a crown.
4· What part of a superficial foot are $\frac{13}{27}$ of a superficial yard?
5· What part of 1· lb. = $\frac{1}{3136}$ of a Ton?
6· What fraction of 1· oz. = $\frac{1}{2016}$ of 1· cwt.?
7· Express $\frac{1}{320}$ of 1· lb. in terms of a dram.
8· Reduce $\frac{9}{19360}$ of a mile to the fraction of a yard.
9· Reduce $\frac{1}{3900}$ of an hour to the fraction of a second.
10· What fraction of a cubic foot are $\frac{11}{12}$ of a cubic yard?
11· What fraction of a gill are $\frac{13}{176}$ of a gallon?
12· What fraction of a day are $\frac{2}{49}$ of a week?
13· What fraction of a lineal inch are $\frac{4}{111}$ of a Scotch Ell?
14· What fraction of a groat are $\frac{3}{16}$ of a florin?
15· What fraction of a pint is $\frac{1}{64}$ of a peck?
16· What fraction of a scruple is $\frac{1}{1000}$ of 1· lb.?
17· What fraction of an E. Ell are $\frac{7}{8}$ of a Fr. Ell?
18· What fraction of a nail are $\frac{3}{64}$ of a yard?
19· What fraction of a perch is $\frac{1}{480}$ of an acre?
20· What fraction of £1· are $\frac{11}{12}$ of a guinea?
21· What fraction of 1· peck are $\frac{5}{80}$ of a bushel?
22· Reduce to shillings, &c. £$\frac{137}{960}$; £$\frac{7}{12}$; £$\frac{5}{12}$; £$\frac{1}{12}$; £$\frac{1}{8}$; £$\frac{3}{8}$; £$\frac{5}{8}$.

Find the value of each of the following Fractions.

23· Of a cwt.; $\frac{7}{8}, \frac{3}{7}, \frac{15}{16}, \frac{49}{56}, \frac{23}{28}, \frac{1459}{1792}, \frac{1059}{4096}, \frac{62743}{28672}, \frac{7733}{86016}$.

24· Of £1·; $\frac{5}{8}, \frac{7}{8}, \frac{1}{16}, \frac{3}{16}, \frac{5}{16}, \frac{7}{16}, \frac{9}{16}, \frac{11}{16}, \frac{13}{16}, \frac{15}{16}, \frac{4}{15}, \frac{7}{15}, \frac{11}{15}, \frac{14}{15}$.

25· Of a shilling; $\frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{3}{16}, \frac{5}{16}, \frac{7}{16}, \frac{9}{16}, \frac{11}{16}, \frac{13}{16}, \frac{15}{16}, \frac{7}{24}, \frac{19}{48}, \frac{11}{96}$.

26· Of a guinea; $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{9}, \frac{11}{12}, \frac{13}{16}, \frac{5}{18}, \frac{11}{21}, \frac{19}{24}, \frac{17}{28}, \frac{23}{36}, \frac{37}{42}, \frac{41}{56}$.

27· Of a lb. Troy; $\frac{2}{3}, \frac{5}{8}, \frac{13}{14}, \frac{11}{18}, \frac{19}{30}, \frac{29}{36}, \frac{29}{48}, \frac{45}{64}, \frac{59}{72}, \frac{83}{90}, \frac{89}{96}, \frac{113}{180}, \frac{125}{144}$.

28· Of a mile; $\frac{17}{32}, \frac{1413}{1980}, \frac{9}{11}, \frac{2117}{2880}, \frac{23}{24}, \frac{619}{2640}, \frac{41}{72}, \frac{679}{880}, \frac{1567}{2880}, \frac{949}{5760}$.

29· Of an acre; $\frac{7}{8}$, $\frac{11}{20}$, $\frac{115}{242}$, $\frac{39}{121}$, $\frac{19}{40}$, $\frac{13}{55}$, $\frac{75}{88}$, $\frac{23}{32}$, $\frac{143}{156}$, $\frac{61}{72}$, $\frac{37}{66}$, $\frac{15}{19}$, $\frac{34}{41}$.
30· Of a common year; $\frac{11}{15}$, $\frac{311}{384}$, $\frac{973}{2190}$, $\frac{7}{8}$, $\frac{13}{16}$, $\frac{24169}{525600}$, $\frac{959}{8760}$, $\frac{729}{730}$.
31· Of a gallon; $\frac{19}{32}$, $\frac{11}{16}$, $\frac{5}{8}$, $\frac{35}{64}$, $\frac{67}{88}$, $\frac{13}{16}$, $\frac{329}{352}$, $\frac{3}{17}$, $\frac{19}{29}$, $\frac{187}{643}$.
32· Of a lineal yard; $\frac{3}{8}$, $\frac{7}{16}$, $\frac{5}{8}$, $\frac{9}{16}$, $\frac{11}{32}$, $\frac{47}{64}$, $\frac{15}{16}$, $\frac{13}{18}$, $\frac{109}{128}$, $\frac{7}{11}$, $\frac{13}{49}$.
33· Of a moidore; $\frac{11}{18}$, $\frac{149}{162}$, $\frac{199}{288}$, $\frac{65}{81}$, $\frac{43}{72}$, $\frac{91}{108}$, $\frac{8}{9}$, $\frac{23}{27}$, $\frac{461}{576}$, $\frac{1011}{1296}$.
34· Of a crown; $\frac{5}{7}$, $\frac{13}{19}$, $\frac{16}{41}$, $\frac{23}{60}$, $\frac{39}{40}$, $\frac{7}{10}$, $\frac{4}{5}$, $\frac{13}{15}$, $\frac{17}{20}$, $\frac{29}{30}$, $\frac{16}{21}$, $\frac{37}{90}$.
35· Of a square mile; $\frac{481}{1536}$, $\frac{9161}{15488}$, $\frac{31}{32}$, $\frac{13}{16}$, $\frac{5}{8}$, $\frac{46158307}{4014489600}$.
36· Of a cubic yard; $\frac{21307}{46656}$, $\frac{11}{12}$, $\frac{7}{9}$, $\frac{13}{15}$, $\frac{65}{729}$, $\frac{45}{98}$.
37· Of a pound Apoth.; $\frac{13}{96}$, $\frac{51}{60}$, $\frac{19}{48}$, $\frac{341}{576}$.
38· Of an Eng. Ell; $\frac{19}{20}$, $\frac{85}{160}$, $\frac{31}{54}$, $\frac{37}{45}$, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{21}{30}$, $\frac{23}{27}$.
39· Of a lineal chain; $\frac{4}{5}$, $\frac{71}{88}$, $\frac{163}{198}$, $\frac{195}{242}$, $\frac{141}{150}$, $\frac{7}{18}$, $\frac{19}{45}$.
40· Of a tun; $\frac{943}{1008}$, $\frac{13}{63}$, $\frac{1118}{2016}$, $\frac{7103}{8064}$, $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, $\frac{9}{11}$, $\frac{13}{15}$.
41· Of a bushel; $\frac{63}{64}$, $\frac{121}{128}$, $\frac{9}{16}$, $\frac{13}{14}$, $\frac{114}{119}$, $\frac{6}{13}$, $\frac{11}{15}$, $\frac{8}{9}$, $\frac{19}{21}$.
42· Of a ton; $\frac{4157}{4480}$, $\frac{2163}{3584}$, $\frac{7}{8}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{9}{25}$, $\frac{17}{24}$, $\frac{13}{48}$, $\frac{79}{80}$.
43· Of a hogshead; $\frac{405}{504}$, $\frac{2001}{2016}$, $\frac{213}{252}$, $\frac{160}{189}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$, $\frac{9}{11}$.

DESCENDING REDUCTION OF DECIMAL FRACTIONS.

Concrete Decimals, like all other Concrete Fractions, are reduced to lower denominations precisely as Concrete Integers are.

FIFTH EXAMPLE WORKED OUT. Reduce ·9 of £1· to shillings.

Example completed.

£.
·9
20·
18· *shillings.*

Here, as before, we implicitly change the ·9 of £1· into ·9 of 1·*s.*, and then multiply by 20· because 20·*s.* = £1·

SIXTH EXAMPLE WORKED OUT. Find the value of £·928125.

Example completed.

£.
·928125
20·
18·5625 *shillings* in £·928125
12·
6·75 *pence* in *s.*·5625
4·
3· *farthings* in *d.*·75

∴ *£·928125* = *s.*18· *d.*6$\frac{3}{4}$.

Observe, that we only reduce the *Decimals*, the units in each line being of the required denomination. There are, of course, three implicit changes of unit in this Example.

SEVENTH EXAMPLE WORKED OUT. Find the value of ·279296875 of a *cwt.*

Example completed.

$\overset{Cwt.}{\cdot279296875}$
4·

1·1171875 *qr.* in $\overset{Cwt.}{\cdot279296875}$
4·

·46875 of 7· *lb.* in $\overset{qr.}{\cdot1171875}$
7·

3·28125 *lb.* in $\overset{qr.}{\cdot1171875}$
16·

4·5 *oz.* in $\overset{lb.}{\cdot28125}$
16·

8· *drs.* in $\overset{oz.}{\cdot5}$

∴ ·279296875 of a *cwt.* = $\overset{qr.}{1\cdot}\ \overset{lb.}{3\cdot}\ \overset{oz.}{4\cdot}\ \overset{drs.}{8\cdot}$

The 7· and 4· are employed as Factors of the Composite Multiplier 28· PRINCIPLE XI.

EXERCISE LXXXV.

EXAMPLES IN DESCENDING REDUCTION OF DECIMALS.

Find the value

1· Of ·659375 of £1·
2· Of ·5625 of 1·*s.*
3· Of ·875 of a *cwt.*
4· Of ·475 of a *mile.*
5· Of ·6875 of a *lin. yd.*
6· Of ·56875 of £1·
7· Of ·48125 of £1·
8· Of ·04375 of £1·
9· Of ·040625 of £1·
10· Of ·6875 of 1· *s.*
11· Of ·1875 of a *crown.*
12· Of ·625 of a *moidore.*
13· Of ·015625 of a *cwt.*
14· Of ·0078125 of a *cwt.*
15· Of ·4453125 of a *lin. mile.*
16· Of ·53125 of a *cwt.*
17· Of ·021875 of £1·
18· Of ·8125 of 1·*s.*
19· Of ·025 of £1·
20· Of ·575 of a *lin. yd.*
21· Of ·0525 of an *hour.*
22· Of ·15625 of a *day.*
23· Of ·83125 of an *acre.*
24· Of ·0625 of a *guinea.*
25· Of ·37890625 of a *lin. mile.*
26· Of ·034375 of £1·
27· Of ·0166015625 of a *lin. mi.*
28· Of ·7578125 of a *cwt.*
29· Of ·3765625 of a *lb. troy.*
30· Of ·61328125 of a *qr. cap.*
31· Of ·56875 of an *acre.*
32· Of ·984375 of a *bushel.*
33· Of ·0859375 of a *tun.*
34· Of ·125 of a *cubic yard.*
35· Of ·0212890625 of a *sq. mi.*
36· Of ·0003125 of a *week.*

To find the value of an Interminate Decimal of any Concrete Quantity.

First. Reduce the Interminate Decimal to its Equivalent Vulgar Fraction. (EXERCISE LXXIII. Page 143·)

Second. Find the value of this Fraction. (EXERCISE LXXXIV. Page 189·)

First Example worked out. What is the value of $\cdot\dot{4}7619\dot{0}$ of a bushel?

$$\overset{\textit{bush.}}{\cdot\dot{4}7619\dot{0}} = \overset{\textit{bush.}}{\tfrac{476190}{999999}} = \overset{\textit{bush.}}{\tfrac{10}{21}} = \overset{\textit{pk.}}{1\cdot}\quad \overset{\textit{gal.}}{1\cdot}\quad \overset{\textit{qt.}}{3\cdot}\quad \overset{\textit{pt.}}{0\cdot}\quad \overset{\textit{gill.}}{1\tfrac{19}{21}}$$

Second Example worked out. Find the value of £$\cdot708\dot{3}$

$$\overset{\textit{£.}}{\cdot708\dot{3}} = \overset{\textit{£.}}{\tfrac{6375}{9000}} = \overset{\textit{£.}}{\tfrac{17}{24}} = \overset{\textit{s.}}{14\cdot}\quad \overset{\textit{d.}}{2\cdot}$$

EXERCISE LXXXV. (*continued.*)

Find the value

37· Of $\cdot0208\dot{3}$ of a *shilling*.
38· Of $\cdot2\dot{6}$ of £1·
39· Of $\cdot5\dot{4}7619\dot{0}$ of a *guinea*.
40· Of $\cdot82\dot{1}4285\dot{7}$ of a *cwt*.
41· Of $\cdot3053\dot{5}7142\dot{8}$ of a *ton*.
42· Of $\cdot67\dot{8}5714\dot{2}$ of a *week*.
43· Of $\cdot07291\dot{6}$ of a *league*.
44· Of $\cdot27329\dot{5}\dot{4}$ of a *mile*.
45· Of $\cdot669791\dot{6}$ of a *moidore*.
46· Of $\cdot9\dot{1}$ of a *lb. Troy*.
47· Of $\cdot2\dot{1}4285\dot{7}$ of a *lb. Avoir.*
48· Of $\cdot158482\dot{1}4285\dot{7}$ of a *tun*.
49· Of $\cdot\dot{6}\dot{3}$ of an *acre*.
50· Of $\cdot6597\dot{2}$ of a *yard*.
51· Of $\cdot2785\dot{4}93827160\dot{}$ of a *square yard*.
52· Of $\cdot71739969135\dot{8}02469135\dot{8}$ of a *cubic yard*.

ASCENDING REDUCTION.

To Reduce any Concrete Quantity to units of a higher Denomination.

Divide by the number of smaller units which make one of the greater.

First Example worked out. Integers of Money-Value. In 34215· farthings how many pence, shillings, and pounds?

Example completed.

$$\begin{array}{l} 4\cdot\overline{)34215\cdot}\ \textit{farthings.} \\ 12\cdot\overline{)8553\tfrac{3}{4}}\ \textit{pence.} \\ \quad 20\cdot\overline{)712\cdot}\ \textit{shillings},\ 9\tfrac{3}{4}\ \textit{pence.} \\ \qquad £35\cdot\ \ 12\cdot\textit{s}.\ \ 9\tfrac{3}{4}\textit{d}. \end{array}$$

In Descending Reduction, the change of unit is made before Multiplying. (See P. 184·) But in Ascending Reduction, the unit is changed *after Dividing*. In the present Example, by Dividing the 34215· farthings by 4· we find that they are equal to 8553¾ *lots of four farthings each.* (See Principle xiv. P. 99·) But each such lot being equal in value to one penny, we change the unit, and call the 8553¾ lots of four farthings, 8553¾ pence. If we were *actually* to exchange any number of farthings for their equivalent in pence, we should take these two steps. For, we should first count the farthings into fours. That is the Division by four. We should then exchange each lot of four farthings for one penny. That is the change of unit.

EXERCISE LXXXVI.

EXAMPLES IN ASCENDING REDUCTION OF INTEGERS.

1· In 341653· pence how many groats?
2· In 6134298· farthings how many florins?
3· How many sixpenny loaves will 64321· farthings buy?
4· How many sovereigns in one million groats?
5· How many moidores in ten thousand pence?
6· Reduce 436145· shillings to guineas, and to crowns.
7· In 3472651· oz. how many lb. qrs. cwts. and tons?
8· In 4365428· grs. how many lb. Troy?

Reduce

9· 42515424· hf. pence to moi.
10· 2718125· hf. crs. to £.
11· 4453025· farthings to £.
12· 3584621· dwts. to lb.
13· 8000· scruples to lb.
14· 643756· nails to yds.
15· 34216854· lin. ins. to yds.
16· 8342165· qrs. to Ells E.
17· 4316521· qrs. to Ells Fr.
18· 7218642· qrs. to Ells Fl.
19· 3421864· sec. to days.
20· 4381697· hours to weeks.
21· 948312· pints to galls.
22· 5841632· galls. to tuns.
23· 6000498· gills to bushels.
24· 415843· groats to guineas.
25· 7349862· grs. to lb. Avoir.
26· 8431684· cu. ins. to galls.
27· 714863· threepences to gs.
28· 614832· lin. ins. to yds.

SECOND EXAMPLE WORKED OUT. Supposing a man to step uniformly one yard, how far would he travel in 1026559· steps?

We have to reduce 1026559· yds. to miles.

Lin. yds.

3·)1026559·

5½)342186· *yds.* 1· *ft.*

2·　　2·

11·)684372· *half-yds.* Page 153·

40·)62215· *po.* 3· *yds.* 2· *ft* 6· *ins.*

8·)1555· *fur.* 15· *po.* 3· *yds.* 2 *ft.* 6· *ins.*

194· *mi.* 3· *fur.* 15· *po.* 3· *yds.* 2· *ft.* 6· *ins.*

In dividing the half-yards by 11· we get a Remainder of $\frac{7}{2}$ yds. = $3\frac{1}{2}$ yds. or 3· yds. 1· ft. 6· ins. The 1· ft· 6 ins. we add to the 1· ft. previously remaining, making together 2· ft. 6· ins.

In the work of this Example there are five Changes of Unit.

Method of Proof. Ascending and Descending Reduction being exactly contrary processes, each may be employed to test the accuracy of the other. Hence, to prove the foregoing work, we must Reduce 194· mi. 3· fur. 15· po. 3· yds. 2· ft. 6· ins. to yards.

EXERCISE LXXXVI. *continued.*

Reduce

29· 4135468· lines to poles.
30· 7184632· inches to miles.
31· 936184365· inches to Ells E.
32· 31642817· sup. ins. to poles.
33· 41381643· sup. ft. to acres.
34· 91438642· sup. yds. to sq. m.
35· 34862185· cu. ins. to yds.
36· 9543216· cu. ft. to yds.
37· 95843621· lines to Sc. Ells.
38· 714321806· sup. ins. to acres.
39· 34218643· sup. ft. to sq. mi.
40· 859436285· cu. ins. to qrs.
41· 74183625· lin. ins. to chs.
42· 359642184· seconds to years.
43· 16842185· grs. to lb. Avoir.
44· 1843976· cu. ins. to galls.

ASCENDING REDUCTION OF FRACTIONS.

Concrete Fractions are Reduced to higher Denominations precisely as Concrete Integers are, namely, by Dividing by the number of smaller units which make one of the greater.

But, by Principle XVII., Dividing by this number will produce the same result (in figures) as Multiplying by its Reciprocal. And this Reciprocal will express the Fraction which the smaller unit is of the greater.

No. of pence in 1·s. = 12·.
Fraction which 1·d. is of 1·s. $= \frac{1}{12}$; the Reciprocal of 12·.

Hence we have the following simple Rule.

To express a Fraction of any Concrete Unit in terms of a higher Unit.

Multiply given Fraction by the Fraction which the smaller unit is of the greater.

FIRST EXAMPLE WORKED OUT. Express $\frac{7}{15}$ of a penny as a Fraction of a shilling, of a half-crown, and of a florin.

First. $1\cdot d. = \frac{1}{12}$ of $1\cdot s.$

$\therefore \frac{7}{15}$ of $1\cdot d. = \frac{7}{15}$ of $\frac{1}{12}$ of $1\cdot s. \overset{s.}{=} \frac{7}{15} \times \frac{1}{12}$ (PRIN. IX.) $= \frac{7}{180}$ of a *shilling.*

Second. $1\cdot d. = \frac{1}{30}$ of half a crown.

$\therefore \frac{7}{15}$ of $1\cdot d. = \frac{7}{15}$ of half a crown, $\overset{h.c.}{=} \frac{7}{15} \times \frac{1}{30}$ (PRIN. IX.) $= \frac{7}{450}$ of *half a crown.*

Third. $1\cdot d. = \frac{1}{24}$ of a florin.

$\therefore \frac{7}{15}$ of $1\cdot$d. $= \frac{7}{15}$ of $\frac{1}{24}$ of a florin, $\overset{flor.}{=} \frac{7}{15} \times \frac{1}{24}$ (PRIN. IX.) $= \frac{7}{360}$ of a *florin.*

SECOND EXAMPLE WORKED OUT. What part of a moidore are $\frac{3}{8}$ of a guinea?

$$1\cdot \text{guinea} = \tfrac{7}{9} \text{ of a moidore.}$$

$\therefore \frac{3}{8}$ of a guin. $= \frac{3}{8}$ of $\frac{7}{9}$ of a moid. $\overset{\textit{moid.}}{=} \frac{3}{8} \times \frac{7}{9}$ (PRIN. IX.) $= \frac{7}{24}$ of a *moidore.*

Note the change of unit in these Examples.

THIRD EXAMPLE WORKED OUT. Reduce $\frac{7}{8}$ of a lineal yard to the Fraction of a pole.

$$1\cdot \text{lineal yard} = \tfrac{1}{5\frac{1}{2}} \text{ of a pole}, = \tfrac{2}{11} \text{ of a pole.}$$

$\therefore \frac{7}{8}$ of $1\cdot$ yd. $= \frac{7}{8}$ of $\frac{2}{11}$ of a pole, $= \frac{7}{8}$ po. $\times \frac{2}{11}$ (PRIN. IX.) $= \frac{7}{44}$ of a pole.

FOURTH EXAMPLE WORKED OUT. Reduce $\frac{3}{16}$ of a lb. to the Fraction of a ton.

$$1\cdot \textit{lb.} = \tfrac{1}{28} \text{ of } \tfrac{1}{4} \text{ of } \tfrac{1}{20} \textit{ of a ton.}$$

$$\therefore \tfrac{3}{16} \text{ of } 1\cdot \textit{lb.} = \tfrac{3}{16} \text{ of } \tfrac{1}{28} \text{ of } \tfrac{1}{4} \text{ of } \tfrac{1}{20} \textit{ of a ton}$$

$$= \frac{3}{16 \times 28 \times 4 \times 20} \text{ or } = \frac{3}{35840} \textit{ of a ton.}$$

EXERCISE LXXXVII.

1· Reduce $\frac{14}{35}$ lb. to the fraction of a qr. and to the fraction of a stone.

2· Express $\frac{31}{108}$ guin. as a fraction of a moidore, and of a Joannes.

3· What fraction of a moidore is equivalent to $\frac{1}{9}$ of $\frac{3}{7}$ of a guinea?

4· Reduce $4\frac{3}{8}$d. to the fraction of a shilling, of a groat, and of a crown.

5· What part of a ton are $11\frac{1}{5}$ lb.?

6· Reduce $\frac{3}{8}$ of $9\frac{3}{5}$ oz. to the fraction of a pound Troy.

7· In $\dfrac{4\frac{1}{2}}{5\frac{6}{7}}$ of a dram what part of a pound?

8· Express in terms of a tun $3\frac{5}{8}$ galls. $+ \dfrac{6\frac{1}{4} - 1\frac{3}{4}}{2\frac{1}{3} \times 4\frac{2}{3}}$ galls.

9· Reduce $\frac{12}{133}$ of £3· 6·s. to the fraction of £1.

10· Express $\frac{3}{15}$ of $3\frac{1}{3}$ sixpences in terms of a crown.

11· Express the sum of $\frac{7}{8}$s. and $\frac{11}{14}$s. in terms of half a guinea.

12· What fraction of an Ell E. is equivalent to $5\frac{5}{8}$ yds. — $3\frac{7}{8}$ yds.?

13· What part of an hour are $2\frac{3}{16}$ times $\frac{11}{12}$ of 14·5 seconds equal to?

14· Reduce $7\frac{3}{4}$ yds. to the fraction of a furlong.

15· Find what fraction of an acre $= 19\frac{1}{3}$ times $\frac{4}{272\frac{1}{4}}$ of a sup. pole.

16· What portion of a Fl. Ell remains after cutting off $11\frac{2}{3}$ of $\frac{3}{2\frac{1}{4}}$ of a nail?

17· Express "A yard and a half and a half quarter" in Ells E.

18· What part of a week is $\frac{3}{8}$ of $\frac{4}{5}$ of $4\frac{1}{4}$ days?

ASCENDING REDUCTION OF DECIMALS.

To express a Simple Concrete Quantity as Decimal of a higher unit of the same kind. PRINCIPLE V.

A Simple Quantity contains units all of the same magnitude. Page 36·

Divide by the number of the smaller units which make one of the greater, carrying out Quotient in Decimal Places. See EXERCISE LVIII. Page 110·

FIRST EXAMPLE WORKED OUT. Reduce 9·d. to the Decimal of 1· shilling.

12·) 9·d.
·75 *of a shilling.*

This process is precisely the same as that employed in Ascending Reduction of Integers. Dividing by 12· shews us that 9·d. = ·75 of 12· *pence;* and, ∴ by change of unit, 9·d. = ·75 of *a shilling.*

To express a Compound Quantity as Decimal of a higher unit of the same kind. PRINCIPLE V.

A Compound Quantity contains units of various magnitudes, but all of the same kind. Page 36·

First. Reduce lowest units to next higher name, as directed above, annexing Quotient to given units (if any) of that higher name.

Second. Reduce the Quantity thus formed to next higher name; and so on, until the whole is reduced to the required denomination.

It is convenient to arrange the different units of given quantity in a vertical column, those of least value being placed uppermost. The successive Quotients can then be set, as found, in their places beside the units to which they belong.

SECOND EXAMPLE WORKED OUT. What Decimal Fraction of £1· is equivalent to 18·s. 4½d.?

4· | 2· *farthings*
12· | 4·5 *pence* = 4½d.
20· | 18·375 *shillings* = 18·s. 4½d.
£·91875 = 18·s. 4½d.

The work may be proved by reversing the process, and finding the value of £·91875, by Descending Reduction. EXERCISE LXXXV. Page 101·

EXERCISE LXXXVIII.

EXAMPLES IN ASCENDING REDUCTION OF DECIMALS.

Express each of the following amounts as a Decimal of £1·

	s.	*d.*		*s.*	*d.*		*s.*	*d.*		*s.*	*d.*
1·	3·	4½	4·	1·	11¼	7·	12·	5¼	10·	7·	2¼
2·	5·	6·	5·	13·	8¼	8·	15·	0¾	11·	16·	3¾
3·	11·	8¼	6·	17·	9¾	9·	10·	10½	12·	2·	1¼

Important Remark. Any number of farthings is equivalent to a Decimal of a groat, a penny, or a halfpenny.[*]

But, Multiples of three farthings are the only sums of money which are equivalent to exact Terminate Decimals of any of our other current coins.

Remembering this will prevent useless attempts to express as an exact Decimal of £1· &c., any sum of money which has no such equivalent. For, we may at once ascertain whether such an equivalent exists by merely reducing the odd pence and farthings of the given sum to farthings, and dividing by 3·. If there be no remainder, the amount in question is equal to an exact Decimal of *any* of our current coins; otherwise, it is equal only to a Decimal of a groat, a penny, or a halfpenny.

As £1· = 960· farthings, there are 959· sums of current money less than £1·. But of these 959· amounts, only 319· are multiples of three farthings; and, consequently, only that number of them can be expressed as Terminate Decimals of £1·, leaving 640· amounts which cannot be so expressed.

If, in place of our existing farthings, new ones were coined, each $\frac{24}{25}$ of the value of our present farthing, so that a sovereign would be equal in value to 1000· such farthings (or mils) instead of to 960· as now, then *any* sum of money under £1· would be equivalent to an exact Decimal[*] of each of our present current coins, except of a groat, a penny, and a halfpenny; which last, as we have seen, are the only ones of which *any* sum of our present money is an exact Decimal. These three, then, and the farthing, would be the only coins thrown out of currency by substituting 1000· mils for 960· farthings. Eight of our coins out of twelve would remain unaffected by the change. Each of these, excepting the threepenny-piece, would also be equal to an exact number of mils. The threepenny-piece would be = 12·5 mils, or £·0125.

EXERCISE LXXXVIII. (*continued.*)

Reduce to Decimals

13· Of a cwt., 3· qr. 14· lb.
14· Of a mile, 3· fur. 32· p.
15· Of a yard, 2· ft. 9· li.
16· Of a cwt., 1· lb. 12· oz.
17· Of a ton, 14· oz.
18· Of a lin. yd., 1· ft. 8·7 in.
19· Of an hour, 3· min. 9· sec.
20· Of a cwt., 2· qr. 3½ lb.
21· Of a day, 3· ho. 45· m.
22· Of an acre, 3· r. 13· po.
23· Of a guin., 1· s. 3¾d.
24· Of a moid., 16·s. 10½d.
25· Of a mi., 3· f. 22· p. 2· yd. 2¼ ft.
26· Of a ton, 3· qr. 14· oz.
27· Of a lb., 4· oz. 10· dwts. 9· grs.
28· Of a qr., 4· bu. 3· pe. 1· gal. 1· qt.
29· Of a tun, 21· gal. 2· qt. 1· pt. 1· gi.
30· Of a cu. yd., 3· c. ft. 4·′ 6·″
31· Of a sq. mi., 13· ac. 2· r. 20· per.
32· Of a week, 3· min. 9· sec.
33· Of a bush., 3· p. 1· gal. 3· qt. 1· pt.
34· Of a crown, 11¼d.
35· Of an acre, 2· r. 11· p.
36· Of a com. yr., 10· we. 3· da.

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[*] The Decimal may, of course, be an Improper Fraction.

EXAMPLES PRODUCING INTERMINATE DECIMALS.

Reduce to Interminate Decimals

37· Of a sov., 5·s. 4·d.
38· Of a guin., 11·s. 6·d.
39· Of a cwt., 3· qr. 8· lb.
40· Of a crown, 1·s. 2½d.
41· Of a ton, 6· cwt. 12· lb.
42· Of a sq. yd., 2· ft. 73·″
43· Of a week, 4· da. 18· ho.
44· Of a leag., 1· fur. 30· po.
45· Of a mile, 2· fur. 41· yd.
46· Of a moid., 13·s. 4¾d.
47· Of a lb., 10· oz. 18· dwts. 16· gr.
48· Of a lb. Av., 1500· grains.
49· Of a tun, 39· gal. 3· qt. 1½ pt.
50· Of an acre, 2· r. 21· p. 24· yd. 6¾ft.
51· Of a yd., 1· ft. 11· in. 9· li.
52· Of a cu. yd., 19· c. ft. 4·′ 5·″ 3·‴

To express any Concrete Quantity in terms of any other Concrete Quantity of the same kind. PRINCIPLE V.

First. Reduce both to the same denomination.

Second. Make the naming Quantity Divisor, and the other Dividend, and Divide, expressing the Quotient as Vulgar Fraction, Integer, Mixed Number, or Decimal, as may be required.

FIRST EXAMPLE WORKED OUT. Express 16·s. 8¼d. in terms of 3·s. 8½d.

Here, *the naming quantity*, 3·s. 8½d. = 178· *farthings*,

∴ *one farthing* = $\frac{1}{178}$ of 3·s. 8½d.

The other quantity, 16·s. 8¼d. = 801· *farthings*,

∴ 16·s. 8¼d. = $\frac{801}{178}$ of 3·s. 8½d.

= 4½ times 3·s. 8½d.

= 4·5 of 3·s. 8½d.

SECOND EXAMPLE WORKED OUT. Divide £44· 13·s. 9·d. by £4· 9·s. 3·d.

£44· 13·s. 9·d. = 3575· *threepences;*

£4· 9·s. 3·d. = 357· *threepences.*

∴ £44· 13·s. 9·d. ÷ £4· 9·s. 3·d. = $\frac{3575}{357}$ of £4· 9·s. 3·d.

= $10\frac{5}{357}$ of £4· 9·s. 3·d.

Hence, £44· 13·s. 9·d. would suffice to pay for $10\frac{5}{357}$ articles at £4· 9·s. 3·d. each.

THIRD EXAMPLE WORKED OUT. Reduce 7·s. 10½d. to a Decimal of 31·s. 6·d.

31·s. 6·d. = 756· *halfpence.*

∴ *One halfpenny* = $\frac{1}{756}$ of 31·s. 6·d.

7·s. 10½d. = 189· *halfpence.*

∴ 7·s. 10½d. = $\frac{189}{756}$ or ¼ or ·25 of 31·s. 6·d.

Fourth Example worked out. Express 1· qr. 4· lb. as a Decimal of 5· stone.

$$5\cdot\text{ stone} = 70\cdot\text{ lb.} \quad \therefore 1\cdot\text{ lb.} = \tfrac{1}{70}\text{ of }5\cdot\text{ stone.}$$

$$1\cdot\text{ qr. }4\cdot\text{ lb.} = 32\cdot\text{ lb.}$$

$$\therefore 1\cdot\text{ qr. }4\cdot\text{ lb.} = \tfrac{32}{70}\text{ or }\cdot4\dot{5}7142\dot{8}\text{ of }5\cdot\text{ stone.}$$

The Example may also be worked out thus:

$$\begin{array}{r|l} 28\cdot & 4\cdot\text{ lb.} \\ 2\cdot5 & 1\cdot\dot{1}4285\dot{7}\text{ qr.} \\ \hline & \cdot4\dot{5}7142\dot{8}\text{ of }5\cdot\text{ stone.} \end{array}$$

Here, we first divide by 28· to reduce 4· lb. to Decimal of 1· qr. We then divide by 2·5 because 5· stone = 2·5 qr.

Fifth Example worked out. Reduce 4½d. to a Decimal of £25·.

$$\pounds25\cdot = 500\cdot\text{s} \quad \therefore \text{ one shilling} = \tfrac{1}{500}\text{ of }\pounds25\cdot$$

$$4\tfrac{1}{2}\text{d.} = \tfrac{3}{8}\text{s.} \quad \therefore 4\tfrac{1}{2}\text{d.} = \tfrac{3}{8}\text{ of }\tfrac{1}{500}\text{ or } = \tfrac{3}{4000}\text{ or }\cdot00075\text{ of }\pounds25\cdot$$

Or thus:

$$\begin{array}{r|l} 4\cdot & 2\cdot\text{ farthings} \\ 12\cdot & 4\cdot5\text{ pence} \\ 500\cdot & 0\cdot375\text{ shillings} \\ \hline & \cdot00075\text{ of }\pounds25\cdot \end{array}$$

Where we first reduce 4½d. to Decimal of *one* shilling; and then reduce this Decimal to that of 500·s. or £25·

Sixth Example worked out. In ten thousand merks how many crowns?

1· merk = 13·s. 4·d. or $13\frac{1}{3}$s.; 1· crown = 5·s.

∴ No. of crowns in 1· merk = $13\frac{1}{3} \div 5\cdot = \frac{40}{3} \div 5\cdot = \frac{8}{3}$ of a crown.

∴ No. of crowns in 10000· merks = 10000· times $\frac{8}{3}$ of a crown = $26666\frac{2}{3}$ crowns.

Or thus:

1· merk = 40· groats; 10000· merks = 400000· groats; and 1· crown = 15· groats.

∴ 10000· merks = $\frac{400000}{15}$ of a crown, or $26666\frac{2}{3}$ crowns.

Seventh Example worked out. How much beef may be bought for £20·, at £3· 10·s. per cwt.?

To ascertain this, we must find how often £3· 10·s. can be paid out of £20·; and this is found by Dividing £20· by £3· 10·s.

£20· = 40· half sovs.; £3· 10·s. = 7· half sovs.

∴ £20· ÷ £3· 10·s. = 40· half sovs. ÷ 7· half sovs.

= $5\frac{5}{7}$ lots of £3· 10·s. each.

∴ Quantity of beef, for which £20· will pay, $= 5\frac{4}{7}$ cwt.
= 5·cwt. 2·qr. 24·lb. EXERCISE LXXXIV.

EXERCISE LXXXIX.

EXAMPLES IN DIVISION OF ONE CONCRETE QUANTITY BY ANOTHER OF THE SAME KIND, QUOTIENT BEING EXPRESSED AS INTEGER, VULGAR FRACTION, OR DECIMAL.

A. EASY EXAMPLES, TO BE WORKED MENTALLY.

Answers, when Improper Fractions, to be given also as Mixed Numbers; and all Fractions to be given in their lowest terms.

1· Express as Fractions of 3½d., ¼d.; ½d.; ¾d.; 1·d.; 1¼d.; 1½d.; 1¾d.; 2·d.; 2¼d.; 2½d.; 2¾d.; 3¼d.; 4½d.; 7¼d.; 11½d.

2· How many pounds, or what part of 1·lb., at 3½d., can be bought for each of the above amounts; also for 1·s. 3·d.; 14·d.; 18·d.; 20·d.; 2·s.; half-a-crown; a florin; half a sov.; half a guinea; £1·; a guinea?

3· At 1¼d. per pint, how much milk for 2·d.; 3·d.; 4·d.; 6·d.; ¼d.; ½d.; ¾d.; 1·d.; 1¼d.; 1½d.; 1¾d.; 2¼d.; 2½d.; 2¾d.; 5·d.; 7½d.; 8¾d.; 10·d.; 11·d.; 1·s.; 13·d.?

How many yards, or what part of 1·yd. can be bought

4· For 6·d., at 1½d. per yd.?
5· For 10½d., at 1¾d. „ „ ?
6· For 20·d., at 1¼d. „ „ ?
7· For 2·s., at 9·d. „ „ ?
8· For 19¼d., at 2¾d. „ „ ?
9· For 30·d., at 3¾d. „ „ ?
10· For 22¾d., at 3½d. „ „ ?
11· For 2·s. 6·d., at 3¼d. per yd.?
12· For 25·s. 6·d., at 8.s. 6·d. „ „ ?
13· For 2·s. 7½d., at 2·d. „ „ ?
14· For 4·s. 1½d., at 4·d. „ „ ?
15· For 6·d., at ⅜d. „ „ ?
16· For 2½d., at $\frac{5}{16}$d. „ „ ?
17· For 8¾d., at $\frac{7}{16}$d. „ „ ?

18· Express in terms of £1·, each of the following amounts: 8·d.; 1·s. 6·d.; 3·s.; 4·s. 4·d.; 10·s. 6·d.; 2·s. 6·d.; 7·s. 6·d.; 12·s. 6·d.; 15·s.; 17·s. 6·d.; 2·s. 8·d.; 1·s. 10·d.; 3·s. 5·d.

B. MORE DIFFICULT EXAMPLES.

1· How many articles worth £2· 10·s. 7½d. each will cost £35· 8·s· 9·d.?

2· Expended £78· 19·s. 4½d. in the purchase of sheep, which cost, on an average, £1· 7·s. 8½d. each. How many were there?

3· How many persons can be paid £4· 3·s. 7½d. each, out of a fund amounting to £413· 18·s. 10½d.?

4· Into how many farms, each 47·ac. 2·r. 13·po. 29·yds., can an estate of 428·ac. 1·r. 5·po. 19·yds. be divided?

5· Supposing a man to travel uniformly at the rate of 37·mi. 5·fur. 18·po. per day, how long will he be completing a distance of 1620·mi. 2·fur. 14·po.?

NOTE TO TEACHER. It may be well here to caution the pupil against the egregious blunder inculcated by many books as to the result of Dividing one Compound Quantity by another. I have before me, at this moment, a manual, published by government authority, containing the astounding statement that "£9368· 14·s. 3¾d. ÷ £42· 7·s. 0¼d. = 4·s. 7¼d. and a fraction of a farthing"!
This error betrays an utter forgetfulness of what Division really is. It has been corrected in recent editions of the work in question. A twin absurdity is the venerable crambo, "Multiply £19· 19·s. 11¾d. by £19· 19·s. 11¾d."

6· How many bins each containing 7· qrs. 3· bu. 2· pks. 1· gal., can be filled with flour, from a store containing 104· qrs. 2· bu. 3· pks.?

7· How many scholars are there in a school of which the gross expenses amount to £143· 10·s. 1·d., and the average cost per scholar £1· 12·s. 7⅜d.?

8· How many fleeces of wool, each 10⅜ lb., will weigh 3· cwt. 2· qrs.?

9· From a tun of wine, how many dozen bottles can be filled, each bottle containing 1⅓ pint?

10· If a cwt. of sugar cost £2· 6·s. 4·d., how much can be bought for 5·s. 9⅓d.?

Find what fraction 5·*s.* 9⅓*d. is of* £2· 6·*s.* 4·*d.*

Then, the same Fraction of 1· *cwt. may be bought for* 5·*s.* 9⅓*d.*

11· Express £226· 11·s. 0⅓d. as a Vulgar Fraction of £317· 3·s. 5⅓d.

12· What is the ratio of £407· 16·s· 5¼d. to £317· 3·s. 10¾d.?

13· Reduce 13·s. 6¼d. to a Fraction of £1· 8·s. 10⅓d.

14· Reduce 41· qrs. 5· gals. to a Fraction of 56· qrs. 1· gal.

15· What Decimal Fraction of £105· 19·s. 6·d. is equal to £66· 4·s. 8¼d.?

16· By what Decimal Fraction must 8· lb. be multiplied to produce 4· lb. 7· oz. 7· drs.?

17· What part of a ton is 13· cwt. 3· qrs. 21· lb.?

18· What part of 11· guineas is £1· 13·s.?

19· How many times can 11· lb. 6· oz. be taken out of 1· cwt. 2· qrs. 8· lb. 5· oz.?

20· If a coat require 3⅝ yds. of cloth, how many such may be made from a piece measuring 60· yards, and how much cloth will there be in the remnant?

Find, and accurately describe, the Quotients

21· Of £2171· 5·s. 7⅓d. ÷ £56· 10·s. 10⅓d.

22· Of 28· Tons. 8· cwt. 1· qr. 16· lb. ÷ 1· Ton. 17· cwt. 3· qrs. 16· lb.

23· Of 1500· mi. 6· po. ÷ 27· mi. 6· fur. 9· po.

24· Of 61· qrs. 4· bu. 2· pe. 1⅔ pt. ÷ 8· qr. 4· bu. 1· pe. 1· gal. 1· qt.

25· Of £312· 18·s. 5⅔d. ÷ £40· 7·s. 6½d.

26· Of 84· ac. 2· r. 34· per. ÷ 5· ac. 3· r. $12\frac{2}{7}$ per.

27· Of 1621· lb. 4· oz. 15· dwts. 13· grs. ÷ 21· lb. 13· dwts. 17· grs.

28· Of 240· yds. 2· qrs. 3· na. ÷ 14· yds. 2· qrs. 3· na.

29· Reduce £68· 3·s. 3¼d. to the Decimal of £104· 3·s. 4·d.

30· Reduce 3· lb. $6\frac{3}{64}$ oz. to the Decimal of 3· qrs. 13· lb. 10· oz. 8· drs.

31· Reduce 68· yds. 2· ft. 4· in. 6· li. to the Dec. of 183· yds. 1· ft. 4· in.

32· Reduce £1· 5·s. 2¼d. to an Interminate Dec. of £2· 18·s. $10\frac{5}{12}$d.

33· What Interminate Decimal of £536· 4·s. 3¾d. ⅜ q. is equivalent to £417· 1·s. 1½d. ⅜ q.?

34· By what Decimal Fraction must 9·s. 5¾d. be divided to yield a Quotient equal to 10·s. 10·d.?

35· Express 46· lb. 7· ℥ 2· ʒ 1· ℈ 15· grs. as a Vulgar Fraction of 111· lb. 10· ℥ 3· ʒ.

THE COMPOUND RULES.

Addition, Subtraction, Multiplication, and Division of Compound Quantities are usually termed "Compound Addition," "Compound Subtraction," &c., and, collectively, "The Compound Rules." Each of them is, however, nothing more than a combination of the simple process with Reduction.

Thus, Compound Addition consists of several Simple Additions and Reductions.

COMPOUND ADDITION.

To find the Sum of any number of Compound Quantities, of the same kind. (PRINCIPLE V.)

First. Arrange the given Quantities so that like units, or fractions of like units, shall stand in the same vertical column; placing the columns in the order of value of their units, ascending from right to left.

Second. Add the units or fractions in right-hand column. Reduce the Sum to next higher name. Carry units found of that name, and set down Remainder under its own column.

Third. Find the Sum of next column and units carried. Reduce this Sum to next higher name. Set down and carry as before. Proceed similarly with all the other columns.

Method of Proof. The same as in Simple Addition. (Page 30·)

EXAMPLES WORKED OUT AND PROVED. I. Find the Sum of £295· 11·s. 8¾d., 18·s. 11½d., £31045· 16·s. 5¾d., £41· 10·s. 4¾d., £9642· 1·s. ¼d., and £70000· 9½d.

£.	s.	d.	
295·	11·	8¾	
	18·	11½	
31045·	16·	5¾	
41·	10·	4¾	
9642·	1·	0¼	
70000·	0·	9½	
111025·	19·	4½	= *Sum of the six amounts.*
110730·	7·	7¾	= *Sum of lower five amounts, top one* [*omitted.*
111025·	19·	4½	= *Sum of the six amounts. Proof.*

In adding, we call ½d. two farthings.

Then, Sum of farthings-column = 14· farthings = 3½d.

Set down ½d. *Carry* 3·d.

Sum of pence-column, with 3·d. carried, = 40·d. = 3·s. 4·d.

Set down 4·d. *Carry* 3·s.

Sum of shillings-column, with 3·s. carried, = 59·s. = £2· 19·s.

Set down 19·s. *Carry* £2·

The work of pounds-column is a Simple Addition.

II. Add together £614· 17·s. $9\frac{5}{16}$d., £22· 13·s. $6\frac{3}{8}$d., £134· 14·s. $5\frac{1}{2}$d., £84· 6·s. $11\frac{1}{6}$d., £5946· 12·s. $8\frac{7}{12}$d., £521· 19·s. $10\frac{3}{4}$d. and £34168· $7\frac{11}{24}$d.

£.	s.	d.	
614·	17·	$9\frac{5}{16}$	
22·	13·	$6\frac{3}{8}$	
134·	14·	$5\frac{1}{2}$	
84·	6·	$11\frac{1}{6}$	
5946·	12·	$8\frac{7}{12}$	
521·	19·	$10\frac{3}{4}$.	
34168·	0·	$7\frac{11}{24}$	
41493·	5·	$11\frac{7}{48}$	= *Sum of the seven amounts.*
40878·	8·	$1\frac{5}{6}$	= *Sum of lower six amounts; top one omitted.*
41493·	5·	$11\frac{7}{48}$	= *Sum of the seven amounts. Proof.*

L. C. D. of the seven fractions of a penny $= 48\cdot$ (See Exercise LXX.)

∴ Sum of the fractions

$$= \left(\tfrac{15}{48} + \tfrac{18}{48} + \tfrac{24}{48} + \tfrac{8}{48} + \tfrac{28}{48} + \tfrac{36}{48} + \tfrac{22}{48}\right) \text{d.}$$

$$= \tfrac{151}{48} \text{d.} = 3\tfrac{7}{48} \text{d.}$$

Set down $\frac{7}{48}$d. *Carry* 3·d.

Then proceed as in former Example.

EXERCISE XC.

EXAMPLES IN ADDITION OF COMPOUND QUANTITIES.

Find the Sum of each of the following sets of like Compound Quantities.

I. MONEY.

Ex. 1·			Ex. 2·			Ex. 3·			Ex. 4·		
£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
4351·	18·	$4\frac{1}{4}$	376·	15·	6·	9186·	14·	$10\frac{1}{2}$	5416·	15·	$5\frac{6}{7}$
695·	3·	11·	4218·	17·	$5\frac{1}{2}$	759·	15·	$9\frac{1}{8}$	325·	8·	$7\frac{4}{7}$
2·	15·	$6\frac{3}{4}$	5473·	9·	$4\frac{1}{2}$	1843·	16·	$6\frac{5}{12}$	8546·	11·	$4\frac{3}{8}$
15406·	12·	$9\frac{1}{2}$	368·	11·	$3\frac{1}{4}$	7568·	18·	$4\frac{1}{6}$	3271·	16·	$3\frac{1}{2}$
87·	6·	$4\frac{1}{4}$	9485·	14·	$7\frac{3}{4}$	4325·	17·	$8\frac{3}{4}$	9·	13·	$6\frac{3}{4}$
98643·	17·	$5\frac{1}{2}$	3786·	5·	$8\frac{1}{4}$	964·	11·	$5\frac{2}{3}$	84·	12·	$10\frac{11}{14}$
7482·	9·	$8\frac{1}{4}$	4584·	11·	$5\frac{1}{2}$	35·	13·	$11\frac{1}{4}$	13·	14·	$11\frac{23}{28}$

Ex. 5·		Ex. 6·		Ex. 7·		Ex. 8·			Ex. 9·			Ex. 10·	
s.	d.	s.	d.	s.	d.	£.	s.	d.	s.	d.	q.	s.	d.
14·	$11\frac{1}{2}$	8·	4·	3·	4·5	8453·	1·	4·5	4·	$6\frac{1}{4}$,	$\frac{1}{5}$	29·	$11\frac{3}{4}$
8·	$6\frac{1}{4}$		$10\frac{1}{2}$	6·	8·75	9642·	5·	11·25	13·	$11\frac{1}{2}$,	$\frac{3}{7}$	48·	$10\frac{1}{2}$
9·	$5\frac{1}{8}$		$11\frac{3}{4}$	2·	11·25	7493·	14·	9·75	18·	$4\frac{3}{4}$,	$\frac{4}{5}$	16·	$4\frac{1}{12}$
17·	$11\frac{3}{4}$	16·	$1\frac{1}{2}$	4·	5·125	3185·	13·	4·5	11·	$5\frac{1}{2}$,	$\frac{6}{7}$	59·	$5\frac{7}{8}$
4·	$6\frac{1}{12}$		$8\frac{3}{4}$	8·	4·375	6431·	6·	6·75	13·	$6\frac{1}{4}$,	$\frac{2}{5}$	12·	$8\frac{5}{6}$
18·	$5\frac{3}{4}$	5·	6·	2·	11·875	4365·	9·	5·5	14·	$10\frac{0}{4}$,	$\frac{3}{25}$	27·	$11\frac{3}{7}$
9·	$6\frac{2}{3}$	17·	$5\frac{1}{2}$	4·	8·125	2184·	15·	9·5	6·	$8\frac{0}{4}$,	$\frac{7}{10}$	14·	$6\frac{5}{16}$
11·	$10\frac{5}{6}$	6·	$4\frac{3}{4}$	3·	11·5	7956·	14·	4·5	15·	$6\frac{3}{4}$		33·	$9\frac{1}{4}$
2·	$2\frac{1}{2}$	1·	6·	8·	9·25	3418·	9·	10·25	5·	$11\frac{1}{2}$,	$\frac{3}{14}$	9·	$10\frac{11}{21}$

II. AVOIRDUPOIS WEIGHT.

Ex. 11·

T.	cwt.	qr.	lb.	oz.	dr.
14·	13·	3·	15·	11·	7·
25·	9·	1·	4·	10·	9·
47·	12·	2·	13·	5·	$0\frac{1}{2}$
31·	17·	0·	25·	12·	$12\frac{1}{5}$
95·	14·	2·	9·	7·	$8\frac{3}{4}$
58·	3·	3·	27·	6·	$10\frac{5}{8}$
19·	16·	1·	12·	0·	$15\frac{3}{20}$
67·	0·	0·	23·	15·	6·
74·	17·	3·	21·	8·	$14\frac{3}{5}$

Ex. 12·

cwt.	qr.	lb.
34·	3·	$19\frac{1}{3}$
59·	2·	$7\frac{2}{9}$
41·	3·	$22\frac{5}{12}$
75·	1·	$25\frac{2}{3}$
69·	3·	$18\frac{1}{3}$
94·	2·	$27\frac{3}{4}$
18·	2·	$12\frac{5}{6}$
23·	1·	$24\frac{7}{18}$
88·	3·	$16\frac{5}{6}$

Ex. 13·

qr.	lb.	oz.
3·	14·	$6\frac{1}{3}$
2·	15·	$8\frac{1}{4}$
3·	26·	$13\frac{3}{4}$
1·	17·	$12\frac{1}{3}$
2·	23·	$8\frac{5}{16}$
1·	8·	$14\frac{1}{3}$
3·	25·	$13\frac{7}{8}$
2·	19·	$15\frac{3}{16}$
1·	25·	$11\frac{3}{4}$

Ex. 14·

lb.	oz.	dr.
34·	4·	$5\frac{5}{6}$
45·	13·	$7\frac{7}{8}$
76·	14·	$11\frac{5}{6}$
59·	15·	$12\frac{7}{16}$
36·	4·	$9\frac{3}{4}$
81·	7·	$10\frac{7}{18}$
19·	9·	$14\frac{5}{24}$
64·	13·	$11\frac{11}{36}$
95·	11·	$12\frac{13}{72}$

III. TROY WEIGHT.

Ex. 15·

lb.	oz.	dwt.	gr.
14·	7·	11·	13·
9·	5·	16·	21·
25·	11·	4·	3·
12·	9·	17·	19·
3·	6·	15·	7·
41·	3·	8·	15·
16·	10·	12·	14·
7·	11·	19·	23·

Ex. 16·

lb.	oz.	dwt.	gr.
16·	1·	13·	16·45
31·	9·	16·	3·085
4·	5·	14·	4·75
	11·	17·	13·875
	6·	12·	9·12
15·	3·	10·	3·65
2·	4·	19·	17·015
		2·	3·005

Ex. 17·

lb.	oz.	dwt.	gr.
2.	9·	14·	$22\frac{1}{3}$
9·	8·	12·	$19\frac{3}{4}$
18·	7·	15·	$13\frac{5}{6}$
4·	6·	13·	$17\frac{7}{16}$
13·	5·	11·	$21\frac{11}{12}$
65·	4·	16·	$14\frac{19}{24}$
12·	3·	12·	$16\frac{5}{6}$
7·	2·	10·	$5\frac{2}{3}$

Ex. 18·

lb.	oz.	dwt.	gr.
4·	3·	16·	23·5
12·	0·	4·	14·
5·	2·	13·	11·75
13·	1·	17·	22·25
6·	11·	9·	16·5
14·	6·	18·	4·5
7·	5·	15·	18·25
15·	8·	17·	9·25

IV. APOTHECARIES' WEIGHT.

Ex. 19·

lb.	℥	ʒ	℈	gr.
3·	4·	5·	2·	19·
4·	7·	2·	1·	15·
6·	9·	3·	0·	13·
1·	5·	7·	1·	8·
5·	11·	6·	2·	14·

Ex. 20·

℥	ʒ	℈	gr.
1·	7·	0·	5·6
8·	6·	2·	18·08
5·	5·	2·	3·75
11·	4·	1·	9·14
7·	3·	2·	18·63

Ex. 21·

ʒ	℈	gr.
6·	1·	14·005
4·	2·	3·784
3·	0·	16·316
1·	1·	4·072
2·	2·	17·46

Ex. 22·

ʒ	℈	gr.
1·	2·	$17\frac{1}{3}$
3·	1·	$15\frac{3}{4}$
4·	1·	$11\frac{1}{3}$
6·	2·	$6\frac{1}{4}$
5·	0·	$12\frac{7}{10}$

V. LINEAL MEASURE.

Ex. 23·

yds.	ft.	in.	li.
34·	2·	11·	4·
17·	2·	10·	5·
65·	0·	5·	9·
14·	1·	8·	8·
22·	2·	9·	6·
15·	2·	6·	5·

Ex. 24·

mi.	fur.	po.	yds.
13·	7·	21·	4.
4·	5·	35·	3·
	3·	19·	2·
26·	2·	7·	1·
33·	6·	38·	5·
17·	4·	15·	4·

Ex. 25·

leag.	mi.	fur.	po.
20·	2·	7·	$19\frac{4}{5}$
4·	1·	4·	$32\frac{3}{8}$
19·	0·	3·	$6\frac{5}{12}$
36·	2·	2·	$28\frac{7}{18}$
24·	2·	5·	$37\frac{1}{2}$
37·	1·	6·	$18\frac{3}{4}$

Ex. 26·

fur.	ft.	in.
26·	39·	11·875
12·	45·	9·005
37·	130·	7·65
15·	62·	8·435
29·	194·	6·025
11·	200·	8·01

VI. CLOTH MEASURE. (LINEAL.)

Ex. 27·

yds.	qr.	na.	in.
5·	3·	2·	$1\frac{1}{2}$
14·	2·	1·	$1\frac{3}{4}$
16·	1·	3·	2·
23·	2·	2·	$2\frac{1}{8}$
17·	3·	1·	$1\frac{1}{4}$
25·	3·	3·	$2\frac{3}{16}$

Ex. 28·

E.E.	qr.	na.	in.
8·	3·	3·	$1\frac{3}{8}$
7·	4·	2·	$2\frac{3}{16}$
4·	2·	1·	2·
13·	1·	3·	1·
11·	3·	1·	$0\frac{3}{4}$
6·	2·	2·	$1\frac{1}{2}$

Ex. 29·

Fr.E.	qr.	na.
8·	3·	2·
16·	2·	3·5
4·	5·	1·25
23·	4·	2·15
17·	1·	3·3
15·	3·	2·7

Ex. 30·

Fl.E.	qr.	na.
14·	2·	3·
11·	1·	2·
6·	2·	1·
33·	0·	3·
17·	1·	2·
64·	3·	1·

Ex. 31·

S.E.	qr.	na.
4·	3·	2·
16·	2·	1·
15·	3·	3·
9·	1·	2·
31·	1·	2·
15·	3·	1·

VII. SUPERFICIAL MEASURE.

Ex. 32·

s.yds.	s.ft.	s.pr.	s.in.
34·	8·	2·	$8\frac{1}{2}$
15·	7·	5·	$9\frac{1}{4}$
49·	6·	3·	$3\frac{3}{4}$
36·	5·	6·	$7\frac{1}{2}$
54·	4·	11·	$5\frac{3}{8}$
43·	3·	9·	$11\frac{1}{8}$

Ex. 33·

s.ft.	′	″
849·	11·	$4\frac{1}{12}$
36·	9·	$6\frac{3}{4}$
574·	10·	$11\frac{1}{8}$
851·	5·	$9\frac{2}{3}$
378·	8·	$10\frac{1}{2}$
435·	7·	$8\frac{5}{8}$

Ex. 34·

ac.	r.	s.po.	s.yds.
14·	3·	38·	$29\frac{1}{2}$
36·	2·	11·	$10\frac{1}{4}$
18·	1·	25·	$5\frac{1}{2}$
55·	3·	14·	$11\frac{3}{4}$
96·	2·	26·	$25\frac{1}{2}$
34·	1·	17·	30·

Ex. 35·

sq.mi.	ac.	r.	s.po.	s.yds.
48·	321·	2·	12·	$15\frac{1}{8}$
37·	410·	1·	24·	$24\frac{1}{16}$
45·	316·	3·	32·	$9\frac{1}{4}$
39·	595·	3·	16·	$18\frac{3}{4}$
64·	631·	2·	18·	$19\frac{1}{2}$
22·	84·	1·	25·	$21\frac{1}{4}$

VIII. CUBIC MEASURE, OR MEASURE OF SOLIDITY.

Ex. 36·

c.yds.	c.ft.	c.in.
14·	17·	456·
23·	23·	349·
15·	15·	884·
47·	4·	1356·
35·	22·	45·
21·	16·	986·

Ex. 37·

c.ft.	′	″	‴
39·	5·	9·	3·
84·	4·	8·	4·
37·	10·	6·	11·
881·	11·	5·	9·
367·	6·	10·	5·
595·	7·	11·	6·

Ex. 38·

c.yds.	c.ft.	′	″	‴
41·	21·	9·	11·	5·
131·	18·	6·	5·	9·
89·	25·	5·	10·	6·
346·	4·	3·	8·	10·
58·	17·	8·	6·	5·
563·	13·	10·	7·	9·

IX. MEASURE OF CAPACITY.

Ex. 39·

gal.	qts.	pts.
14·	3·	1·
23·	2·	0·
35·	1·	1·
28·	3·	$0\frac{1}{2}$
19·	0·	$1\frac{1}{2}$
37·	2·	$1\frac{1}{2}$

Ex. 40·

bu.	pks.	gal.	qts.
19·	3·	1·	3·
3·	0·	1·	2·
15·	2·	0·	2·
28·	1·	1·	3·
17·	3·	0·	1·
34·	2·	1·	2·

Ex. 41·

qrs.	bu.	pks.	gal.
7·	7·	3·	1·
3·	3·	2·	0·
4·	2·	1·	1·
12·	5·	2·	1·
6·	6·	1·	0·
8·	4·	3·	1·

Ex. 42·

gal.	qts.	pt.	gi.
16·	3·	1·	3·
14·	2·	0·	2·
33·	1·	1·	2·
47·	3·	0·	1·
18·	1·	1·	3·
12·	2·	1·	2·

Ex. 43·

pks.	gal.	qts.
3·	1·	3·
4·	0·	2·
5·	1·	3·
9·	1·	1·
6·	1·	2·
4·	0·	3·

Ex. 44·

bu.	gal.	qts.
43·	7·	3·
5·	5·	2·
16·	1·	1·
19·	3·	1·
8·	2·	3·
17·	5·	2·

Ex. 45·

lds.	qrs.	bu.	pks.
17·	4·	7·	3·
28·	3·	4·	2·
13·	2·	3·	1·
15·	4·	2·	0·
6·	1·	5·	3·
34·	2·	6·	2·

X. APOTHECARIES' FLUID MEASURE.

Ex. 46·

gal.	O	℥
1·	7·	13·5
	5·	8·01
2·	6·	14·25
3·	2·	11·
1·	4·	9·75
2·	3·	5·34

Ex. 47·

O	℥	ʒ
3·	13·	7·875
4·	11·	1·125
7·	15·	6·5
1·	14·	5·45
6·	12·	4·25
5·	9·	3·125

Ex. 48·

O	℥	ʒ	♏
14·	13·	5·	42·
11·	9·	7·	18·
7·	11·	4·	29·
34·	5·	3·	53·
8·	12·	6·	46·
7·	9·	4·	19·

XI. TIME.

Ex. 49·

da.	ho.	min.	sec.
41·	14·	41·	41·
39·	11·	23·	17·
65·	16·	37·	25·
18·	5·	54·	43·
23·	4·	38·	29·
42·	13·	31·	51·

Ex. 50·

mo.	we.	da.	ho.
11·	3·	6·	21·
9·	2·	1·	14·
8·	1·	4·	13·
5·	2·	5·	19·
7·	2·	2·	18·
3·	3·	3·	23·

Ex. 51·

we.	da.	ho.	min.
3·	6·	15·	30·
2·	4·	13·	45·
9·	0·	21·	15·
5·	3·	11·	28·
4·	1·	16·	19·
16·	2·	22·	35·

Ex. 52·

c. yrs.	da.	ho.	min.
7·	315·	11·	30·
4·	49·	18·	21·
11·	163·	21·	58·
5·	342·	5·	32·
9·	54·	6·	28·
18·	283·	14·	19·

Ex. 53·

c. yrs.	we.	da.	ho.
18·	49·	6·	13·
4·	36·	4·	11·
33·	21·	3·	16·
12·	50·	2·	24·
1·	12·	5·	18·
24·	13·	3·	15·

Ex. 54·

c. yrs.	da.	ho.	mi.
15·	100·	14·	32·5
4·	84·	23·	21·25
3·	241·	11·	4·75
62·	358·	16·	36·845
8·	205·	19·	4·005
45·	193·	21·	17·35

COMPOUND SUBTRACTION.

There are two methods of working Compound as well as Simple Subtraction. That commonly employed supposes Minuend and Subtrahend to be both equally increased, the former by borrowing, and the latter by carrying. This, as shewn on pages 82· to 85·, is an artificial method, which could not be employed with the actual units of the Minuend before us. The natural method, which we adopt, neither increases the Minuend nor Subtrahend preparatorily to finding their Difference. It merely rearranges parts of the Minuend.

To find the Difference of any two Compound Quantities of the same kind. (PRINCIPLE V.)

First. Arrange as for Compound Addition, the less under the greater.

Second. Commencing with right-hand column, Subtract, if possible, each Subtrahend-quantity from the Minuend-quantity above it.

Third. When requisite, borrow one unit from the next column in which there is a significant figure. Then:

I. Reckon the Minuend-quantity whence you borrow as one less than it is written.

II. Reckon the Minuend-quantity for which you borrow as being increased by the number of its units which make one of the next higher name.

III. If any columns intervene, each will, of course, be occupied by a cipher. Reckon each such cipher as one less than the number of units of its column which make one of the next higher name. Then Subtract.

This distribution will exactly dispose of the borrowed unit. For, supposing that I borrow for the farthings, then:

1·d. borrowed	is converted into	$\frac{4}{4}$d.
1·s. „	is converted into	11$\frac{4}{4}$d.
£1· „	is converted into	19·s. 11$\frac{4}{4}$d.

If I borrow for drams, then:

		cwt.	qrs.	lb.	oz.	drs.
1· oz. borrowed	becomes					16·
1· lb. „	becomes				15·	16·
1· qr. „	becomes			27·	15·	16·
1· cwt. „	becomes		3·	27·	15·	16·
1· ton „	becomes	19·	3·	27·	15·	16·

Examples worked out. I. Subtract £495· 11·s. 6$\frac{3}{4}$d. from £629· 3·s. 8$\frac{1}{4}$d.

	£.	s.	d.
From	629·	3·	8$\frac{1}{4}$
Take	495·	11·	6$\frac{3}{4}$

The subtraction in the highest name, which here is "pounds," is mere Simple Subtraction, of which no more need be said. Apart from this, two borrowings are necessary. First, 1·d. is borrowed for the farthings, and thus the 8$\frac{1}{4}$d. become 7$\frac{5}{4}$d. Then £1· is borrowed for the shillings, so that the £9· 3·s. are converted into £8· 23·s. Hence, by borrowing and rearrangement, the entire Minuend is converted into £628· 23·s. 7$\frac{5}{4}$d., and the operation to be performed is

	£.	s.	d.
From	628·	23·	7$\frac{5}{4}$
Take	495·	11·	6$\frac{3}{4}$
Remr.	133·	12·	1$\frac{1}{2}$

In the work of this Example we have *two borrowings and no carrying.*

By the usual method we should have *two borrowings* and *two carryings.*

II. What amount remains after taking £843· 7·s. $6\frac{1}{4}$d. out of £10000·?

	£.	s.	d.
From	10000·	0	0
Taking	843·	7·	$6\frac{1}{4}$
Leaves	9156·	12·	$5\frac{3}{4}$

For, after borrowing and rearranging, the Example becomes,

	£.	s.	d.
From	9999·	19·	$11\frac{4}{4}$
Take	843·	7·	$6\frac{1}{4}$

In this Example we have *one borrowing* and *no carrying.* By the ordinary method there would be *seven borrowings* and *six carryings.* By these operations, the Minuend would be changed into £11111· 1·s. 1·d., and the Subtrahend into £1954· 8·s. $7\frac{1}{4}$d!

It is very evident that if 10000· sovereigns lay on the table before us, there would be no necessity to call for £1111· 1·s. 1·d. more, in order to take away £843· 7·s. $6\frac{1}{4}$d. We should exchange *one sovereign* for 20· shillings, then *one* of those shillings for 12· pence, then *one* of those pence for 4· farthings. This being done, we should have, instead of 10000· sovs., the *same amount* in a more convenient form, namely, 9999· sovs. 19·s. $11\frac{4}{4}$d. Then we should subtract farthings from farthings, pence from pence, and so on. Our method, therefore, being strictly that which must be followed in handling the *real* coins, weights, &c., which are represented by the figures of the Minuend, we are justified in terming it *The Natural Method.*

III. By how much does 16· tons 5· cwt. 2· qrs. 13· lb. 7· oz. 9· dwts. fall short of 32· tons 4· cwt. 8· lb. 3· drs.?

	Tons.	cwt.	qrs.	lb.	oz.	drs.
Minuend	32·	4·	0	8·	0	3·
Subtrahend	16·	5·	2·	13·	7·	9·
Difference	15·	18·	1·	22·	8·	10·

Here, we first borrow 1· lb., converting 8· lb. 3· drs. into 7· lb. 15· oz. 19· drs. This gives us Remainders 10· drs. and 8· oz. We next have to borrow 1· cwt., converting 4· cwt. and 7· lb. into 3· cwt. 3· qrs. and 35· lb. This gives us Remrs. 22· lb. and 1· qr. We then borrow 1· ton, converting 2· tons and 3· cwt. into 1· ton and 23· cwt., which gives Remr. 18· cwt. Lastly, subtracting 16· tons from 31· tons, we obtain Remr. 15· tons.

IV. Find the Diff. of £6· 13·s. $8\frac{5}{8}$d. and £19· 2s· $11\frac{1}{4}$d.

	£.	s.	d.
Minuend	19·	2·	$11\frac{1}{4}$
Subtrahend	6·	13·	$8\frac{5}{8}$
Difference	12·	9·	$2\frac{5}{8}$

First, Convert $11\frac{1}{4}$d. into $10\frac{12}{8}$d.

Second, Convert £9· 2·s. into £8· 22·s.

EXERCISE XCI.

EXAMPLES IN SUBTRACTION OF COMPOUND QUANTITIES.

I. MONEY.

	Ex. 1·			Ex. 2·			Ex. 3·			Ex. 4·		
	£.	s.	d.	£.	s.	d.·	£.	s.	d.	£.	s.	d.
From	400·	7·	0·	746·	0·	$0\frac{1}{4}$	845·	2·	$3\frac{1}{4}$	3000·	0·	0·
Take	164·	13·	$8\frac{1}{2}$	388·	11·	$6\frac{3}{4}$	7·	18·	$7\frac{1}{2}$		19·	$5\frac{3}{4}$

5· How much must be added to 11·s. $5\frac{1}{2}$d. to make up £20·?

6· Find the Difference of £3· 2·s. 4·d. and £1· 15·s.

7· By how much does £28· 3·s. 2·d. fall short of £150·?

8· What amount must be subtracted from £35· 12·s. to leave 17·s. $6\frac{1}{2}$d.?

9· By how much does £12· 3·s. 5·d. exceed 4·s. $2\frac{1}{4}$d.?

10· What amount is less than £3· by 16·s. $11\frac{3}{4}$d.?

11· After paying for some butter out of half a sovereign, 3·s. $10\frac{1}{2}$d. remained. What was the cost of the butter?

12· What amount does £135· 4·s. exceed by £22· 13·s. $9\frac{1}{2}$d.?

13· A person desirous of accumulating £45·, has saved £7· 3·s. 6·d. How much more must he save?

14· Supposing a tradesman's stock to be worth £1453· 11·s. 6·d., the debts due to him to amount to £451· 13·s. 8·d., and that he owes £1600· 19·s. $5\frac{1}{4}$d.; how much does he really possess?

15· A woman sold, at market, for ready money, butter to the amount of 18·s. $7\frac{1}{4}$d., eggs 1·s. $10\frac{1}{2}$d., and potatoes 17·s. 8·d. She then purchased draperies to the value of £1· 13·s. $6\frac{3}{4}$d., groceries 11·s. $4\frac{1}{4}$d., a pair of shoes costing 5·s. 6·d., and a tea-set for 8·s.

I. How much did she receive?

II. How much did she expend?

III. What amount had she remaining, supposing she brought with her a five-pound note?

IV. How much more did she lay out in draperies than in groceries?

V. How much less did the shoes cost than the tea-set?

VI. When did she first require to change her note, supposing her transactions to have proceeded in the above order?

16· Find the exact position of a person's affairs to whom is owed £43· 11·s. 7·d. + £301· 5·s. $4\frac{1}{2}$d. + £17· 13·s. $5\frac{1}{4}$d. + 18·s. $7\frac{3}{4}$d., who owes £354· 3·s. $6\frac{1}{4}$d. + £411· 14·s. $3\frac{1}{4}$d. + £215· 13·s. $6\frac{1}{4}$d. + £300· 15·s., and whose cash and other effects amount to £750· 15·s. 8·d.

EXAMPLES HAVING OTHER FRACTIONS THAN FARTHINGS IN ONE, OR BOTH, OF THE QUANTITIES.

In the first of the following examples 1·d. is borrowed and turned into $\frac{8}{8}$ of a penny, so that the $3\frac{5}{8}$d. are converted into $2\frac{13}{8}$d.

In example 18· £600· are converted into £599· 19·s. $11\frac{7}{7}$d.

In example 19· the first amount borrowed is 1·d., which is converted into 3· farthings and $\frac{40}{40}$ of a farthing, so that $1\frac{0}{4}$d. $\frac{1}{5}$q. becomes $\frac{3}{4}$d. $\frac{48}{40}$q.

In such examples as these, the first step is always to reduce the two fractions to a common denominator. It is then at once perceived whether borrowing will or will not be required. Page 89·

	Ex. 17·			Ex. 18·			Ex. 19·				Ex. 20·			
	£.	s.	d.	£.	s.	d.	£.	s.	d.	q.	£.	s.	d.	q.
From	214·	4·	$3\frac{5}{8}$	1600·	0·	0·	3154·	2·	$1\frac{0}{4}$	$\frac{1}{5}$	3561·	6·	$5\frac{0}{4}$	$\frac{3}{11}$
Take	186·	17·	$1\frac{3}{4}$	958·	4·	$5\frac{6}{7}$	5·	18·	$4\frac{1}{4}$	$\frac{1}{8}$	1004·	8·	$11\frac{3}{4}$	$\frac{5}{6}$

	Ex. 21·	Ex. 22·	Ex. 23·	Ex. 24·
	£. s. d. q.	£. s. d. q.	£. s. d. q.	£. s. d.
Min.	3265· 3· 4·	8561· 2· $5\frac{1}{8}$	3000· 0· 0·	120· 1· 8·
Subtd.	1695· 12· $11\frac{2}{4}\frac{3}{7}$	395· 14· $3\frac{2}{4}\frac{1}{3}$	14· $7\frac{3}{4}\frac{5}{8}$	35· 15· 11·75

II. AVOIRDUPOIS WEIGHT.

	Ex. 25·	Ex. 26·	Ex. 27·
	T. cwt. qr. lb. oz. dr.	T. cwt. qr. lb. oz. dr.	T. cwt. qr. lb. oz. dr.
From	13· 12· 3· 15· 2· 11·	11· 0· 0· 0· 0· 0·	7· 0· 2· 0· 0· $0\frac{6}{7}$
Take	9· 18· 1· 24· 1· 13·	3· 15· 7· 13·	15· 3· 27· 12· 9·

	Ex. 28·	Ex. 29·	Ex. 30·
	T. ct. qr. lb. oz.	T. ct. qr. lb. oz. dr.	T. ct. qr. lb. oz. dr.
Min.	4· 19· 0· 0· $13\frac{1}{5}$	18· 0· 0· 0· 0· 0·	300· 14· 0· 13· 8· $3\frac{1}{2}$
Subtd.	2· 16· 3· 15· $15\frac{3}{4}$	18· 2· 5· 9· $11\frac{6}{7}$	64· 19· 3· 22· 11· $0\frac{3}{4}$

III. TROY WEIGHT.

	Ex. 31·	Ex. 32·	Ex. 33·	Ex. 34·
	lb. oz. dt. gr.	lb. oz. dt. gr.	lb. oz. dt. gr.	lb. oz. dt. gr.
Min.	12· 2· 0· 1·	24· 11· 16· 21·	5· 10· 19· $20\frac{3}{11}$	11· 0· 0· 0·
Subtd.	3· 8· 5· 6·	19· 13· 18· $22\frac{1}{4}$	4· 3· 7· $0\frac{5}{8}$	11· 11· $11\frac{10}{11}$

IV. APOTHECARIES' WEIGHT.

	Ex. 35·	Ex. 36·	Ex. 37·	Ex. 38·
	lb. ℥ ʒ ℈ gr.	℥ ʒ ℈ gr.	lb. ℥ ʒ ℈	℥ ʒ ℈ gr.
Min.	4· 0· 0· 0· 0·	11· 0· 2· $5\frac{5}{9}$	1· 0· 0· 0·	10· 0· 1· 0·5
Subtd.	1· 9· 5· 1· $17\frac{1}{5}$	4· 6· 2· $8\frac{3}{7}$	5· 4· 2·75	9· 7· 2· 14·75

V. LINEAL MEASURE.

	Ex. 39·	Ex. 40·	Ex. 41·	Ex. 42·
	yds. ft. in. li.	mi. fur. po. yds.	lg. mi. fur. po.	po. ft. in.
From	310· 0· 5· 0·	22· 5· 21· $3\frac{3}{4}$	2· 2· 0· 15·	37· 68· 11·
Take	149· 2· 8· 7·5	18· 7· 39· $4\frac{1}{2}$	1· 0· 7· $31\frac{3}{16}$	12· 130· 0·

VI. CLOTH MEASURE. (LINEAL.)

	Ex. 43·	Ex. 44·	Ex. 45·	Ex. 46·	Ex. 47·
	yds. qr. na. in.	E.E. qr. na. in.	Fr.E. qr. na.	Fl.E. qr. na.	S.E. qr. na.
Min.	14· 2· 1· 1·	8· 0· 0· 0·	4· 5· $3\frac{4}{5}$	7· 0· $0\frac{1}{2}$	9· 1· 2·
Subtd.	9· 3· 3· $1\frac{3}{4}$	1· 4· 3· $0\frac{1}{2}$	2· 3· $2\frac{1}{10}$	3· 1· $3\frac{4}{5}$	4· 3· 3·

VII. SUPERFICIAL MEASURE.

	Ex. 48·	Ex. 49·	Ex. 50·	Ex. 51·
	s.y. s.f. s.p. s.in.	s.f. ′ ″	ac. r. po. yd.	sq.m. ac. r. po.
From	34· 2· 8· 11·	95· 3· 6·	84· 2· 18· $27\frac{1}{2}$	14· 201· 3· 18·2
Take	18· 7· 11· 5·2	89· 5· 9·	36· 3· 32· $98\frac{3}{5}$	11· 543· 0· 31·125

VIII. CUBIC MEASURE, OR MEASURE OF SOLIDITY.

	Ex. 52·	Ex. 53·	Ex. 54·	Ex. 55·
	c.y. c.f. c.p. ″	T.shp. ft. ′	c.yds. ft. in.	c.ft. ′ ″ ‴
From	4· 21· 11· 5·	300· 0· 0·	31· 24· 743·5	341· 1· 0· 0·
Take	1· 26· 6· 7·	164· 38· 7·	19· 26· 1005·75	68· 7· 9· 3·

IX. MEASURE OF CAPACITY.

	Ex. 56·	Ex. 57·	Ex. 58·	Ex. 59·
	qr. bu. pe. gal.	bu. pe. gal. qts.	gal. qt. pt. gi.	ld. qr. bu. pe.
From	13· 5· 2· 1⅓	11· 2· 0· 1⅛	18· 1· 0· 1¼	7· 3· 2· 1·
Take	7· 6· 3· 1¾	9· 3· 1· 1⅛	16· 3· 2· 3⅛	5· 4· 6· 3·

X. TIME.

	Ex. 60·	Ex. 61·	Ex. 62·	Ex. 63·
	c.yr. da. ho.	we. da. ho. min.	da. ho. min. sec.	c.mo. we. da.
From	3· 142· 5·	15· 2· 5· 11·	43· 11· 2· 5·	11· 1· 5⅛
Take	1· 264· 11$\frac{5}{12}$	4· 5· 19· 20·	38· 14· 18· 29·8	8· 3· 6⅑

COMPOUND MULTIPLICATION.

To Multiply any Compound Quantity by an Integer less than 13·

First. Multiply the Quantity of the lowest name in the Multiplicand. Reduce Product to next higher name. Set down and carry as in Compound Addition.

Second. Proceed similarly with the other quantities of the Multiplicand.

First Example worked out. What will 9· articles cost at £3· 11·s. 8¼d. each?

They will evidently cost 9· times £3· 11·s. 8¼d. or £3· 11·s. 8¼d. × 9·

£.	s.	d.	
3·	11·	8¼	= *cost of* 1· *article.*
		9·	
32·	5·	2¼	= *cost of* 9· *articles.*

9·q. = 2¼d. *Set down* ¼d. *Carry* 2·d.
74·d. = 6·s. 2·d. *Set down* 2·d. *Carry* 6·s.
105·s. = £5· 5·s. *Set down* 5·s. *Carry* £5·

This process is founded on Principle xxv. (Page 172·)

Second Example worked out. Find the extent of 5· fields, each being 4· ac. 2· r. 35· po.

ac.	r.	po.	
4·	2·	35·	= *extent or area of* 1· *field.*
		5·	
23·	2·	15·	= *extent or area of* 5· *fields.*

175· po. = 4· r. 15· po. *Set d.* 15·po. *Car.* 4·r.
14·r. = 3· ac. 2·r. *Set d.* 2·r. Car. 3· ac.

THIRD EXAMPLE WORKED OUT. If 3· lb. 11· oz. 13· dwts. cost a crown, what weight of goods, at the same rate, would cost £1·?

Now, a crown = $\frac{1}{4}$ of £1·; hence 4· times as much may be bought for £1· as for 1· crown.

lb. oz. dwts.
3· 11· $13\frac{4}{5}$ = *weight which costs* 1· *crown.*
4·
15· 10· $15\frac{1}{5}$ = *weight which costs* £1·

FOURTH EXAMPLE WORKED OUT. If 4· cwt. of sugar cost £8· 17·s. 6·d., what is the rate per Ton?

4· cwt. = $\frac{1}{5}$ of a Ton. ∴ A Ton will cost 5· times as much as 4· cwt. will cost.

£. s. d.
8· 17· 6· = *value of* 4· *cwt.*
5·
44· 17· 6· = *value of* 20· *cwt., or* 1· *Ton.*

The tables of Aliquot Parts will be of service in Examples like these last two.

EXERCISE XCII.

EXAMPLES IN COMPOUND MULTIPLICATION BY INTEGERS LESS THAN 13·

Find the 2nd, 3rd, 4th, 5th, 6th, 7th, 8th, 9th, 10th, 11th, and 12th Multiples of each of the following sixteen Compound Quantities, describing each Multiple in writing.

1· £415· 13·s. $8\frac{1}{4}$d.
2· £743· 5·s. $11\frac{5}{8}$d.
3· £1542· 16·s. 5·d. $\frac{3}{5}$q.
4· £59· 11·s. 7·d. $\frac{3}{8}$q.
5· 14·s. $6\frac{3}{4}$d.
6· 11·s. $9\frac{5}{12}$d.
7· 3·s. $4\frac{3}{4}$d. $\frac{1}{2}$q.
8· 15·s. 11·d. $\frac{7}{8}$q.
9· $8\frac{3}{4}$d.
10· $11\frac{5}{6}$d.
11· $14\frac{1}{4}$d.
12· $20\frac{3}{4}$d.
13· 1·s. $4\frac{1}{4}$d.
14· 3·s. $6\frac{1}{8}$d.
15· 5·s. $7\frac{3}{4}$d.
16· 8·s. $4\frac{11}{12}$d.

How much rice will cost one shilling,

17· If 3·lb. 4·oz. 8·dr. cost 6·d.?
18· If 2·lb. 8·oz. 2·dr. cost 4·d.?
19· If 1·lb. 12·oz. cost 3·d.?
20· If 1·lb. 1·oz. 2·dr. cost 2·d.?
21· If 11·oz. 4·dr. cost $1\frac{1}{2}$d.?
22· If 7·oz. 3·dr. cost 1·d.?

What is the rate per cwt.

23· When 56·lb. cost £3· 14·s. $11\frac{1}{2}$d.?
24· When 28·lb. cost £2· 19·s. $7\frac{3}{4}$d.?
25· When 14·lb. cost £1· 15·s. $8\frac{3}{4}$d.?
26· When 28·lb. cost £1· 2·s. $7\frac{1}{2}$d.?

What weight of goods will cost £1·

27· If 1· cwt. 2· qr. 21· lb. cost 10·s.?
28· If 3· qr. 13· lb. $7\frac{1}{2}$ oz. cost 6·s. 8·d.?
29· If 3· lb. 11· oz. $15\frac{1}{5}$ dwt. cost 5·s.?
30· If 4· lb. 5· oz. 14·875 gr. cost 3·s. 4·d.?
31· If 7· ℥ 2· ʒ 1· ℈ cost 2·s. 6·d.?
32· If 4· lb. 15· oz. cost 1·s. 8·d.?

What is the rent per acre,

33· At 29·s. 6·d. for 2·roods?
34· At 21·s. 9·d. for 1·rood?
35· At 17·s. 6·d. for 20·po.?
36· At 10·s. 9·d. for 16·po.?

What will 1· *quarter of a cwt. cost,*

37· If 14·lb. cost 3·s. 11⅝d.?
38· If 7·lb. cost 2·s. 5⅜d.?
39· If 4·lb. cost 1·s. 10¾d.?
40· If 3·lb. 8·oz. cost 12¼d.?

What is the cost per ton,

41· At £11· 14·s. 10·d. for 10·cwt.?
42· At £7· 17·s. 9·d. for 4·cwt.?
43· At £1· 6·s. 8·d. for 2·cwt. 2·qr.?
44· At £5· 14·s. 8·d. for 2·cwt.?

45· How much is corn per bushel when a peck costs 1·s. 11¼d.?
46· If a gallon of beans cost 1·s. 1½d. what will half a bushel cost?
47· If 2·oz. of tea cost 11½d. what will 1·lb. cost?
48· If 4·oz. of cochineal cost 1·s. 4½d. what is the price per lb.?
49· What will a doz. lb. of flour cost when 4·lb. are worth 5¼d.?
50· If a parcel be carried 7·mi. 6·fur. for 1·s. 8·d. how far should it be carried for 5·s.?
51· When 1·yd 3·na. of ribbon cost 7½d. what length will cost a crown?

FIFTH EXAMPLE WORKED OUT. FORM OF SHOP-BILL. Make out a bill for the undermentioned articles, purchased at Millport, November 30th, 1858·, by Mrs. Railton, of Willis & Co., Drapers and Haberdashers: 8·yds. damask, at 2·s. 9·d. per yard; 12·yds. alpaca, at 1·s. 7½d. per yd.; 11·yds. ribbon, at 1·s. 1·d. per yd.; 2·pairs gloves, at 2·s. 8·d. per pair; 4·doz. buttons, at 7½d. per doz.; 12·yds. shirting, at 6¾d. per yd.; 9·yds. ditto, at 6½d. per yd. The parcel was delivered by an assistant, John Froude, who received the amount for his employers, and discharged the bill in their name.

Millport, *Nov.* 30*th*, 1858·

Mrs. Railton,

Bought of Willis & Co.,

DRAPERS AND HABERDASHERS.

				£.	s.	d.
8·	*yds. damask,*	@ 2/9*	*per yd.*	1·	2·	0·
12·	„ *alpaca,*	„ 19½*d.*	„		19·	6·
11·	„ *ribbon,*	„ 13·*d.*	„		11·	11·
2·	*pr. gloves,*	„ 2/8	*per pr.*		5·	4·
4·	*doz. buttons,*	„ 7½*d.*	*per doz.*		2·	6·
12·	*yds. shirting,*	„ 6¾*d.*	*per yd.*		6·	9·
9·	„ *do.*	„ 6½*d.*	„		4·	10½
	Paid, Nov. 30th, 1858·			£3·	12·	10½

John Froude,

For Willis & Co.

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* Shillings and pence are thus expressed by men of business.

EXERCISE XCII. (*continued.*)

Make a bill for each of the following sets of commodities, supplying names of buyer, seller, &c.

52· A Grocer's Bill. 3· lb. coffee, at 20·d.; 2· lb. tea, at 5·s. 4·d.; 12· lb. moist sugar, at 5¾d.; 10· lb. rice, at 3½d.; 8· lb. loaf sugar, at 7¼d.; 9· lb. raisins, at 6½d.; 7· lb. currants, at 5¾d.

53· A Butcher's Bill. Ribs of beef, 9· lb. at 7½d.; shoulder of veal, 8· lb. at 6½d.; loin of pork, 11· lb. at 6¼d.; leg of mutton, 9· lb. at 6¾d.; breast of veal, 8· lb. at 7·d.; quarter of lamb, 9· lb. at 8½d.; 3· lb. of suet, at 6·d.

54· A Stationer's Bill. 5· quires letter paper, at 10·d.; 5· quires black-edged note paper, at 7·d.; 3· quires pink blotting, at 1·s. 6·d.; 2· boxes envelopes, at 2·s. 8·d.; 10· boxes steel pens, at 1·s. 9·d.; ½ gross* pen-holders, at 8·d. per doz.; 1· doz. slates, at 9½d.; ½ quire† drawing, at 4·d. per sheet.

55· A Draper's Bill. 6· yds. flannel, at 17¾d.; 11· yds. lawn, at 14·d.; 4· yds. chintz, at 16½d.; 9· yds. diaper, at 8¾d.; 4· pr. kid gloves, at 3·s. 10·d.; 12· yds. ribbon, at 12½d.; 8· yds. shirting, at 7¾d.; 11· yds. damask, at 2·s. 4½d.; 7· yds. sheeting, at 13·d.; 7· yds. Russia ditto, at 2·s. 1·d.; 5· pr. cotton stockings, at 1·s. 5½d.; 12· yds. print, at 7¾d.

Find the 2nd, 3rd, 4th, 5th, 6th, 7th, 8th, 9th, 10th, 11th, and 12th Multiples of each of the following Compound Quantities, describing each Multiple in writing.

56· Of 14· T. 13· cwt. 15· lb. 11· oz. 7· dr.
57· Of 14· lb. 11· oz. 17· dwts. 13· gr.
58· Of 9· lb. 3· ℥ 6· ʒ 2· ℈ 15· gr.
59· Of 22· yd. 2· ft. 9· in. 6· li.
60· Of 17· mi. 4· fur. 15· po. 3¾ yd.
61· Of 16· yd. 1· qr 3· na. 2· in.
62· Of 8· E. E. 3· qr. 3· na. 1⅜ in.
63· Of 34· su. yd. 8· ft. 2·′ 8½″
64· Of 14· ac. 38· po. 29½ yd.
65· Of 5· ld. 3· qr. 6· bu. 3· pe.
66· Of 4· O 11· ℥ 5· ʒ 48· m.
67· Of a Julian year.
68· Of a tropical year.
69· Of a lunar month.

To Multiply any Compound Quantity by a Composite Number, having Factors each less than 13·

Multiply continually by any set of Corresponding Factors of the Multiplier. (Principle xi. and pp. 63·, &c.)

Examples worked out. I. Mult^{y}· £18· 14·s. 6¼d. by 42·.
II. Find the weight of 63· hhd. each 3· cwt. 2· qr. 21· lb.

First Example completed.

$$\begin{array}{rrrl}
\text{£.} & \text{s.} & \text{d.} & \\
18\cdot & 14\cdot & 6\frac{1}{4} & \\
 & & 7\cdot & \qquad\qquad\qquad \text{£.}\ \ \text{s.}\ \ \text{d.} \\
\hline
131\cdot & 1\cdot & 7\frac{3}{4} & = 7\cdot\text{ times } 18\cdot\ 14\cdot\ 6\frac{1}{4} \\
 & & 6\cdot & \\
\hline
\underline{\underline{786\cdot}} & \underline{\underline{9\cdot}} & \underline{\underline{10\frac{1}{2}}} & = 42\cdot\text{ times } 18\cdot\ 14\cdot\ 6\frac{1}{4}
\end{array}$$

Second Example completed.

$$\begin{array}{rrrl}
\text{cwt.} & \text{qr.} & \text{lb.} & \\
3\cdot & 2\cdot & 21\cdot & =\text{weight of } 1\cdot\text{ hhd.} \\
 & & 9\cdot & \\
\hline
33\cdot & 0\cdot & 21\cdot & =\text{weight of } 9\cdot\text{ hhd.} \\
 & & 7\cdot & \\
\hline
\underline{\underline{232\cdot}} & \underline{\underline{1\cdot}} & \underline{\underline{7\cdot}} & =\text{weight of } 63\cdot\text{ hhd.}
\end{array}$$

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* A gross = 12· doz. † A quire contains 24· sheets.

EXERCISE XCIII.

COMPOUND MULTIPLICATION BY COMPOSITE INTEGERS.

Describe each line of the work in writing.

What is the value

1· Of 21· cwt. at £2· 10·s. 7½d.?
2· Of 72· yds. at 9·s. 8¼d.?
3· Of 28· hhds. at £7· 5·s. 10½d.?
4· Of 16· tuns at £13· 13·s. 6·d.?
5· Of 108· tons at £143· 10·s.?
6· Of 24·lb. at 16·s. 6⅛d.?
7· Of 36· sheep at 35·s. 6·d.?
8· Of 70· pigs at 42·s.?
9· Of 44· oxen at £25· 17·s. 6·d.?
10· Of 132· bales at £74· 15·s. 8·d.?
11· Of 144· casks at £13· 9·s. 5¼d.?
12· Of 32· coats at £3· 18·s. 6·d.?
13· Of 50· hats at 17·s. 5·d.?
14· Of 27· watches at £7· 17·s. 6·d.?
15· Of 110· days' work at 5·s. 10·d.?
16· Of 22· flutes at £3· 18·s. 6·d.?
17· Of 100· packages at 39·s. 9·d.?
18· Of 18· tea-sets at £2· 12·s. 6·d.?
19· Of 60· scores at 13·s. 1⅙d. *each*?
20· Of 49· ingots at £416· 12·s. 8·d.?

What quantity will cost £1·,

21· If 3·qrs. 12·lb. 7·oz. cost 1·s. 4·d.?
22· If 3·qrs. 9·lb. 5·oz. cost 1·s. 3·d.?
23· If 1·qr. 22·lb. 6·oz. cost 1·s.?
24· If 1·qr. 18·lb. 11·oz. cost 10·d.?
25· If 22·lb. 13·oz. 8·drs. cost 8·d.?
26· If 7·lb. 9·oz. 15·drs. cost 7½d.?
27· If 4·lb. 9·oz. 14·dwts. cost 6·d.?
28· If 3·lb. 5·oz. 21·grs. cost 5·d.?
29· At 2·qts. 1·pt. 2·gi. for 4·d.?
30· At 1·pt. 2·gi. for 3¾d.?
31· At 7·su. ft. 9′ for 3·d.?
32· At 1·qt. 1⅛pt. for 2·d.?
33· At 1·pt. $1\frac{1}{16}$gi. for 1½d.?
34· At 3⅜qts. for 1·d.?
35· At 1·℈ 18·grs. for ¾d.?
36· At 2·ʒ 2·℈ for ½d.?

At what rate per cwt.

37· Will 8·lb. cost 11·s. $7\frac{3}{16}$d.?
38· Will 7·lb. cost 5·s. $9\frac{5}{32}$d.?
39· Will 4·lb. cost 2·s. 7⅜d.?
40· Will 3½ lb. cost 1·s. 5¾d.?
41· Will 2· lb. cost 3·s. 11½d.?
42· Will 1· lb. cost 2·s. 4¾d.?

What is the price per ton

43· At £1· 17·s. 11·d. for 1¼ cwt.?
44· At £3· 8·s. 6·d. for 1· cwt.?
45· At £1· 15·s. 9·d. for 2· qrs.?
46· At £1· 5·s. 8½d. for 1· qr.?
47· If 28·lb. cost 7·s. 10⅝d.?
48· If 8·lb. cost 13·s. $4\frac{3}{16}$d.?
49· If 4·lb. cost 9·s. $11\frac{3}{10}$d.?
50· If 3½ lb. cost 14·s. $6\frac{5}{16}$d.?

How much cloth for a crown

51· If 3· yds. 3⅛ na. cost 4·d.?
52· If 7· E. E. 3· qrs. cost 3¾d.?
53· If 3· Fr. E. 2· qrs. cost 3·d.?
54· If 1⅝ yds. cost ¾d.?
55· At 2·d. for 2· yds. 3· qrs.?
56· At 1½d. for 1· yd. 1 qr.?
57· At 1·d. for 3· yd. 3½ na.?
58· At ½d. for 3· qrs. 2⅛ na.?

What would it cost to dig 1· *acre*

59· At 1·s. 4·d. for 8· po.?
60· At 10½d. for 4· po.?
61· At 1·s. 3·d. for 5· po.?
62· At 4¾d. per po.?
63· At 2·s. 10½d. for 10· po.?
64· At 7½d. for 2· po.?

Multiply

65· 13· T. 5· cwt. 2· qrs. 15· lb. 13· oz. 10⅝ drs. by 120·
66· 14· lb. 7· oz. 11· dwts. 13· grs. by 25·
67· 6· lb. 9· ℥ 3· ʒ 2· ℈ 19· grs. by 96·
68· 54· su. yds. 8· ft. 11·′ 5·″ by 15·
69· 4· qrs. 5· bu. 2· pe. 1· gal. 3· qts. by 42·

70· The fortieth part of a field measured 2·r. 12·po. 15⅛ yds. What was the extent of the whole field?

71· What sum of money will suffice to pay 35· persons £4· 13·s. 7½d. each?

72· What will 42· cwt. of beef cost at £3· 2·s. 6·d. per cwt.?

73· If 2· oz. tea cost 9·d. what will 5· lb. cost?

To Multiply a Compound Quantity by any Integer greater than 12·

First. Multiply by the Corresponding Factors of the nearest Composite Number above or below the Multiplier.

Second. Add or Subtract such Multiples of the Multiplicand as will make up, or leave, the Multiple required.

This process is precisely the same, in principle, as that explained on p. 71·

FIRST EXAMPLE WORKED OUT. Multiply £614· 18·s. 4⅛d. by 151·

The nearest Composite Number, under 151· is 150·, of which the Corresponding Factors are 3·, 5·, and 10·.

Example completed.	Or thus:
£. s. d.	£. s. d.
614· 18· 4⅛	614· 18· 4⅛
3· × 5· × 10· + 1· = 151·	12·
£. s. d.	£. s. d.
1844· 15· 0⅜ = 3·t. 614· 18· 4⅛	7379· 0· 1½ = 12·t. 614· 18· 4⅛
5·	12·
£. s. d.	£. s d.
9223· 15· 1⅞ = 15·t. 614· 18· 4⅛	88548· 1· 6· = 144·t. 614· 18· 4⅛
10·	4304· 8· 4⅞ = 7·t. 614· 18· 4⅛
£. s. d.	£. s. d.
92237· 11· 6¾ = 150·t. 614· 18· 4⅛	92852· 9· 10⅞ = 151·t. 614· 18· 4⅛
614· 18· 4⅛	
92852· 9· 10⅞ = 151·t. 614· 18· 4⅛	

SECOND EXAMPLE WORKED OUT. Find the weight of 118· blocks of granite, each 315· cwt. 3· qrs. 17· lb.

I. Nearest Composite No. *above* 118· is 120·

II. Nearest Composite No. *under* 118· is 112·;* but 110· is more convenient, being composed of *two* Factors, each less than 12·

Example completed.	Or thus:
cwt. qrs. lb.	cwt. qrs. lb.
315· 3· 17· = wt. of 1· block.	315· 3· 17· = wt. of 1· block.
12· × 10· — 2· = 118·	10·
3790· 3· 8· = wt. of 12· blocks.	3159· 0· 2· = wt. of 10· blocks.
10·	11·
37908· 0· 24· = wt. of 120· blocks.	34749· 0· 22· = wt. of 110· blocks.
631· 3· 6· = wt. of 2· blocks.	2527· 0· 24· = wt. of 8· blocks.
37276· 1· 18· = wt. of 118· blocks.	37276· 1· 18· = wt. of 118· blocks.

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* Having Factors each less than 13·

EXERCISE XCIV.

Describe each line in writing.

Find the value of

1· 17·lb. tea, at 4·s. $9\frac{1}{2}$d.
2· 19·yds. crape, at 3·s. $11\frac{3}{4}$d.
3· 23·yds. ribbon, at 1·s. $5\frac{3}{4}$d.
4· 29·pairs gloves, at 3·s. 3·d.
5· 43·lb. butter, at 1·s. $2\frac{1}{2}$d.
6· 137·lb. pork, at $5\frac{3}{4}$d.
7· 113·sheep, at £1· 11·s. 8·d.
8· 97·bales, at £7· 9·s. 4·d.
9· 149·doz. wine, at 5·s. $8\frac{1}{2}$d. pr. b.
10· 161·lb. gold, at £2· 17·s. pr. oz.

Find the weight, extent, &c., of

11· 71·butts, ea. 1·T. 3·cwt. 2·qrs.
12· 89·pieces, ea. 65·yd. 3·qr. $2\frac{1}{8}$ na.
13· 37·days'-journeys, ea. $45\frac{7}{8}$ mi.
14· 7·lb. 8·oz. $13\frac{4}{7}$ grs. × 101·
15· 7·E.E. 3·qrs. 3·na. $1\frac{3}{8}$in. × 143·
16· 3·lb. 7·℥ 2·℈ 5·6 grs. × 13·
17· 34·su. yds. 8·ft. 11·′ $9\frac{5}{8}$″ × 179·
18· 28·ac. 3·r. 26·po. × 31·
19· 69·tropical years.
20· 14·hhds. 32·gal. $1\frac{1}{8}$qt. × 92·

When the Multiplier is large, the best plan, generally, is to multiply by Factors of highest power of 10· contained in it. Whatever other Multiples may be necessary, to make up the one required, can then be readily found.

Example worked out. What would 4356· articles cost at £3· 15·s. $8\frac{3}{4}$d. each?

Highest power of 10· contained in 4356· is 1000· or $10^3 = 10\cdot \times 10\cdot \times 10\cdot$ By these Factors, therefore, we first multiply.

£.	s.	d.	Example completed.
3·	15·	$8\frac{3}{4}$	= cost of 1· article.
		10·	× 10· × 10· × 4· + 300· + 50· + 6· = 4356·
37·	17·	$3\frac{1}{2}$	= cost of 10· articles.
		10·	
378·	12·	11·	= cost of 100· „
		10·	
3786·	9·	2·	= cost of 1000· „
		4·	
15145·	16·	8·	= cost of 4000· „
1135·	18·	9·	= cost of 300· „ being 3· times cost of 100·
189·	6·	$5\frac{1}{2}$	= cost of 50· „ being 5· times cost of 10·
22·	14·	$4\frac{1}{2}$	= cost of 6· „ being 6· times cost of 1·
16493·	16·	3·	= cost of 4356· articles.

EXERCISE XCV.

Each line of the work is to be described in writing.

1· Find the value of 1645· things at £11· 13·s. $4\frac{1}{4}$d. each.
2· What will 34628· uniforms cost at £4· 11·s. 6·d. each?
3· Ascertain the 11011th Multiple of £3· 17·s. $4\frac{5}{8}$d.

Multiply

4· £5· 11·s. $7\frac{1}{8}$d. by 453·
5· £11· 17·s. $6\frac{4}{5}$d. by 3154·
6· £21· 12·s. $4\frac{5}{16}$d. by 30504·
7· 3·cwt. 1·qr. 18·lb. by 97643·
8· 3·lb. 9·oz. 7·5 dwts. by 1751·
9· 24·yds. 3·qrs. 2·na. by 949·

To Multiply a Compound Quantity by a Fraction, we must have recourse to Compound Division. (Principle xvii.)

COMPOUND DIVISION.

This process is made up of Simple Division and Reduction. It is called "Short" when the Reductions and Divisions are worked mentally (only the results being expressed), and "Long" when the entire work is written out at full length.

Division is the distribution of the Dividend into lots each equal to the Divisor. Hence, (as we have already said, on page 105·,) *the Divisor must consist of units of the same kind and value as the Dividend.* And, inasmuch as £2866· 13·s. 10·d. comprises three kinds of units, namely, *pounds*, *shillings*, and *pence*, three kinds of Divisor must be employed to divide £2866· 13·s. 10·d., say, by 4·

For, to divide £2866· 13·s. 10·d. by 4· implies the distribution
of the pounds into lots of £4· each,
of the shillings into lots of 4·s. each,
of the pence into lots of 4·d. each.

So that this example includes three perfectly distinct Simple Divisions. And, performed independently of each other, these Simple Divisions would give us

£2866· ÷ £4· = 716½ lots of £4· each,
13·s. ÷ 4·s. = 3¼ lots of 4·s. each,
10·d. ÷ 4·d. = 2½ lots of 4·d. each.

But, to fulfil the purpose for which Compound Division is always employed, it is necessary to connect the above three processes, by retaining only the *exact lots* of £4· each, and rejecting the half lot (£2·), in order to change it into shillings, to be, together with the 13·s. of Dividend, distributed into lots of 4·s. each. Remaining shillings, if any, will, in like manner, be exchanged for pence, added to the given 10·d. of Dividend, and the Sum of these distributed into lots of 4·d. each. If any pence remain, they will be exchanged into farthings, and distributed into lots of 4· farthings each. We thus obtain the answers to the following questions.

How many exact lots
Of £4· may be obtained from £2866· carrying Remr. to s.?
Of 4·s. may be obtained from remg· £ + 13·s. carrying Remr. to d.?
Of 4·d. may be obtained from remg· s. + 10·d. carrying Remr. to q.?
Of 4·q. may be obtained from remg· d., if any?

Now, working out this example by experiment with the real units (or their representatives), we should distribute £2866· into 716· lots of £4· each, and have £2· left. These £2·, being exchanged for 40·s., would make, with the given 13·s., a sum of 53·s., which we should distribute into 13· lots of 4·s. each., and have 1·s. remaining. Changing this shilling into 12· pence, and adding in the given 10·d., we should have 22· pence, which would give us 5· lots of 4· pence each, and 2· pence remaining. Twopence being changed into farthings would furnish 2· lots of 4· farthings each.

The Division of £2866· 13·s. 10·d. by 4· being now completed, we should have

716· lots, each £4·,	10· lots, each 4·d.
13· lots, each 4·s.	2· lots, each 4·q.

But, the object of Compound Division is, always, to find, not the true Quotient, but the Fraction of the Dividend which coincides with that Quotient. This Fraction is the Reciprocal of Divisor. (PRINCIPLE XVII., and pages 103· and 104·)

In the present example, then, the answer required is *one fourth part* of £2866· 13·s. 10·d.

To obtain this, we recur to true Quotient, and take

$$\left.\begin{array}{lll} \pounds1\cdot & \text{from each lot of } \pounds4\cdot, & \text{making } \pounds716\cdot \\ 1\cdot\text{s.} & \text{from each lot of } 4\cdot\text{s.}, & \quad\text{,,} \qquad 13\cdot\text{s.} \\ 1\cdot\text{d.} & \text{from each lot of } 4\cdot\text{d.}, & \quad\text{,,} \qquad 10\cdot\text{d.} \\ 1\cdot\text{q.} & \text{from each lot of } 4\cdot\text{q.}, & \quad\text{,,} \qquad \tfrac{1}{2}\text{d.} \end{array}\right\} \begin{array}{c} \text{making in all,} \\ \pounds716\cdot\ 13\cdot\text{s. } 10\tfrac{1}{2}\text{d.} \\ \text{which is the 4th part} \\ \text{of } \pounds2866\cdot\ 13\cdot\text{s. } 10\cdot\text{d.} \end{array}$$

$\therefore\ \frac{1}{4}$ of £2866· 13·s. 10·d. $\doteq$ £2866· 13·s. 10·d. $\div$ 4· (PRINCIPLE XVII.)

The Divisor 4·, not being named as a Concrete Number, must, nevertheless, not be taken for an Abstract Number. It cannot conveniently be named, because it represents, *in succession*, four different Divisors. (See First Example worked out, in Compound Long Division, page 224·)

No Concrete Quantity, Simple or Compound, can be divided by an Abstract Number. (PRINCIPLE V.)

This is evident from the nature of Division.

To Divide a Compound Quantity by an Integer.

First. Arrange Dividend and Divisor as in Simple Division.

Second. Divide Quantity of highest name, and set down Quotient.

Third. Reduce remaining units to next lower name, adding in given units of that name. Divide the Sum, and set down Quotient.

Fourth. Reduce remainder, and so on.

The Compound Quotient (its units being taken as Simple Units,) represents also that Aliquot Part of Dividend which is expressed by the Reciprocal of Divisor. (PRINCIPLES XIV. and XVII.)

FIRST EXAMPLE WORKED OUT. Find $\frac{1}{8}$ of £346· 10·s. $5\frac{3}{7}$d.

Example completed.

$$\begin{array}{r|lll} & \pounds. & \text{s.} & \text{d.} \\ 8\cdot) & 346\cdot & 10\cdot & 5\tfrac{3}{7} \\ \hline & 43\cdot & 6\cdot & 3\tfrac{1}{2},\ \tfrac{5}{7}\text{q.} = \tfrac{1}{8}\,\textit{Dividend.} \end{array}$$

From the Division of the pence, 5·d. remain.

We have, next, therefore, to divide $5\frac{3}{7}$d. by 8·

$5\frac{3}{7}\text{d.} \div 8\cdot = \frac{38}{7}\text{d.} \div 8\cdot \doteq \frac{38}{7}\text{d.} \times \frac{1}{8} = \frac{19}{28}\text{d.}$

And $\frac{19}{28}\text{d.} = \frac{19}{7}\text{q.} = 2\frac{5}{7}\text{q.}$

SECOND EXAMPLE WORKED OUT. Eleven equal masses of metal weighed in all, 2·T. 12·cwt. 3·qr. $15\frac{5}{6}$ lb. How much did each weigh?

Each weighed $\frac{1}{11}$ of their joint weight.

Example completed.

T. cwt. qr. lb.

11·) 2· 12· 3· 15$\frac{5}{6}$ = weight of the whole 11· masses.

4· 3· 6$\frac{35}{66}$ = 4·cwt. 3·qr. 6·lb. 8·oz. 7$\frac{25}{33}$dr. = $\frac{1}{11}$ of Dividend = weight of 1· mass.

Third Example worked out. Nine equal-sized gardens occupy together an area of 11·ac. 3·r. 22·p. 4·yds. What is the extent of each?

Find $\frac{1}{9}$ of 11·ac. 3·r. 22·po. 4·yds.

Example completed.

ac. r. po. yds.

9·) 11· 3· 22· 4· = area of 9· gardens.

1· 1· 11· 2· 2·ft. 72·in. = $\frac{1}{9}$ of Dividend = area of ea. garden.

Fourth Example worked out. If 1· cwt. cost £11· 18·s. 7·d., what will 28·lb. cost?

28·lb. = $\frac{1}{4}$ of 1·cwt. ∴ Divide £11· 18·s. 7·d. by 4·

Example completed.

£. s. d.

4·) 11· 18· 7· = cost of 1· cwt. or 112·lb.

2· 19· 7$\frac{3}{4}$ = cost of $\frac{1}{4}$ cwt. or 28·lb.

Fifth Example worked out. If 2·cwt. 2·qr. 12·lb. 6·oz. 8·dr. cost £1·, what quantity will cost 6·s. 8·d.?

Example completed.

cwt. qr. lb. oz. dr.

3·) 2· 2· 12· 6· 8· = weight worth £1·

3· 13· 7· 8· = weight worth 6·s. 8·d. or $\frac{1}{3}$ of £1·

In each of the above examples, what does the Divisor successively represent?

What is the true meaning of each Quotient?

What is meant by an Aliquot Part?

For what purpose is Compound Division employed?

EXERCISE XCVI.

EXAMPLES IN DIVISION OF COMPOUND QUANTITIES BY INTEGERS LESS THAN 13·

Find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$ *and* $\frac{1}{12}$, *of each of the following six Compound Quantities. Describe results.*

1· Of £3461· 11·s. 9·d.
2· Of £7345· 0·s. 4$\frac{3}{4}$d.
3· Of £6143· 12·s. 8$\frac{1}{8}$d.
4· Of £73· 5·s. 6·d. $\frac{3}{8}$q.
5· Of £2000·
6· Of £931· 13·s. 7$\frac{1}{4}$d. $\frac{7}{11}$q.
7· If 6·lb. 9·oz. of rice cost 1·s. how much will cost 6·d.?
8· What weight for 4·d. if 7·lb. 8·oz. 6·dr. cost 1·s.?
9· If 1·cwt. cost £14· 5·s. 10·d. what will 14·lb. cost?
10· If the rent of 1· acre be £4· 7·s. what is the rent of 2· roods?
11· If 26·lb. 6·oz. 3·dwts. 17·25 grs. cost £1·, what weight will cost 3·s. 4·d.?

12· If 5·lb. 6·oz. 4·dr. cost one shilling, how much may be bought for 1·d.?

13· What weight may be bought for half a crown, when 20·s. will purchase 4·lb. 10·℥ 2·ʒ 2·℈?

14· Required the rent of 16· perches of ground, at the rate of £5· 7·s. 6·d. per acre?

15· At £7· per acre, what is the rent of half a rood?

16· What will 14·lb. of tea cost, at £14· 5·s. 10·d. per cwt.?

17· If half a stone of fine flour cost 1·s. how much is threepenny worth?

18· What weight will cost 1$\frac{1}{2}$d. when 5·lb. 10·oz. cost 1·s.?

19· How much sugar is worth 1·s. 8·d. when 2·qr. 3·lb. 4·oz. cost £1·?

Find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$, and $\frac{1}{12}$, *of each of the following nineteen Compound Quantities.*

20· Of 3·T. 13·cwt. 2·qr. 17·lb. 10·oz. 11·dr.
21· Of 5·T. 17·cwt. 1·qr. 19·lb. 14·oz. 4$\frac{1}{5}$ dr.
22· Of 34·lb. 7·oz. 8·dwt. 13·gr.
23· Of 45·lb. 11·oz. 14·dwt. 22$\frac{1}{15}$ gr.
24· Of 13·lb. 5·℥ 7·ʒ 2·℈ 17$\frac{3}{14}$ gr.
25· Of 82·yds. 2·ft. 10·in. 7·li.
26· Of 13·mi. 7·fur. 21·po. 4·yds.
27· Of 16·yds. 1·qr. 3·na. 2$\frac{1}{8}$ ins.
28· Of 13·E.E. 3·qr. 1·na. $\frac{3}{4}$ in.
29· Of 14·Fr.E. 5·qr. 3·na. 1$\frac{7}{16}$ in.
30· Of 33·Fl.E. 1·qr. 2·na.
31· Of 15·Sc.E. 3·qr. 1·na. 2$\frac{3}{16}$ ins.
32· Of 37·su.yds. 8·ft. 2·′ 8·″ 6·‴
33· Of 64·sq.mi. 631·ac. 2·r. 18·po.
34· Of 34·qr. 5·bu. 2·pe. 1·gal. 1·qt. 1·pt.
35· Of 9·gal. 3·O 5·℥ 7·ʒ 31·♏.
36· Of a Julian year.
37· Of a lunar month.
38· Of a tropical year.

To Divide a Compound Quantity by a Composite Number, composed of any number of Factors, each less than 13·

Divide continually by any set of Corresponding Factors of the Divisor.

First Example worked out. If a cwt. of tea cost £24· 15·s. 3·d. what will 1·lb. cost?

The cost of 1·lb. being $\frac{1}{112}$ of that of 1·cwt., we must divide £24· 15·s. 3·d. by 112·, of which Number 4·, 7·, and 4· are one set of Corresponding Factors.

Example completed.

$$\begin{array}{rl} & \quad\;\; \pounds.\quad\;\; s.\quad\; d. \\ 112\cdot\left\{\begin{array}{l} 4\cdot)\underline{\,24\cdot\;\;15\cdot\;\;3\cdot\qquad} \\ \;7\cdot)\underline{\,6\cdot\quad\;\;3\cdot\;\;9\tfrac{3}{4}\;\,} \\ \;\;4\cdot)\underline{\quad\;\;17\cdot\;\;8\tfrac{1}{4}\;\,} \end{array}\right. & \begin{array}{l} =\text{cost of } 1\cdot\text{cwt., or } 112\cdot\text{lb.} \\ =\text{cost of } 28\cdot\text{lb.; being } \tfrac{1}{4} \text{ cost of } 1\cdot\text{cwt.} \\ =\text{cost of } 4\cdot\text{lb.; being } \tfrac{1}{7} \text{ cost of } 28\cdot\text{lb.} \end{array} \\ \qquad\qquad\quad\;\; 4\cdot\;\;5\tfrac{1}{16} & =\text{cost of } 1\cdot\text{lb.; being } \tfrac{1}{4} \text{ cost of } 4\cdot\text{lb.} \end{array}$$

SECOND EXAMPLE WORKED OUT. Find $\frac{1}{56}$ part of 13·T. 11·cwt. 2·qr. 15·lb. $3\frac{1}{5}$ oz.

Example completed.

$$
56\cdot\left\{\begin{array}{l}
\begin{array}{rrrrrrr}
 & \text{T.} & \text{cwt.} & \text{qr.} & \text{lb.} & \text{oz.} & \\
8\cdot) & 13\cdot & 11\cdot & 2\cdot & 15\cdot & 3\frac{1}{5} & \text{dr.} \\ \hline
7\cdot) & 1\cdot & 13\cdot & 3\cdot & 22\cdot & 14\cdot & 6\frac{2}{5} \\ \hline
 & & 4\cdot & 3\cdot & 11\cdot & 4\cdot & 5\frac{17}{35} \\ \hline\hline
\end{array}
\end{array}\right.
\begin{array}{l}
\\ \\
=\frac{1}{8} \text{ of Dividend.} \\
=\frac{1}{56} \text{ of Dividend; being} \\
\quad \frac{1}{7} \text{ of } \frac{1}{8} \text{ of Dividend.}
\end{array}
$$

THIRD EXAMPLE WORKED OUT. If 82·yds. 2·qr. cost 5·s. what length will cost 2·d.?

2·d. = $\frac{1}{30}$ of a crown, or = $\frac{1}{6}$ of $\frac{1}{5}$ of a crown.

Example completed.

$$
30\cdot\left\{\begin{array}{l}
\begin{array}{rrr}
 & \text{yds.} & \text{qr.} \\
6\cdot) & 82\cdot & 2\cdot \\ \hline
5\cdot) & 13\cdot & 3\cdot \\ \hline
 & 2\cdot & 3\cdot \\ \hline\hline
\end{array}
\end{array}\right.
\begin{array}{l}
\\
= \text{quantity worth 5·s. or 60·d.} \\
= \text{quantity worth } \frac{1}{6} \text{ of 5·s., or 10·d.} \\
= \text{quantity worth } \frac{1}{30} \text{ of 5·s., or 2·d.}
\end{array}
$$

FOURTH EXAMPLE WORKED OUT. What is the rent of 10· poles of garden-ground, at 46·s. per acre?

10·po. = $\frac{1}{16}$ (or $\frac{1}{2}$ of $\frac{1}{8}$) of an acre.

Example completed.

$$
16\cdot\left\{\begin{array}{l}
\begin{array}{rrrr}
 & \text{£.} & \text{s.} & \text{d.} \\
8\cdot) & 2\cdot & 6\cdot & 0\cdot \\ \hline
2\cdot) & & 5\cdot & 9\cdot \\ \hline
 & & 2\cdot & 10\frac{1}{2} \\ \hline\hline
\end{array}
\end{array}\right.
\begin{array}{l}
\\
= \text{rent of 1· acre, or 160· poles.} \\
= \text{rent of } \frac{1}{8} \text{ of an acre, or 20· poles.} \\
= \text{rent of } \frac{1}{16} \text{ of an acre, or 10· poles.}
\end{array}
$$

EXERCISE XCVII.

EXAMPLES IN DIVISION OF COMPOUND QUANTITIES BY COMPOSITE DIVISORS.

Every line of the work to be accurately described.

1· Find $\frac{1}{25}$ of £415· 11·s. $6\frac{3}{4}$d.
2· „ $\frac{1}{32}$ of 3·T. 17·cwt. 3·qr.
3· „ $\frac{1}{55}$ of 84·lb. 9·oz. 16·dts.
4· „ $\frac{1}{96}$ of 346· c.yds. 4·ft. 3·′
5· „ $\frac{1}{30}$ of 6·lb. 10·℥ 3·ʒ
6· Find $\frac{1}{99}$ of 436·mi. 6·fur. 4·yds.
7· „ $\frac{1}{42}$ of 200·bus. 2·pe. 1·gal.
8· „ $\frac{1}{54}$ of 34·O 13·℥ 5·ʒ
9· „ $\frac{1}{110}$ of 95·lo. 4·qr. 7·bus.
10· „ $\frac{1}{144}$ of 748·da. 11·ho. 31·m.

Find the value of 1· article

11· When 21· cost £53· 3·s. $1\frac{1}{2}$d.?
12· When 72· cost £34· 17·s. 6·d.?
13· When 28· cost £204· 4·s. 6·d.?
14· When 16· cost £218· 16·s.?
15· When 108· cost £15598·?
16· When 24· cost £19· 16·s. 3·d.?
17· When 36· cost £63· 18·s.?
18· When 70· cost £147·?
19· When 44· cost £1138· 10·s.?
20· When 132· cost £9871· 8·s.?
21· When 144· cost £1939· 19·s.?
22· When 32· cost £125· 12·s.?
23· When 50· cost £43· 10·s. 10·d.?
24· When 27· cost £212· 12·s. 6·d.?
25· When 110· cost £32· 1·s. 8·d.?
26· When 22· cost £86· 7·s.?
27· When 100· cost £198· 15·s.?
28· When 18· cost £47· 5·s.?
29· When 60· cost £39· 6·s.?
30· When 49· cost £20415· 8·d.?

What quantity may be bought

31· For 16·d., when 12· cwt. 3· qr. 18· lb. 9· oz. cost £1·?
32· For 15·d., when 13· cwt. 1· qr. 9· lb. cost £1·?
33· For 10·d., when 10· cwt. 8· oz. cost £1·?
34· For 7$\frac{1}{2}$d., when 2· cwt. 19· lb. 14·oz. cost £1·?
35· For 6·d., when 192· lb. 4· oz. Troy cost £1·?
36· For 4·d., when 41· gal. 1· qt. cost £1·?
37· For 3$\frac{3}{4}$d., when 12· gal. cost £1·?
38· For 2·d., when 46· gal. 3· qt. 1· pt. cost £1·?
39· For 3·d., when 68· su. yds. 8· ft. cost £1·?
40· For 1$\frac{1}{2}$d., when 25· gal. 1· qt. 2· gi. cost £1·?
41· For $\frac{3}{4}$d., when 2· lb. 1· ℥ 2· ʒ 2· ℈ cost £1·?
42· What will 8· lb. cost at £8· 2·s. 4$\frac{5}{8}$d. per cwt.?
43· What will 7· lb. cost at £4· 12·s. 2$\frac{1}{2}$d. per cwt.?
44· What will 4· lb. cost at £3· 13·s. 2$\frac{1}{2}$d. per cwt.?
45· What will 3$\frac{1}{2}$ lb. cost at £2· 7·s. 4·d. per cwt.?
46· What will 2· lb. cost at £11· 1·s. 8·d. per cwt.?
47· What will 1· lb. cost at £13· 8·s. 4·d. per cwt.?
48· How much will 1· cwt. 1· qr. cost at £30· 6·s. 8·d. per ton?
49· How much will 1· cwt. cost at £68· 10·s. per ton?
50· How much will 3$\frac{1}{2}$ lb. cost at £464· 16·s. 8·d. per ton?
51· What length of ribbon for 4·d. if 47· yds. 3· qr. 2$\frac{1}{8}$ na. cost 5·s.?
52· At 121· E. E. 3· qr. for a crown, how much for 3$\frac{3}{4}$d.?
53· If 66· Fr. E. 4· qr. cost 5·s., what length will 3·d. buy?

EXAMPLES IN WHICH EACH DIVISOR IS COMPOSED OF THREE OR MORE FACTORS, EACH LESS THAN 13·

54· What sum of money is contained 960· times in £3768·?
55· If 1152· articles of equal value cost £15519· 12·s., what is that each?
56· Find the capacity of a cask which will contain $\frac{1}{420}$ of 288·gal. 3·qts.
57· What weight must be repeated 256· times to produce 206·cwt. 2·qrs. 17·lb.?
58· Multiplier being 168·, and Product 1593·lb. 9·oz. 15·dwt., what is Multiplicand?
59· Find $\frac{1}{504}$ of 21071·cu. yd. 12·feet 11·primes 6·seconds.
60· Of what quantity are 36697·gal. 2·qts. the 2520th Multiple?
61· What weight is $\frac{1}{210}$ of 97·Tons 11·cwt. 26·lb. 10·oz. 2·dr.?
62· What weight must be multiplied by 180· to produce 8251·lb. 6·oz. 15·5 drs.?
63· Find $\frac{1}{126}$ of 563·lb. 11·oz. 8·dwt. 22·5 grs.
64· Find $\frac{1}{105}$ of 478·lb. 6·oz. 1·dwt. 6$\frac{15}{16}$ grs.
65· Find $\frac{1}{756}$ of 1426·℥ 2·ʒ 1·℈ 13·6 grs.
66· Find $\frac{1}{882}$ of 3998·leag. 2·mi. 1·fur. 34$\frac{3}{4}$ po.

COMPOUND LONG DIVISION.

The process assumes this form, when the Divisor is either a Prime Number, or a Composite Number whose Factors are inconveniently large or numerous.

Long Division enables us to exhibit the successive changes of unit in the Divisor, and thus to display clearly the true nature of the operation, as explained on pages 218· and 219·

FIRST EXAMPLE WORKED OUT. If the cost of 352· articles of equal value amount to £6594· 14·s. 8·d., what is each worth?

Each is worth $\frac{1}{352}$ of £6594· 14·s. 8·d.

And $\frac{1}{352}$ of £6594· 14·s. 8·d. $\doteq$ £6594· 14·s. 8·d. $\div$ 352· (PRIN. IX. and XVII.)

Example completed. *Ciphers expressed.* (See page 116· &c.)

£. £. s. d.
Divr. of £ = 352·) 6594· 14· 8· (18· lots of £352· each.
3520· (PRIN. XIV.)
3074·
2816·
£258· remain.
s. 20·
Divr. of s. = 352·) 5174· s. (14· lots of 352·s. ea. (PR. XIV.)
3520·
1654·
1408·
246· s. remain.
d. 12·
Divr. of d. = 352·) 2960· d. (8· lots of 352·d. ea. (PR. XIV.)
2816·
144· d. remain.
q. 4·
Divr. of far. = 352·) 576· q. (1$\frac{7}{11}$ lots of 352· farthings each.
352· (PRIN. XIV.)
224· q. = $\frac{224}{352}$ or $\frac{7}{11}$ of a lot of 352· q.
(PR. XV. and XIV.)

Now, supposing £1·, 1·s., 1·d., 1·far. and $\frac{7}{11}$ of a farthing to be taken, respectively, from each lot named in the several Quotients, the collection would make up $\frac{1}{352}$ of Dividend.

£. s. d. £. s. d. q. £. s. d.
∴ $\frac{1}{352}$ of 6594· 14· 8· = 18· 14· 8$\frac{1}{4}$, $\frac{7}{11}$ $\doteq$ 6594· 14· 8· $\div$ 352·

The connection of the two results may appear still clearer, if we suppose the collection just spoken of to be repeated as often as possible, namely, (in this example) 352· times. Then,

The 18· lots of £352· each, become 352· lots of £18· each.
The 14· lots of 352·s. each, become 352· lots of 14·s. each.
The 8· lots of 352·d. each, become 352· lots of 8·d. each.
The 1· lot of 352· fars., becomes 352· lots of $\frac{1}{4}$d. each.
The $\frac{7}{11}$ of a lot of 352· fars., becomes 352· lots of $\frac{7}{11}$q. each.

This being done, we should have the Dividend £6594· 14·s 8·d. distributed into 352· equal amounts, each being $\frac{1}{352}$ of £6594· 14·s. 8·d. and = £18· 14·s. $8\frac{1}{4}$d. $\frac{7}{11}$q. In the actual experimental distribution of the money, if it lay before us in real coins, the whole of the steps detailed above must be gone through.

The *redistribution* of the lots furnishes a striking instance of change of unit, and of coincidence of numbers, the 18· which at first represents lots, afterwards standing for £, &c.

PRINCIPLES IX. and VI. are also involved in this part of the process.

18· lots *of* £352· = 352· lots *of* £18·

Changing the word "*of*" for the sign ×, by PRINCIPLE IX., and reversing the order of the quantities, under PRINCIPLE VI., we have

£352· × 18· = £18· × 352·

EXERCISE XCVIII.

Describe each Dividend, Divisor, and Quotient, in writing.

1· Find $\frac{1}{17}$ of £89· 1·s. $5\frac{1}{4}$d., and $\frac{1}{19}$ of £98· 15·s. $7\frac{1}{4}$d.
2· If 23· yds. of ribbon cost £1· 14·s. $0\frac{1}{4}$d. what will 1· yd. cost?
3· If 29· pairs of gloves cost £4· 14·s. 3·d. what is that per pair?
4· What is the value of a Dutch cheese of which 43· cost £2· 11·s. $11\frac{1}{3}$d.?
5· How much per lb. is pork, when 6· score and 17· lb. cost £3· 5·s. $7\frac{3}{4}$d.?
6· A flock of 113· sheep cost £178· 18·s. 4·d., what is their average price?
7· Find that amount of which £724· 5·s. 4·d. is the 97th Multiple?
8· What is gold per oz. when 161· lb. cost £5546· 9·s.?
9· How much per bottle is that wine of which 149· doz. cost £510· 6·s. 6·d.?
10· Find $\frac{1}{71}$ of 83·T. 8· cwt. 2· qr., and $\frac{1}{89}$ of 5863· yds. 2· qr. $1\frac{1}{8}$ na.
11· At what uniform rate per day will a coach travel 1697· mi. 3· fur. in 37· days?
12· Divide 780· lb. 10· oz. 12· dwt. $2\frac{4}{9}$ grs. into 101· equal parts.
13· Divide 47· lb. 4· ℥ 2· ʒ 2· ℈ 12·8 grs. by 13·
14· Divide 1112· E.E. 3· qr. $\frac{7}{8}$ in. by 143·
15· What area must be repeated 179· times to cover a space occupying 896· ac. 1· r. 22· po. $13\frac{15}{16}$ yd.?
16· What is the sixty-ninth part of 25201· da. 17· ho. 9· min. 9·3 sec.?
17· If 92· equal vessels contain collectively 1335· hhd. 10· gal. $2\frac{2}{3}$ qt., how much will each hold?
18· When Multiplier is 643·, and Product £17352· 11·s. 8·d., what is Multiplicand?

To find any Fraction of a Compound Quantity.

Proceed as directed on pages 151· and 152·

FIRST EXAMPLE WORKED OUT. What sum of money is $\frac{7}{11}$ of £346· 18·s. $4\frac{3}{4}$d. $\frac{1}{9}$q.?

Example completed.

By First Method.	By Second Method.
£. s. d. q.	£. s. d. q.
11·)346·18·4$\frac{3}{4}$,$\frac{1}{9}$	346·18·4$\frac{3}{4}$,$\frac{1}{9}$ =$\frac{11}{11}$ of Given Sum
31·10·9$\frac{0}{4}$,$\frac{64}{99}$=$\frac{1}{11}$ of Given Sum	7·
7·	11·)2428· 8·9$\frac{1}{4}$,$\frac{7}{9}$ =$\frac{77}{11}$ of Given Sum
220·15·4$\frac{0}{4}$,$\frac{52}{99}$=$\frac{7}{11}$ of Given Sum	220·15·4$\frac{0}{4}$,$\frac{52}{99}$=$\frac{7}{11}$ of Given Sum

SECOND EXAMPLE WORKED OUT. What is the area of a park which is exactly five ninths of an estate containing 348·ac. 3·r. 36·po.?

Example completed.

By First Method.	By Second Method.
ac. r. po.	ac. r. po.
9·)348· 3· 36· = $\frac{9}{9}$ of estate.	348· 3· 36· = $\frac{9}{9}$ of estate.
38· 3· 4· = $\frac{1}{9}$ of estate.	5·
5·	9·)1744· 3· 20· = $\frac{45}{9}$ of estate.
193· 3· 20· = $\frac{5}{9}$ of estate.	193· 3· 20· = $\frac{5}{9}$ of estate.

∴ *Area of the park* = 193·*ac.* 3·*r.* 20·*po.*

The Second Method is more *convenient,* as it avoids Multiplication of Fractions, unless there be one in the given Quantity.

But the First Method is the more *natural* of the two, and is, consequently, the easier to trace.

THIRD EXAMPLE WORKED OUT. What will 1·qr. 21·lb. cost, at £12· 12·s. 6·d. per cwt.?

1·qr. 21·lb. = 49·lb. = $\frac{49}{112}$ or $\frac{7}{16}$ of 1·cwt. (EXERCISE LXXXIX.)

∴ 1·qr. 21·lb. will cost $\frac{7}{16}$ of £12· 12·s. 6·d.

Example completed.

By First Method.	By Second Method.
£. s. d. lb. cwt.	£. s. d. cwt. lb.
16·{ 4·)12· 12· 6· cost of 112· or 1·	12· 12· 6· =c. of $\frac{4}{4}$, or 112·
4·)3· 3· 1$\frac{1}{2}$=c. of 28· or $\frac{1}{4}$	7·
15· 9$\frac{1}{4}$,$\frac{1}{2}$=c. of 7· or $\frac{1}{16}$	16·{ 4·)88· 7· 6· =c. of $\frac{28}{4}$, or 784·
7·	4·)22· 1· 10$\frac{1}{2}$=c. of $\frac{7}{4}$, or 196·
5· 10· 5$\frac{1}{2}$,$\frac{1}{2}$=c. of 16· or $\frac{7}{16}$	5· 10· 5$\frac{1}{2}$,$\frac{1}{2}$=c. of $\frac{7}{16}$, or 49·

FOURTH EXAMPLE WORKED OUT. What weight of goods will 8·s. 4·d. pay for, if 3·cwt. 1·qr. 14·lb. cost £1·?

8·s. 4·d. = $\frac{5}{12}$ of £1·, ∴ 8·s. 4·d. will pay for $\frac{5}{12}$ of 3·cwt. 1·qr. 14·lb.

Example completed.

By First Method.	By Second Method.
cwt. qr. lb.	cwt. qr. lb.
12·)3. 1. 14· =wt. worth 20/ or £1·	3· 1· 14· =wt. worth £$\frac{12}{12}$, or 20/
1· 3$\frac{1}{2}$=wt. worth 1/8 or £$\frac{1}{12}$	5·
5·	12·)16· 3· 14· =wt. wth. £$\frac{60}{12}$, or 100/
1· 1· 17$\frac{1}{2}$=wt. worth 8/4 or £$\frac{5}{12}$	1· 1· 17$\frac{1}{2}$=wt. wth. £$\frac{5}{12}$, or 8/4

EXERCISE XCIX.

EXAMPLES IN MULTIPLICATION OF COMPOUND QUANTITIES BY FRACTIONS.

A. *Work each of the following twelve examples by both methods.*

Describe each line of the work in writing.

What is the value

1· Of $\frac{3}{8}$ of £3164· 11·s. $5\frac{1}{4}$d.?
2· Of $\frac{7}{12}$ of £914· 7·s. $6\frac{1}{8}$d.?
3· Of $\frac{9}{17}$ of £653· 3·s. $10\frac{2}{4}$d. $\frac{4}{7}$?
4· Of $\frac{11}{21}$ of 16·cwt. 3·qr. 8·lb.?
5· Of $\frac{5}{6}$ of 14·lb. 5·oz. 13·dwt.?
6· Of $\frac{4}{5}$ of 23·yd. 1·qr. 3·na.?
7· Of 5·lb. 11·℥ 6·ʒ 2·℈ 14·grs. × $\frac{9}{16}$?
8· Of 26·mi. 2·fur. 7·po. 1·yd. × $\frac{11}{25}$?
9· Of 31·Sc. E. 1·qr. 2·na. × $\frac{6}{13}$?
10· Of 849·su. yd. 7·ft. 8·′ × $\frac{10}{17}$?
11· Of 17·bu. 3·pe. 1·gal. 1·qt. × $\frac{4}{9}$?
12· Of 7·O 11·℥ 4·ʒ 29·♏ × $\frac{3}{16}$?

B. *Work the next twenty examples by the first method only.*

Describe each line in writing.

What is the value

s. d.
13· Of 7·oz. Av. at 15· 6· per lb.?
14· Of 5·oz. Av. at 13· 9· per lb.?
15· Of 11·oz. Av. at 12· 8· per lb.?
16· Of 13·oz. Av. at 4· 8· per lb.?
17· Of 15·oz. Av. at 11· 4· per lb.?
18· Of 3·qrs. at 17· 5· per yd.?
19· Of 3·na. at 15· 4· per yd.?
20· Of 5·na. at 12· 6· per yd.?
21· Of 7·na. at 5· 3· per yd.?
22· Of 9·na. at 13· 5· per yd.?

£. s. d.
23· Of 3·r. 20·po. at 2· 3· 0· per ac.?
24· Of 1·r. 20·po. at 5· 5· 6· per ac.?
25· Of 2·r. 10·po. at 4· 11· 6· per ac.?
26· Of 1·r. 10·po. at 3· 17· 9· per ac.?
27· Of 2·q. 7·lb. at 1· 4· 6· per cwt.?
28· Of 3·q. 14·lb. at 2· 16· 0· per cwt.?
29· Of 1·q. 14·lb. at 3· 3· 0· per cwt.?
30· Of 2·q. 14·lb. at 4· 11· 8· per cwt.?
31· Of 1·q. 4·lb. at 6· 9· 5· per cwt.?
32· Of 1·q. 20·lb. at 1· 5· 8· per cwt.?

If 2·cwt. 3·qr. 18·lb. cost 20·s. what weight may be bought

s. d.
33· For 7· 6·?
34· For 12· 6·?
35· For 15· 0·?
36· For 17· 6·?
37· For 13· 4·?
38· For 3· 9·?
39· For 6· 3·?
40· For 8· 9·?

If 4·yd. 3·qr. 2·na. cost 1·s., what length may be bought

41· For $2\frac{1}{2}$d.?
42· For $3\frac{1}{2}$d.?
43· For $4\frac{1}{2}$d.?
44· For 5·d.?
45· For $6\frac{1}{2}$d.?
46· For 7·d.?
47· For $7\frac{1}{2}$d.?
48· For 8·d.?
49· For 9·d.?
50· For 10·d.?
51· For 11·d.?
52· For $3\frac{3}{4}$d.?

If 32·gal. 2·qt. 1·pt. cost £1·, what quantity may be bought

s. d.
53· For 11· 3·?
54· For 13· 9·?
55· For 16· 3·?
56· For 18· 9·?
57· For 13· 6·?
58· For 5· 6·?
59· For 4· 6·?
60· For 9· 6·?

THIRD METHOD.

First. Find the values of the several Aliquot parts of Multiplicand which make up the required Fraction of it.

Second. Add these values together.

FOURTH METHOD. A.

First. Find the value of the Aliquot part of. Multiplicand nearest above the required Fraction of it.

Second. Find the value of the Difference of these, and Subtract.

FOURTH METHOD. B.

When the Difference between the whole Multiplicand and the required Fraction of it is a convenient Aliquot part of the former, find the value of that part, and Subtract.

These methods are extremely useful: for the values of the smaller Aliquot parts can generally be easily derived from those of the larger.

FIFTH EXAMPLE WORKED OUT. Find the value of 12·oz. Avoir. at £1· 16·s. 3·d. per lb.

12· oz. Avoir. = $\frac{3}{4}$ lb. = $\frac{1}{2}$ lb. + $\frac{1}{4}$ lb. = $\frac{1}{2}$ lb. + $\frac{1}{2}$ of $\frac{1}{2}$ lb.

We therefore find

I. The cost of $\frac{1}{2}$ lb., or 8·oz. } and then add these
II. The cost of $\frac{1}{4}$ lb., or 4·oz. } values together.

Example completed.

By Third Method.

	£.	s.	d.	lb.
2·)	1·	16·	3·	= c. of 1·
2·)		18·	1$\frac{1}{2}$	= c. of $\frac{1}{2}$ or 8·oz.
		9·	0$\frac{3}{4}$	= c. of $\frac{1}{4}$ or ($\frac{1}{2}$ of $\frac{1}{2}$) lb. or 4·oz.
	1·	7·	2$\frac{1}{4}$	= c. of $\frac{3}{4}$ or 12·oz.

By Fourth Method. B.

	£.	s.	d.	lb. oz.
4·)	1·	16·	3·	= c. of 1· or 16·
		9·	0$\frac{3}{4}$	= c. of $\frac{1}{4}$, or 4·
	1·	7·	2$\frac{1}{4}$	= c. of $\frac{3}{4}$, or 12·

SIXTH EXAMPLE WORKED OUT. Find the cost of 4· cwt. 3· qrs. of sugar, at £73· 13·s. 4·d. per ton.

Here, for Third Method, we observe that

4·cwt. will cost $\frac{1}{5}$ of what 1 ton costs.
2·qrs. will cost $\frac{1}{8}$ of what 4·cwt. cost.
1·qr. will cost $\frac{1}{2}$ of what 2·qrs. cost.

Example completed; by Third Method.

	Divs.	£.	s.	d.	
For 4·cwt.	5·	73·	13·	4·	= cost of 20·cwt. or 1· ton.
„ 2·qr.	8·	14·	14·	8·	= cost of 4·cwt., being $\frac{1}{5}$ of cost of 20· cwt.
„ 1·qr.	2·	1·	16·	10·	= cost of 2·qr., being $\frac{1}{8}$ of cost of 4·cwt.
			18·	5·	= cost of 1·qr., being $\frac{1}{2}$ of cost of 2· qrs.
		17·	19·	11·	= c. of 4·ct. 3·qr., being sum of above 3 amts.

Again, for Fourth Method, we notice that 4·cwt. 3·qr. are just 1·qr. short of 5·cwt., which is $\frac{1}{4}$ of a ton. And 1·qr. = $\frac{1}{80}$ of 1· ton, or $\frac{1}{20}$ of 5·cwt.

∴ 4·cwt. 3·qr. = $(\frac{1}{4}-\frac{1}{80})$ of a ton, or $\frac{1}{4}$ of a ton $-\frac{1}{20}$ of 5·cwt.

Now, 5·cwt. will cost $\frac{1}{4}$ of what 1· ton costs.

And, 1·qr. will cost $\frac{1}{20}$ of what 5·cwt. cost.

Hence, 4·cwt. 3·qr. will cost the Difference of those two amounts.

Example completed; by Fourth Method. A.

	Divs.	£.	s.	d.	
For 5·cwt.	4·	73·	13·	4·	=cost of 20·cwt., or 1· ton.
„ 1·qr.	20·	18·	8·	4·	=cost of 5·cwt., being $\frac{1}{4}$ of cost of 1· ton.
			18·	5·	=cost of 1·qr., being $\frac{1}{20}$ of cost of 5·cwt.
		17·	9·	11·	=c. of 4·cwt. 3·qr., being Diff. of above two.

Seventh Example worked out. How much beef will 15·s. buy, if 2·qr. 8·lb. cost £1·?

15·s. will pay for $\frac{3}{4}$ of what £1· will pay for.

Example completed.

By Third Method.

	Divs.	qr.	lb.	
For 10·s.	2·	2·	8·	=20·s.-worth.
„ 5·s.	2·	1·	4·	=10·s.-worth.
			16·	= 5·s.-worth.
		1·	20·	=15·s.-worth.

By Fourth Method. B.

	Divr.	qr.	lb.	
For 5·s.	4·	2·	8·	= 20·s.-worth.
			16·	= 5·s.-worth.
		1·	20·	= 15·s.-worth.

EXERCISE XCIX. (*resumed.*)

Return to Examples already given, and (describing each line,) work over again

By 3rd Method: Examples 2·, 7·, 11·, 14·, 15·, 16·, 20·, 22·, 25·, 26·, 27·, 30·, and 31·.

By 4th Method, A: Examples 13·, and 21·.

By 4th Method, B: Examples 5·, 6·, and 36·.

By 3rd Method, and also by 4th Method, A: Examples 1·, 12·, 24·, and 29·.

By 3rd Method, and also by 4th Method, B: Examples 17·, 18·, 23·, and 28·.

To Multiply a Compound Quantity by a Mixed Number.

First. Multiply by the Fraction, by any of above methods.

Second. Multiply by the Integer.

Third. Add the results together.

But, if Aliquot Parts are employed to find required Fraction of Multiplicand, then Multiply *first* by Integer.

Sometimes the work is abbreviated by Multiplying by the Integer next above the Mixed Multiplier, and Subtracting the Difference. (See Fourth Method, B, p. 228·)

Example worked out. Find the value of 9· cwt. 3· qr. 14· lb. at £2· 13·s. 8·d. per cwt.

Amount required = £2· 13·s. 8·d. × $9\frac{7}{8}$.

Example completed.

£.	s.	d.	
2·	13·	8·	=val. of $\frac{8}{8}$ of 1·cwt.
		7·	
8·) 18·	15·	8·	=val. of $\frac{56}{8}$ of 1·cwt.
2·	6·	$11\frac{1}{2}$	=val. of $\frac{7}{8}$ of 1·cwt.
24·	3·	0·	=val. of 9·cwt.
26·	9·	$11\frac{1}{2}$	=val. of $9\frac{7}{8}$ cwt.

Or thus: by Multn. and Subtn.

£.	s.	d.	
2·	13·	8·	=val. of 1·cwt.
		10·	
26·	16·	8·	=val. of 10·cwt.
	6·	$8\frac{1}{2}$	=val. of $\frac{1}{8}$ of 1·cwt.
26·	9·	$11\frac{1}{2}$	=val. of $9\frac{7}{8}$ cwt.

Or thus: by taking Aliquot Parts.

	Div.	£.	s.	d.	
For 2·qr.	2·	2·	13·	8·	= cost of 1·cwt.
				9·	
		24·	3·	0·	= cost of 9·cwt.
„ 1·qr.	2·	1·	6·	10·	= cost of 2·qr., being $\frac{1}{2}$ of cost of 1·cwt.
„ 14·lb.	2·		13·	5·	= cost of 1·qr., being $\frac{1}{2}$ of cost of 2·qr.
			6·	$8\frac{1}{2}$	= cost of 14·lb., being $\frac{1}{2}$ of cost of 1·qr.
		26·	9·	$11\frac{1}{2}$	= cost of 9·cwt. 3·qr. 14·lb.

EXERCISE C.

EXAMPLES IN MULTIPLICATION OF COMPOUND QUANTITIES BY MIXED NUMBERS.

Every line to be described in writing.

Find the value of

1·	$5\frac{1}{2}$ lb.	at	$11\frac{1}{2}$d.
2·	$17\frac{3}{4}$ lb.	at	14·d.
3·	$11\frac{1}{4}$ lb.	at	25·d.
4·	$9\frac{1}{2}$ lb.	at	$7\frac{3}{4}$d.
5·	$8\frac{1}{4}$ lb.	at	$13\frac{1}{2}$d.
6·	$15\frac{3}{4}$ lb.	at	$18\frac{1}{4}$d.
7·	$12\frac{1}{8}$ lb.	at	22·d.
8·	$31\frac{3}{8}$ lb.	at	45·d.
9·	$16\frac{5}{8}$ lb.	at	$63\frac{1}{2}$d.
10·	$9\frac{7}{8}$ lb.	at	$32\frac{1}{4}$d.
11·	$5\frac{1}{16}$ yds.	at	$11\frac{3}{4}$d.
12·	$11\frac{3}{16}$ yds.	at	$14\frac{1}{4}$d.
13·	$14\frac{5}{16}$ yds.	at	$21\frac{3}{8}$d.
14·	$21\frac{7}{16}$ yds.	at	$32\frac{1}{16}$d.
15·	$13\frac{9}{16}$ yds.	at	$29\frac{5}{8}$d.
16·	$15\frac{11}{16}$ yds.	at	$42\frac{1}{16}$d.
17·	$14\frac{13}{16}$ yds.	at	$36\frac{1}{2}$d.
18·	$17\frac{15}{16}$ yds.	at	$21\frac{7}{16}$d.
19·	$31\frac{7}{8}$ yds.	at	$54\frac{7}{8}$d.
20·	$16\frac{5}{8}$ yds.	at	$16\frac{5}{8}$d.

	lb.	oz. (Av.)		£.	s.	d.	
21·	11·	1·	at	2·	11·	8·	per lb.
22·	4·	2·	at		16·	4·	„ „
23·	7·	3·	at		5·	8·	„ „
24·	8·	4·	at	1·	3·	4·	„ „
25·	12·	9·	at		1·	$10\frac{1}{2}$	„ „
26·	18·	11·	at		5·	6·	„ „
27·	7·	12·	at		1·	5·	„ „
28·	3·	13·	at		3·	8·	„ „
29·	15·	14·	at		14·	5·	„ „
30·	21·	15·	at		7·	10·	„ „

	cwt.	qr.	lb.		£.	s.	d.	
31·	7·	0·	12·	at	2·	18·	9·	per cwt.
32·	14·	2·	0·	at	1·	17·	4·	„ „
33·	11·	3·	7·	at	3·	3·	0·	„ „
34·	5·	1·	14·	at	1·	5·	8·	„ „
35·	3·	1·	12·	at	1·	10·	4·	„ „
36·	4·	2·	20·	at	2·	9·	0·	„ „
37·	9·	3·	24·	at		16·	4·	„ „
38·	6·	2·	21·	at	1·	2·	8·	„ „
39·	2·	3·	6·	at	3·	19·	6·	„ „
40·	1·	1·	7·	at	5·	17·	6·	„ „

41· How much cheese would cost £4· 1·s. 8·d., if 1 qr. 24·lb. cost £1·? *Multiply by* $4\frac{1}{12}$.

42· How much beef will £3· 2·s. 6·d. buy, if 1·qr. 3·lb. 8·oz. cost £1·? *Multiply by* $3\frac{1}{8}$.

43· What quantity of beer is worth £7· 3·s. 4·d., when 1·bar. 28·galls. cost £1·? *Multiply by* $7\frac{1}{6}$.

44· If 23·yd. 2·qr. 1·na. of linen cost a sovereign, how much will cost five guineas? *Multiply by 5¼.*

45· Supposing the keep of a horse for 4·we. 5·da. to cost £2·, how long could he be maintained for £3· 10·s.? *Divide by 2·. Multiply Quotient by 3½.*

46· What weight of goods will cost £9· 7·s. 6·d., at the rate of 2·cwt. 1·qr. 24·lb. for £1·?

47· If 20·s. will pay for manuring 3·r. 21·po. of ground, what extent will be manured for £8· 6·s. 3·d.?

48· At the rate of 35·mi. 5·fur. 30·po. per hour, how far will a train travel in 4·ho. 20·min.?

49· If the fare for a distance of 41·mi. 7·fur. be £1·, how far should one be conveyed for £2· 8·s. 9·d.?

50· If a man's wages be 17·s. 6·d. per week, how much will he earn in 9·we. 4·da.?

51· If a man must work 11·da. 5·ho. to earn £2·, in what time will he earn £7· 10·s., counting 10· hours to the day? *Multiply by 3¾.*

52· If 13·yd. 3·qr. 1·na. cost 16·s., how much will cost 40·s.? *Multiply by 2½.*

To Divide a Compound Quantity by a Fraction.

Multiply by Reciprocal of Divisor. (PRINCIPLE XVII.)

This process is employed to ascertain the Quantity of which the given one is a certain Fraction.

For example, if we wish to discover the amount of which £342· 15·s. 2½d. are ⅝, we must Divide £342· 15·s. 2½d. by ⅝, the result of which operation will coincide with the Product of £342· 15·s. 2½d. × $\frac{8}{5}$. For,

∵ ⅝ of *Required Amount* = £342· 15·s. 2½d.
∴ *Required Amount* × ⅝ = £342· 15·s. 2½d. (PR. IX.)

Then, Dividing both sides of the Equation by ⅝ (PRIN. XXIV.)

Required Amount = £342· 15·s. 2½d. ÷ ⅝ (PR. XX.)
∴ *Required Amount* = £342· 15·s. 2½d. × $\frac{8}{5}$ (PR. XVII.)

FIRST EXAMPLE WORKED OUT. Five-eighths of a certain sum of money amount to £342· 15·s. 2½d.; what is that sum?

Example completed.

By First Method.	By Second Method.
£. s. d.	£. s. d.
5·) 342· 15· 2½ = ⅝ of Reqd. Amt.	342· 15· 2½ = ⅝ of Reqd. Amt.
68· 11· 0½ = ⅛ of Reqd. Amt.	8·
8·	5·) 2742· 1· 8· = $\frac{40}{8}$ of Reqd. Amt.
548· 8· 4· = Required Amt.	548· 8· 4· = Required Amt.

SECOND EXAMPLE WORKED OUT. If 2·qr. 24·lb. cost £1· 8·s. 9·d., what is the price per cwt.?

2·qr. 24·lb. $=\frac{5}{7}$ of 1·cwt. (EXERCISE LXXXIX.)
∴ £1· 8·s. 9·d. $=\frac{5}{7}$ *of cost of* 1·*cwt.*
∴ *cost of* 1·*cwt.* = £1· 8·s. 9·d. $\div\frac{5}{7}$. (PRIN. XX.)
$\doteq$ £1· 8·s. 9·d. $\times\frac{7}{5}$. (PRIN. XVII.)

Example completed.

By First Method.	By Second Method.
£. s. d.	£. s. d.
5·) 1· 8· 9·=c. of $\frac{5}{7}$ cwt., or 80·lb.	1· 8· 9·=c. of $\frac{5}{7}$ cwt.
5· 9·=c. of $\frac{1}{7}$ cwt., or 16·lb.	7·
7·	5·) 10· 1· 3·=c. of 5·cwt. (P.XIX.
2· 0· 3·=c. of 1·cwt.	2· 0· 3·=c. of 1·cwt. Cor. i.)

To Divide a Compound Quantity by a Mixed Number.

First. Reduce Divisor to its equivalent Improper Fraction in its lowest terms.

Second. Multiply Given Quantity by Reciprocal of this Improper Fraction.

THIRD EXAMPLE WORKED OUT. If 9·cwt. 3·qr. 14·lb. cost £26· 9·s. $11\frac{1}{2}$d., what will 1·cwt. cost?

9·cwt. 3·qr. 14·lb. $=9\frac{7}{8}$ cwt. $=\frac{79}{8}$ of 1·cwt.
∴ cost of 1·cwt. = £26· 9·s. $11\frac{1}{2}$d. $\div\frac{79}{8}$
$\doteq$ £26· 9·s. $11\frac{1}{2}$d. $\times\frac{8}{79}$

Example completed; except the *work* of the Long Division.

By First Method.	By Second Method
£. s. d.	£. s. d.
79·) 26· 9· $11\frac{1}{2}$=c. of $\frac{79}{8}$ of 1·cwt.	26· 9· $11\frac{1}{2}$=c. of $\frac{79}{8}$ of 1·cwt.
6· $8\frac{1}{2}$=c. of $\frac{1}{8}$ of 1·cwt.	8·
8·	79·) 211· 19· 8·=c. of 79·cwt.*
2· 13· 8·=c. of 1·cwt.	2· 13· 8·=c. of 1·cwt.

EXERCISE CI.

EXAMPLES IN DIVISION OF COMPOUND QUANTITIES BY FRACTIONS OR MIXED NUMBERS.

Describe every line in writing.

1· Of what sum are £1247· 1·s. $0\frac{3}{4}$d. three fifths?
2· Of what amount are £2973· 3·s. $10\frac{1}{2}$d. $=\frac{4}{7}$?
3· If $\frac{3}{4}$ of a yd. cost 2·s. $2\frac{1}{4}$d. what will 1·yd. cost?
4· If 7· articles cost £4· 2·s. $5\frac{11}{12}$d. what will 11· cost?

What will 1·lb. Avoir. cost

5· If $\frac{3}{4}$ lb. cost 3·s. $6\frac{3}{4}$d.?
6· If $1\frac{1}{2}$ lb. cost 16·s. $11\frac{1}{4}$d.?
7· If $2\frac{1}{4}$ lb. cost 12·s. $2\frac{1}{4}$d.?
8· If $2\frac{3}{4}$ lb. cost 19·s. $0\frac{1}{4}$d.?
9· If $4\frac{1}{2}$ lb. cost 8·s. $10\frac{1}{2}$d.?
10· If $7\frac{1}{4}$ lb. cost 24·s. $5\frac{5}{8}$d.?
11· If $3\frac{1}{8}$ lb. cost 17·s. $2\frac{1}{4}$d.?
12· If $5\frac{7}{16}$ lb. cost 14·s. 6·d.?
13· If 11·oz. cost 8·s. $8\frac{1}{2}$d.?
14· If 15·oz. cost 10·s. $7\frac{1}{2}$d.?
15· If 5·oz. cost 4·s. $3\frac{9}{16}$d.?
16· If 6·oz. cost 2·s. $11\frac{1}{2}$d.?
17· If 7·oz. cost 6·s. $3\frac{1}{4}$d.?
18· If 9·oz. cost 15·s. $6\frac{3}{4}$d.?
19· If 13·oz. cost 3·s. $9\frac{1}{2}$d.?
20· If 12·oz. cost 11·s. 10·d.?

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* Principle XIX. Cor. i.

How much is cloth per yard

21· When 3·qr. cost 13·s. $0\frac{3}{4}$d.?
22· When 3·na. cost 2·s. $10\frac{1}{2}$d.?
23· When 5·na. cost 3·s. $10\frac{7}{8}$d.?
24· When 7·na. cost 2·s. $3\frac{9}{16}$d.?
25· When 9·na. cost 7·s. $6\frac{9}{16}$d.?

26· At 2·s. $2\frac{1}{4}$d. for $\frac{7}{8}$ yd.?
27· At 1·s. $3\frac{3}{4}$d. for $\frac{3}{8}$ yd.?
28· At 8·s. $7\frac{3}{4}$d. for $\frac{5}{8}$ yd.?
29· At 10·s. $2\frac{5}{8}$d. for $\frac{3}{4}$ yd.?
30· At 4·s. $11\frac{1}{8}$d. for $\frac{5}{16}$ yd.?

What is the rent per acre

31· At £1· 17·s. $7\frac{1}{2}$d. for 3·r. 20·p.?
32· At £1· 19·s. $6\frac{3}{4}$d. for 1·r. 20·p.?

33· At £2· 11·s. $5\frac{5}{8}$d. for 2·r. 10·p.?
34· At £1· 4·s. $3\frac{9}{16}$d. for 1·r. 10·p.?

What is the price per cwt.

		£.	s.	d.		qr.	lb.
35·	At	5·	1·	$11\frac{1}{4}$	for	3·	14·?
36·	At	1·	15·	$4\frac{1}{4}$	for	1·	14·?
37·	At	3·	19·	$8\frac{3}{32}$	for	2·	14·?

		£.	s.	d.		qr.	lb.
38·	At	1·	2·	$9\frac{3}{8}$	for	2·	7·?
39·	At	1·	16·	$11\frac{6}{7}$	for	1·	4·?
40·	At		11·	0·	for	1·	20·?

At what rate per lb. Avoir. will

41· $5\frac{1}{2}$ lb. cost £3· 2·s. $1\frac{1}{4}$d.?
42· $17\frac{3}{4}$ lb. cost £3· 18·s. $9\frac{3}{16}$d.?
43· $11\frac{1}{4}$ lb. cost £3· 4·s. $5\frac{7}{16}$d.?
44· $9\frac{1}{2}$ lb. cost £3· 2·s. $11\frac{1}{4}$d.?
45· $8\frac{1}{4}$ lb. cost £3· 7·s. $8\frac{5}{8}$d.?

	lb.	oz.		£.	s.	d.
46·	11·	1·	cost	2·	0·	$6\frac{3}{4}$?
47·	4·	2·	cost	1·	6·	$9\frac{3}{4}$?
48·	7·	3·	cost	2·	15·	$8\frac{7}{16}$?
49·	8·	4·	cost	2·	4·	0·?
50·	12·	9·	cost		10·	$11\frac{3}{4}$ $\frac{5}{8}$q.

What is the rate per yard when

51· $5\frac{1}{16}$ yd. cost £1· 2·s. $9\frac{3}{8}$d.?
52· $11\frac{3}{16}$ yd. cost £1· 6·s. $1\frac{1}{4}$d.?
53· $14\frac{5}{16}$ yd. cost £2· 13·s. $8\frac{1}{16}$d.?
54· $21\frac{7}{16}$ yd. cost £12· 1·s. $2\frac{1}{16}$d.?
55· $13\frac{9}{16}$ yd. cost £3· 19·s. $8\frac{5}{32}$d.?

	yd.	qr.	na.		£.	s.	d.
56·	15·	2·	3·	cost	9·	1·	$0\frac{23}{32}$?
57·	14·	3·	1·	cost	2·	14·	$7\frac{29}{64}$?
58·	17·	3·	3·	cost	7·	19·	$11\frac{5}{16}$?
59·	31·	3·	2·	cost	8·	17·	$11\frac{5}{8}$?
60·	16·	2·	2·	cost	3·	13·	$5\frac{1}{8}$?

How much will 1· *cwt. cost*

		cwt.	qr.	lb.		£.	s.	d.
61·	If	7·	1·	12·	cost	4·	19·	$3\frac{6}{7}$?
62·	If	14·	2·	0·	cost	17·	17·	8·?
63·	If	11·	3·	7·	cost	12·	11·	$0\frac{3}{16}$?
64·	If	5·	1·	14·	cost	3·	0·	11·?

		cwt.	qr.	lb.		£.	s.	d.
65·	If	3·	1·	12·	cost	3·	5·	$8\frac{13}{14}$?
66·	If	4·	2·	20·	cost	9·	18·	$10\frac{1}{14}$?
67·	If	9·	3·	24·	cost	25·	16·	$5\frac{11}{14}$?
68·	If	2·	3·	6·	cost	10·	8·	$7\frac{33}{56}$?

The Compound Quantity may, sometimes, with advantage, be put into the form of a Vulgar or Decimal Fraction, previously to Multiplying or Dividing.

This method is of especial utility when the Multiplier or Divisor is a Fraction or a Mixed Number.

When the Multiplier or Divisor is large, the Compound Quantity may be turned into a Simple Quantity by Reducing it to the lowest denomination which it contains.

The result* will, of course, come out in units of this denomination, and will require to be converted, by Ascending Reduction, into the Compound form.

FIRST EXAMPLE WORKED OUT. What will $42\frac{3}{8}$ E. E. of cloth cost, at $5\frac{5}{8}$d. per yard?

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* In Division, the coincident result. (See pp. 218· and 224·)

Example completed.

First. By Vulgar Fractions.

$$\begin{aligned}
&42\tfrac{3}{5}\text{ E. E.} = \tfrac{213}{5}\text{ E. E.} = \tfrac{213}{5}\text{ of } 1\tfrac{1}{4}\text{ yd.} = \tfrac{213}{4}\text{ yd.};\\
&\text{Cost of } \tfrac{213}{4}\text{ yd. at } 5\tfrac{5}{8}\text{d.} = \tfrac{45}{8}\text{d.} = \tfrac{213}{4}\\
&\qquad = \tfrac{9585}{32}\text{d.} = 299\tfrac{17}{32}\text{d.};\\
&\qquad = \pounds 1{\cdot}\ 4{\cdot}\text{s. } 11\tfrac{1}{2}\text{d. } \tfrac{1}{8}\text{q.};
\end{aligned}$$

EXERCISES XLV. & XLVI.
EXERCISE LIX.
EXERCISE LXXXIV.

Second. By Decimal Fractions.

$$\begin{aligned}
&42\tfrac{3}{5}\text{ E. E.} = 42{\cdot}6 \text{ times } 1{\cdot}25\text{ yd.} = 53{\cdot}25\text{ yd.};\\
&\text{Cost of } 53{\cdot}25\text{ yd. at } 5\tfrac{5}{8}\text{d.} = 5{\cdot}625\text{d.} \times 53{\cdot}25\\
&\qquad = 299{\cdot}53125\text{d.};\\
&\qquad = \pounds 1{\cdot}\ 4{\cdot}\text{s. } 11\tfrac{1}{2}\text{d. } \tfrac{1}{8}\text{q.}
\end{aligned}$$

EXERCISES LXXII. & XLIII.
EXERCISE XLIII.
EXERCISES LXXXVI. & LXXXV.

SECOND EXAMPLE WORKED OUT. Multiply £4· 11·s. 6·d. by 9864·

Example completed.

First: By Reduction.

$$\begin{aligned}
\pounds 4{\cdot}\ 11{\cdot}\text{s. } 6{\cdot}\text{d.} \times 9864{\cdot} &= 1098{\cdot}\text{d.} \times 9864{\cdot}\\
&= 108306{\cdot}\text{d.}\\
&= \pounds 45127{\cdot}\ 16{\cdot}\text{s.}
\end{aligned}$$

Second: By Vulgar Fractions.

$$\begin{aligned}
\pounds 4{\cdot}\ 11{\cdot}\text{s. } 6{\cdot}\text{d.} \times 9864{\cdot} &= \pounds 4\tfrac{23}{40} \times 9864{\cdot}\\
&= \pounds\tfrac{183}{40} \times 9864{\cdot}\\
&= \pounds\tfrac{225639}{5} = \pounds 45127\tfrac{4}{5}\\
&= \pounds 45127{\cdot}\ 16{\cdot}\text{s.}
\end{aligned}$$

Third: By Decimal Fractions.

$$\begin{aligned}
\pounds 4{\cdot}\ 11{\cdot}\text{s. } 6{\cdot}\text{d.} \times 9864{\cdot} &= \pounds 4{\cdot}575 \times 9864{\cdot}\\
&= \pounds 45127{\cdot}8\\
&= \pounds 45127{\cdot}\ 16{\cdot}\text{s.}
\end{aligned}$$

PROPOSED DECIMAL COINAGE.

By far the greater part of the time, attention, and labour, expended on the explanations, tables, rules, and examples occupying pages 173· to 234· inclusive, has been exacted from us by the fact that our Concrete Units of Comparison do not proceed by regular Decimal gradations.

That is to say, each such unit has not, exactly, *one tenth* of the value of the next greater, and, consequently, tenfold the value of the next smaller.

If all our Measures, Weights, and Coins were thus decimally arranged, Concrete Quantities of every kind would be Added, Subtracted, Multiplied, and Divided as Simple Quantities are.

For example, if *ten* of any kind of Measure, Weight, or Coin were equivalent to *one* of the next higher Measure, Weight, or Coin, then, whenever, in Addition or Multiplication, we came to, for instance, 73· Concrete Units, of whatever kind, we should have merely to "*Set down three and carry seven.*" Whereas, at present, 73· *farthings* must be divided by 4·; 73· *pence*, by 12·; 73· *shillings*, by 20·; 73· *ounces*, by 16·; 73· *pounds*, by 28·; 73· *quarters*, Av., by 4·; 73· *cwts.* by 20·. In the same way, a Sum or Product of *inches* must be divided by 12·; *feet*, by 3· or by 6·; *lineal yards*, by 5½; *superficial yards*, by 30¼; and so on.

In Subtraction and Division we have to Multiply instead of to Divide by these different numbers.

In all, there are no fewer than one hundred and three numbers which it is necessary to remember and to employ in their turn in the computation of Compound Quantities.

Under a Decimal System all these one hundred and three numbers would be displaced by the constant Divisor or Multiplier *ten*. Concrete Reduction would then be comprised under the two rules for carrying (Page 26·); there would be no "Compound Rules;" and, consequently, the time now demanded for the study and practice of these might be applied to higher and more improving pursuits.

The Decimal System is established in France, the United States of America, and many other countries; and its introduction into England, for the coinage, at least, if not for Measures and Weights, is strenuously advocated by many of our most eminent men, both in the scientific and commercial worlds.

Several schemes of Decimal Coinage have been proposed. The most generally approved is that known as "The pound and mil scheme." By this arrangement, our Coins of account would, as now, be four, namely, the Pound, Florin, Cent, and Mil, each of the latter three being equivalent to *one tenth* of the next higher.

Instead of *twelve* Coins of Exchange, as now, we should have *ten*, namely, the sovereign, halfsovereign, crown, halfcrown, florin, shilling, sixpence, threepenny-piece, cent, and mil. The sixpence and threepenny-piece would receive new names; the former becoming a twenty-five-mil-piece, the latter a twelve-and-a-half-mil-piece. Hence, by the proposed Change of Coinage, the new coins introduced would be *two;* the present coins abolished would be *four;* and the present coins retained, *eight*.

TABLES OF DECIMAL COINAGE.

I. COINS OF ACCOUNT AND EXCHANGE.

10· mils = 1· cent.
10· cents = 1· florin.
10· florins = 1· pound.

II. REDUCTION TABLE OF COINS OF ACCOUNT.

£.		flor.		cent.		mil.
1·	=	10·	=	100·	=	1000·
·1	=	1·	=	10·	=	100·
·01	=	·1	=	1·	=	10·
·001	=	·01	=	·1	=	1·

III. COINS OF EXCHANGE ONLY.

	flor.	cent.	mil.
A half-sovereign	= 5·	= 50·	= 500·
A crown	= 2·5	= 25·	= 250·
A half-crown	= 1·25	= 12·5	= 125·
A shilling	= ·5	= 5·	= 50·
A sixpenny-piece	= ·25	= 2·5	= 25·
A threepenny-piece	= ·125	= 1·25	= 12·5

Unless, as some propose, a half-mil-piece were coined, the present threepenny-piece could not, under the Decimal System, be exchanged for its exact equivalent of smaller coins. It could, therefore, be only employed as an aliquot part of each of the larger coins.

Table II. shews how simply each of the four Decimal Coins of Account could be at once expressed in terms of any of the other three.

For example, any number of mils would be immediately expressed

In Cents, by removing Decimal Point one place to the left;
In Florins, by removing Decimal Point two places to the left;
In Pounds, by removing Decimal Point three places to the left.

So that, of any line of Digits representing mils, we might read

Digit in first place of Integers, as so many *mils;*
Digit in second place of Integers, „ *cents;*
Digit in third place of Integers, „ *florins;*
Digits beyond third place of Integers „ *pounds.*

$$\therefore \overset{\textit{mils.}}{41653\cdot}=\overset{\textit{cents.}}{4165\cdot 3}=\overset{\textit{flors.}}{416\cdot 53}=\overset{\textit{£.}}{41\cdot 653};=\text{also } \overset{\textit{£.}}{41\cdot}\ \overset{\textit{flo.}}{6\cdot}\ \overset{\textit{c.}}{5\cdot}\ \overset{\textit{m.}}{3\cdot}$$

There would, however, be no necessity to write the amount in four separate columns with the name of each at top; for, when expressed as pounds and decimals of £1·,

Digit in first place of Decimals would represent *florins;*
Digit in second place of Decimals ,, ,, *cents;*
Digit in third place of Decimals ,, ,, *mils.*

The Decimal System of Computation has already obtained preference in Public Offices and in many of our leading commercial establishments;* although, with our present coinage, the double labour is necessary of reducing the amounts to Decimals before computation, and of restoring the results to the non-decimal form after it. (See Page 234·.) Moreover, our actual coinage presents these three impediments to this practice of Computation by Decimals:

I. Only 319·, out of the 959· amounts less than £1·, can be accurately expressed in Decimals of £1· (See Page 197·)

II. Of these 319· amounts, decimally expressed, 160· extend to six places of Decimals, 80· to five places, and 40· to four places.

III. No Digit beyond first place of Decimals of £1· represents an exact coin or number of coins.

But, with the Decimal Coinage:

I. Each of the 999· amounts less than £1· would be an exact Decimal of £1·

II. No such Decimal would extend beyond three places.

III. Each Digit would represent an exact coin or number of coins.

The opponents of the proposed Decimal Coinage admit its superiority for account-keeping, but they insist on two objections; namely,

I. That it would be less convenient than our present system for mental calculations.

II. That the several coin-values would have fewer aliquot parts than at present, and that, consequently, the exact division of them would be much more restricted than it is now.

The first of these difficulties would probably disappear as the new system of coinage became familiar.

The second objection does really hit the weak point in the proposed system.

Its force will be appreciated after comparing the Tables of Aliquot Parts of Decimal Coinage, given on next page, with those of the Aliquot Parts of Current Money, on page 183·

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* See Evidence of Mr. Finlaison, in Final Report of Decimal Coinage Commissioners, p. 25·

TABLES OF ALIQUOT PARTS OF DECIMAL COINAGE.

I. Of £1·

mils.

500· = $\frac{1}{2}$
250· = $\frac{1}{4}$
200· = $\frac{1}{5}$
125· = $\frac{1}{8}$
100· = $\frac{1}{10}$
62·5 = $\frac{1}{16}$
50· = $\frac{1}{20}$
40· = $\frac{1}{25}$
25· = $\frac{1}{40}$
20· = $\frac{1}{50}$
12·5 = $\frac{1}{80}$
10· = $\frac{1}{100}$
8· = $\frac{1}{125}$
5· = $\frac{1}{200}$
4· = $\frac{1}{250}$
2·5 = $\frac{1}{400}$
2· = $\frac{1}{500}$
1· = $\frac{1}{1000}$
·5 = $\frac{1}{2000}$

II· Of a hf.-sov.

mils.

250· = $\frac{1}{2}$
125· = $\frac{1}{4}$
100· = $\frac{1}{5}$
62·5 = $\frac{1}{8}$
50· = $\frac{1}{10}$
25· = $\frac{1}{20}$
20· = $\frac{1}{25}$
12·5 = $\frac{1}{40}$
10· = $\frac{1}{50}$
5· = $\frac{1}{100}$
2·5 = $\frac{1}{200}$
2· = $\frac{1}{250}$
1· = $\frac{1}{500}$
·5 = $\frac{1}{1000}$

III. Of a crown.

mils.

125· = $\frac{1}{2}$
62·5 = $\frac{1}{4}$
50· = $\frac{1}{5}$
25· = $\frac{1}{10}$
12·5 = $\frac{1}{20}$
10· = $\frac{1}{25}$
5· = $\frac{1}{50}$
2·5 = $\frac{1}{100}$
2· = $\frac{1}{125}$
1· = $\frac{1}{250}$
·5 = $\frac{1}{500}$

IV. Of a half-cr.

mils.

62·5 = $\frac{1}{2}$
25· = $\frac{1}{5}$
12·5 = $\frac{1}{10}$
5· = $\frac{1}{25}$
2·5 = $\frac{1}{50}$
1· = $\frac{1}{125}$
·5 = $\frac{1}{250}$

V. Of a florin.

mils.

50· = $\frac{1}{2}$
25· = $\frac{1}{4}$
20· = $\frac{1}{5}$
12·5 = $\frac{1}{8}$
10· = $\frac{1}{10}$
5· = $\frac{1}{20}$
2·5 = $\frac{1}{40}$
2· = $\frac{1}{50}$
1· = $\frac{1}{100}$
·5 = $\frac{1}{200}$

VI. Of a shilng.

mils.

25· = $\frac{1}{2}$
12·5 = $\frac{1}{4}$
10· = $\frac{1}{5}$
5· = $\frac{1}{10}$
2·5 = $\frac{1}{20}$
2· = $\frac{1}{25}$
1· = $\frac{1}{50}$
·5 = $\frac{1}{100}$

VII. Of a half-shilling, or 25· mil-piece; (present 6·d.)

mils.

12·5 = $\frac{1}{2}$
5· = $\frac{1}{5}$
2·5 = $\frac{1}{10}$
1· = $\frac{1}{25}$
·5 = $\frac{1}{50}$

VIII. Of a qr.-sh., or 12½ mil-piece; (present 3·d.)

mils.

2·5 = $\frac{1}{5}$
·5 = $\frac{1}{25}$

IX. Of a cent.

mils.

5· = $\frac{1}{2}$
2·5 = $\frac{1}{4}$
2· = $\frac{1}{5}$
1· = $\frac{1}{10}$
·5 = $\frac{1}{20}$

The whole system of Decimal Coinage would have, then, 81· Aliquot Parts. Our present Coinage has 122·

If an entire article cost £1·, there are 27· Aliquot Parts of that article, for each of which our actual Coinage will exhibit the exact value. (See Page 183·)

By the Decimal Coinage, only 19· such Aliquot Parts could be exactly paid for.

This would be a serious drawback from the merits of the Decimal Coinage, were it either necessary or common for an entire article to cost an exact coin-value. But this is not the case, the cost of a measure or weight being quite as frequently a *broken* amount as an exact coin-value. And when the commodity is likely to be sold in Aliquot Parts, a price is fixed which is convenient for division into those particular Aliquot Parts, so that the coin-value of each may be exact.

The retail price of tea, for instance, rises and falls by fourpences per lb., because a difference of 4·d. on *a pound* is exactly one farthing on *an ounce*.

Hence, the Decimal Coinage, superior in all other respects, would be as suitable as the present for business purposes. 48· mils, for example, would furnish the same number of Aliquot Parts as 48· farthings (or 1·s.); and 24· mils would be similarly divisible to 6·d.

Again, when the price of the whole, in farthings, is not a multiple of 2·, the price of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, &c., can only be paid to the nearest *farthing;* but, under the Decimal System, it could be computed to the nearest *mil*, and the loss would be less, for any fraction of a mil is smaller than the same fraction of a farthing.

REDUCTION OF PRESENT COINAGE TO DECIMAL COINAGE.

Supposing the Decimal Currency to be adopted, some time would necessarily elapse before the names and ideas of the present coins would disappear. During this interval, simple methods of Reducing the old coinage-value into the new would be in request.

To find the equivalent in Decimal Coins for any sum of present money.

This may, of course, be done by EXERCISE LXXXVIII. page 196·, but the following method is easier for mental calculation.

First. Express the amount in mils. This may be exactly done, down to threepence, by Tables II. and III., page 236·.

Call each £1·	1000· mils.
For each remaining Florin, add	100· mils.
If one Shilling remain, add	50· mils.
If Sixpence then remain, add	25· mils.
If Threepence then remain, add	12·5 mils.
If *not more* than 6· Farthings remain, add for each Farthing	1· mil.
If *more* than 6· Farthings remain, add 12·5 mils (3·d.), and then Subtract 1· mil for every Farthing said Remr. falls short of 3·d.	

Second. The amount being expressed in mils, may be read in £. flors. cents and mils, as directed on Page 236·.

We lose $\frac{1}{25}$ of a farthing by *adding* it as a mil; and gain the same quantity by *subtracting* it as a mil; but, as never more than 6· farthings need be so computed, the greatest loss arising would be $\frac{6}{25}$ (or less than $\frac{1}{4}$) of a farthing, a smaller sacrifice than is constantly made in small transactions with our present system of Coinage.

With certain odd sums of pence and farthings, the Rule just given may be materially contracted, thus:

For any Remainder

$$\left.\begin{array}{l}\text{Above } 1\tfrac{1}{2}\text{d. and under 3·d., add 12·5; (3·d.)}\\ \text{Above } 4\tfrac{1}{2}\text{d. and under 6·d., add 25·; (6·d.)}\\ \text{Above } 10\tfrac{1}{2}\text{d. and under 1·s., add 50.; (1·s.)}\\ \text{Above } 22\tfrac{1}{2}\text{d. and under 2·s., add 100·; (2·s.)}\end{array}\right\}\begin{array}{c}\text{And, then,}\\ \text{Subtract 1·}\textit{mil}\text{ for}\\ \text{every extra}\\ \text{farthing added.}\end{array}$$

Examples worked out. Find, in the Decimal Coinage, the exact or nearest equivalent to each of the following sums.

$$\begin{array}{rrr|rr|rr|rrr|rrrr} & \text{s.} & \text{d.} & & \text{d.} & & \text{d.} & & \text{s.} & \text{d.} & & \text{£.} & \text{s.} & \text{d.}\\ 1\cdot & 18\cdot & 3\cdot & 3\cdot & 1\tfrac{3}{4} & 5\cdot & 2\tfrac{3}{4} & 7\cdot & 1\cdot & 10\tfrac{3}{4} & 9\cdot & 21\cdot & 13\cdot & 7\tfrac{1}{4}\\ 2\cdot & 19\cdot & 9\cdot & 4\cdot & 5\tfrac{1}{4} & 6\cdot & 11\tfrac{1}{2} & 8\cdot & 7\cdot & 11\tfrac{1}{4} & 10\cdot & 54\cdot & 6\cdot & 4\cdot\end{array}$$

$$\begin{array}{llll|llll} & & \text{I.} & & & & \text{II.} & \\ \text{s.}\ \ \text{d.} & & \text{flo.} & \text{mils.} & \text{s.}\ \ \text{d.} & & \text{flo.} & \text{mils.}\\ 18\cdot\ 0\cdot & = & 9\cdot = & 900\cdot & 18\cdot\ 0\cdot & \text{or} & 9\cdot = & 900\cdot\\ \ \ \ \ \ \ 3\cdot & = & & 12\cdot5 & \ \ 1\cdot\ 9\cdot & = & 50\cdot+25\cdot+12\cdot5 = & 87\cdot5\\ \therefore\ \underline{\underline{18\cdot\ 3\cdot}} & = & & \underline{\underline{912\cdot5}} & \therefore\ \underline{\underline{19\cdot\ 9\cdot}} & = & & \underline{\underline{987\cdot5}}\end{array}$$

III.

$$\overset{\text{d.}}{1\tfrac{3}{4}} = \overset{\text{d.}}{3\cdot} - \overset{\text{far.}}{5\cdot} = \overset{\text{mils.}}{12\cdot5} - \overset{\text{mils.}}{5\cdot}\text{ nearly,} = \overset{\text{mils.}}{7\cdot5}\text{ nearly.}$$

Here, computing 1¾d. as 3·d., is adding 5· farthings: hence, we afterwards subtract 5· mils.

IV.

$$\overset{\text{d.}}{5\tfrac{1}{4}}\text{ being }\overset{\text{far.}}{3\cdot}\text{ less than }\overset{\text{d.}}{6\cdot}\text{, and }\overset{\text{d.}}{6\cdot}\text{ being } = \overset{\text{mils.}}{25\cdot},$$

$$\therefore\ 5\tfrac{1}{4} = (25\cdot - 3\cdot)\text{ mils, nearly,} = \overset{\text{mils.}}{22\cdot}\ (\text{or }\overset{\text{c.}}{2\cdot}\ \overset{\text{m.}}{2\cdot})\text{ nearly.}$$

V.

$$\overset{\text{d.}}{2\tfrac{3}{4}} = \overset{\text{mils.}}{12\cdot5} - \overset{\text{mil.}}{1\cdot}\text{, nearly,} = \overset{\text{mils.}}{11\cdot5}\ (\text{or }\overset{\text{c.}}{1\cdot}\ \overset{\text{m.}}{1\tfrac{1}{2}})\text{ nearly.}$$

Because, 2¾d. = 1· far. less than 3·d.

VI.

$$\overset{\text{d.}}{11\tfrac{1}{2}} = \overset{\text{mils.}}{50\cdot} - \overset{\text{mils.}}{2\cdot}\text{, nearly,} = \overset{\text{mils.}}{48\cdot}\ (\text{or }\overset{\text{c.}}{4\cdot}\ \overset{\text{m.}}{8\cdot})\text{ nearly.}$$

Because 11½d. ≡ 2· fars. less than 1·s.

VII.

$$\overset{\text{s.}}{1\cdot}\ \overset{\text{d.}}{10\tfrac{3}{4}} = \overset{\text{s.}}{2\cdot} - \overset{\text{fars.}}{5\cdot} = (100\cdot - 5\cdot)\text{m. nearly} = 95\cdot\text{m. (or }\overset{\text{c.}}{9\cdot}\ \overset{\text{m.}}{5\cdot})\text{ nearly.}$$

VIII.

$$\overset{\text{s.}}{7\cdot}\ \overset{\text{d.}}{11\tfrac{1}{4}} = \overset{\text{flo.}}{4\cdot} - \overset{\text{fars.}}{3\cdot} = \overset{\text{mils.}}{400\cdot} - \overset{\text{mils.}}{3\cdot}\text{, nearly} = \overset{\text{m.}}{397\cdot}\ (\text{or }\overset{\text{f.}}{3}\cdot\overset{\text{c.}}{9}\cdot\overset{\text{m.}}{7\cdot})\text{ nearly.}$$

IX.

	£.	s.	d.		mils.
	21·	0·	0·	=	21000.
		12·	0·	=	600·
		1·	0·	=	50·
			6·	=	25·
			1¼	=	nearly 5·
∴	21·	13·	7¼	=	21680·

X.

	£.	s.	d.		mils.
	54·	0·	0·	=	54000·
		6·	0·	=	300·
			3·	=	12·5
			1·	=	nearly 4·
∴	54·	6·	4·	=	54316·5

Note, that any number of shillings are equivalent to *half as many hundreds* of mils.

EXERCISE CII.

EXAMPLES IN NUMERATION AND REDUCTION OF DECIMAL COINAGE.

A. *Read each of the following numbers of Mils:* I. *In Cents.* II. *In Florins.* III. *In Pounds.* IV. *In as many of the four names as possible.*

	mils.		mils.		mils.		mils.
1·	84316·	6·	907·	11·	843·	16·	543106·
2·	231842·	7·	3030·	12·	17008·	17·	200453·
3·	6143·	8·	16104·	13·	10507·	18·	71054·
4·	9004·	9·	18000·	14·	40058·	19·	130105·
5·	16010·	10·	31526·	15·	604070·	20·	7060080·

B. *Read the following:* I. *In Mils.* II. *In Cents.* III. *In Florins.* IV. *In Pounds.*

1· £4· 8·f. 3·c. 7·m.
2· £17· 3·f. 9·m.
3· £18· 8·c. 7·m.
4· £345· 5·c.
5· £74· 20·m.
6· 3·f. 8·c.
7· 9·f. 6·m.
8· 98·c. 7·m.
9· 7·f. 5·m.
10· 54·f. 8·m.
11· £9· 7·c.
12· £400· 4·m.
13· £301· 5½ f.
14· 84·f. 4½ c.
15· £16· 3·f. 8¾ c.

C. *Reduce the following to the Decimal Coinage. When the result is not exactly true, state the amount of error.*

	s.	d.		s.	d.		s.	d.		s.	d.
1·	17·	5·	6·	13·	2·	11·	67·	10½	16·	45·	5¾
2·	18·	4¾	7·	19·	5¼	12·	39·	9·	17·	58·	6¾
3·	11·	3½	8·	43·	8½	13·	47·	2¾	18·	12·	1¾
4·	24·	7¾	9·	16·	5¾	14·	25·	9¾	19·	14·	4½
5·	3·	8·	10·	28·	8½	15·	7·	11·	20·	17·	10½

To Reduce Decimal to present Coinage.

First. Express the amount in Pounds and Decimals of £1·

Second. Reduce the Decimals to shillings, pence, and farthings. (Exercise lxxxv. Page 191·)

Examples worked out. Find the value, in Actual Coinage, of £84· 3·f. 4·c. 9·m.; and of £3· 7·c. 5·m.

I.

$$\begin{array}{ccccccccccc} \text{£.} & \text{f.} & \text{c.} & \text{m.} & & \text{£.} & & \text{£.} & \text{s.} & \text{d.} & \text{q.} \\ 84\cdot & 3\cdot & 4\cdot & 9\cdot & = & 84\cdot349 & = & 84\cdot & 6\cdot & 11\frac{3}{4}, & \frac{1}{25} \end{array}$$

II.

$$\begin{array}{cccccccccc} \text{£.} & \text{f.} & \text{c.} & \text{m.} & & \text{£.} & & \text{£.} & \text{s.} & \text{d.} \\ 3\cdot & 0\cdot & 7\cdot & 5\cdot & = & 3\cdot075 & = & 3\cdot & 1\cdot & 6\cdot \end{array}$$

EXERCISE CIII.

Find the value, in Actual Coinage,

1· Of £743· 2·f. 5·c. 3·m.
2· Of £910· 4·c. 7½ m.
3· Of £54· 8·f. 6·m.
4· Of £36· 9·f. 8·c. 8·m.
5· Of £954· 6·f. 4·c.
6· Of £14· 5·m.; and 98·m.
7· Of 16·f. 43·m.; and 31462·m.
8· Of 84317½ c.; and 9463·m.
9· Of £43· 9¾ c.; and 415·c.
10· Of £17· 8½ f.; and 1742·f.

Decimal Coinage would be added by Simple Addition.

Example worked out. Find the Sum of 43765·m., 91·f. 54·m., £3· 6·m., £17· 543·m., £520· 6·f. 4·c. 7·m.

This may be done in three ways.

Example completed.

I. In Mils.	II. In Pounds.	III. In the four Denoms.
m.	£.	£. f. c. m.
43765·	43·765	43· 7· 6· 5·
9154·	9·154	9· 1· 5· 4·
3006·	3·006	3· 0· 0· 6·
17543·	17·543	17· 5· 4· 3·
520647·	520·647	520· 6· 4· 7·
594115·	594·115	594· 1· 1· 5·

The first two of these processes are purely Simple Additions, because "The units added are of the same kind and value. (See page 36·)

The first may be employed by a person who knows nothing of Arithmetic beyond Simple Addition of Integers.

The second, which is the best method, demands only an acquaintance with the Notation and Numeration of Decimal Fractions. (See pages 16· to 21·)

The third is, strictly speaking, a Compound Addition, because the units added have different names. (See page 36·) But, the carrying being invariably *one for ten*, this also is, virtually, a Simple Decimal Addition. (Prin. iv.)

Decimal Coinage would be Subtracted by Simple Subtraction.

Example worked out. What would remain after expending £4· 6·f. 3·c. 9·m. out of £17· 5·c. 8·m.?

Example completed.

I. In Mils.		II. In Pounds.		III In the four denoms.	
	m.		£.		£. f. c. m.
From	17058·	*From*	17·058	*From*	17· 0· 5· 8·
Take	4639·	*Take*	4·639	*Take*	4· 6· 3· 9·
Diff.	12419·	*Rem.*	12·419	*Rem.*	12· 4· 1· 9·

Decimal Coinage would be Multiplied by Simple Multiplication.

Example worked out. What would 58· articles cost at 8·f. 4·c. 3·m. each?

Example completed.

I. In Mils.			II. In Pounds.		III. In the four Denoms.
m.			£.		£. f. c. m.
843·	= cost of 1.	=	·843	=	0· 8· 4· 3·
58·			58·		58·
6744·	= cost of 8·	=	£6·744	=	6· 7· 4· 4·
42150·	= cost of 50·	=	42·15	=	42· 1· 5· 0·
48894·	= cost of 58·	=	48·894	=	48· 8· 9· 4·

The Multiplication of the Decimal Coinage would be much more concise than Compound Multiplication of £. s. d. This is evident from the Example just completed, where we have but *two* Multiplications and *one* Addition; whereas, to Multiply £. s. d. by 58· requires *three* Multiplications and an Addition (or Subtraction). (Exercise xciv., p. 217·) Moreover, these Multiplications, being effected by Factors, employ different Multiplicands, but the Decimal Multiplication employs only one.

In order to arrive at an accurate estimate of the relative merits of the two systems as to economy of time and figures in Multiplication, we will compare the Decimal with the Compound process, when Multiplier is:

i. *Any Power of* 10·, as: 100·, 1000·, 100,000· &c. (Page 53·)

Decimal Coinage is Multiplied by any Power of ten in *one line*, by merely removing digits of Multiplicand to the left. (Prin. viii.)

But, £. s. d. must be Multiplied continually by as many tens as Multiplier has ciphers.

D. C. *saves as many lines, all but one, as Multiplier has ciphers.*

II. *Any Multiple of a power of* 10·, as: 50·, 700·, 432000·

Decimal Coinage is Multiplied in the same number of lines (+ one line of Addition) as there are Significant Figures in Multiplier.

The £. s. d. Multiplication occupies all those lines, together with as many (all but one) for the Multiplications by 10·, as the Multiplier has places.

Dec. Coin. × 432000· in four lines.
£. s. d. × 432000· in nine lines.

D. C. *saves as many lines, all but one, as Multiplier has places.*

III. *Any other Integer, greater than* 20· (and not being the Product of two Factors each less than 13·), as: 3528·, 74396· &c.

Multiplication of D. C. by 74396· occupies *six* lines.
Multiplication of £. s. d. by 74396· occupies *ten* lines.

D. C. *saves as many lines, all but one, as Multiplier has places.*

With D. C., too, a saving of figures may frequently be effected by employing "The Back Figure Contraction," (Page 74·,) which cannot be applied to Multiplication of £. s. d.

Thus, D. C. is Multiplied in one line by 13·, 14· &c., to 19·; by 101·, 102·, &c. to 119·, and similar Numbers.

IV. *Any Power of* ·1, as: ·01, ·001, ·000001, &c. (Page 53·)

The result of Multiplying by $\frac{1}{10} \doteq$ the result of Dividing by 10· (PRINCIPLE XVII.) Hence, To Multiply £. s. d. by ·1, we Divide by 10·; and to Multiply £. s. d. by ·0001, (that is, by $\frac{1}{10000}$,) we Divide by 10000·, which, being effected by the continued Division by 10·, 10·, 10·, and 10·, occupies *four* lines.

But, D. C. is Multiplied by any Power of ·1 in *one* line, by merely removing Digits of Multiplicand to the right. (PRIN. X., page 60·)

D. C. *saves as many lines, all but one, as Multiplier has places.*

£. s. d. may be Divided by any Power of 10·, as Simple Integers and Decimals are, by removing D. P. of £. as many places to the left as there are ciphers in Divisor, (this is usually termed "cutting off" so many figures from the right hand of Dividend;) the £. to right of D. P. being then reduced to shillings, will be similarly Divided; then the pence; then the farthings. The Decimal process will, however, still be the more concise of the two.

V. *Any Decimal Fraction*, as: ·045, ·47632, &c.

Here, D. C. is Multiplied, in succession, by each Significant Figure of the Multiplier, and the several Products are then added. (Page 71·)

But, £. s. d. must be first *Divided* continually by as many tens as the Multiplier has places. Then follow as many Multiplications

as Multiplier has Significant Figures; and, lastly, an Addition of the several Products.

The Multiplication of D. C. by ·47632 occupies *six* lines.

The Multiplication of £. s. d. by ·47632 occupies *eleven* lines.

D. C. *saves as many lines as Multiplier has places.*

FIRST SET OF EXAMPLES WORKED OUT: ILLUSTRATING THE FIRST THREE OF THE FOREGOING CASES. Find the value of 10000·, of 70000·, and of 74396· articles at £9· 5·f. 7·c. 4·m. each; and also at £9· 11·s. 5¾d. each.

Examples completed.

I.

£.	
9·574	= c. of 1·
10000·	
95740·	= c. of 10000·*

II.

£.	
9·574	= c. of 1·
70000·	
670180·	= c. of 70000·†

III.

£.	
9·574	= c. of 1·
74396·	
57·444	= c. of 6·
861·66	= c. of 90·
2872·2	= c. of 300·
38296·	= c. of 4000·
670180·	= c. of 70000·
712267·304	= c. of 74396·‡

IV.

£.	s.	d.	
9·	11·	5¾	= c. of 1·
		10·	
95·	14·	9½	= c. of 10·
		10·	
957·	7·	11·	= c. of 100·
		10·	
9573·	19·	2·	= c. of 1000·
		10·	
95739·	11·	8·	= c. of 10000·*
		7·	
670177·	1·	8·	= c. of 70000·†
38295·	16·	8·	= c. of 4000·
2872·	3·	9·	= c. of 300·
861·	13·	1½	= c. of 90·
57·	8·	10½	= c. of 6·
712264·	4·	1·	= c. of 74396·‡

* D. C. *one* line: £. s. d. *four* lines. † D. C. *one* line: £. s. d. *five* lines.
‡ D. C. *six* lines: £. s. d. *ten* lines.

SECOND SET OF EXAMPLES WORKED OUT: ILLUSTRATING FOURTH AND FIFTH OF THE FOREGOING CASES.

I. The joint stock of a Company being divided into 10000· shares, and expenses being incurred to the amount of £16979· 3·s. 4·d., it is required to find how much of this sum should be paid by the holder of 2437· shares.

II. Find the same person's liability, supposing the debt to be £16979· 1·f. 6·c.

The Debt being distributed into the same number of shares as the Stock, namely, 10000·, the shareholder in question must pay 2437· of these, or ·2437 of the Debt = Debt × ·2437. (PRIN. IX.)

Examples completed.

I.	£.	s.	d.		
10·)	16979·	3·	4·	=Whole Debt.	
10·)	1697·	18·	4·	=·1	of D.
10·)	169·	15·	10·	=·01	of D.
10·)	16·	19·	7·	=·001	of D.
	1·	13·	11½	=·0001	of D.
			7·		
	11·	17·	8½	=·0007	of D.
	50·	18·	9·	=·003	of D.
	679·	3·	4·	=·04	of D.
	3395·	16·	8·	=·2	of D.
	4137·	16·	5½	=·2437	of D.

II.	£.		sh.
	16979·16	=Whole D.	
	·2437		
	11·885412	=	7·
	50·93748	=	30·
	679·1664	=	400·
	3395·832	=	2000·
	£4137·821292	=	2437·
=	£4137· 8·f. 2·c. 1·292m.		

If the second method of finding ·2437 of £16979. 3·s. 4·d. (see page 226·) had been employed, the work would have occupied *twelve* lines. Try it.

Division of Decimal Coinage would be Simple Division.

This is so evident, that no "Example worked out" is requisite.

If Divisor be, or contain, a Decimal Fraction, then Division of £. s. d. will require as many more lines than Division of D. C. as Divisor has Decimal Places. For, (besides the Division,) £. s. d. must be *Multiplied* continually by as many tens as Divisor has Decimal places. For example,

£41· 3·s. 4·d. ÷ ·9857 ≐ £41· 3·s. 4·d. × $\frac{10000}{9857}$ PRIN. XVII.

= £41· 3·s. 4·d. × 10· × 10· × 10· × 10· ÷ 9857· PRIN. XV.

Also, £41· 3·s. 4·d. ÷ 9·857 ≐ £41· 3·s. 4·d. × $\frac{1000}{9857}$ PRIN. XVII.

= £41· 3·s. 4·d. × 10· × 10· × 10· ÷ 9857· PRIN. XV.

When Divisor contains no Decimal Fraction, the two processes are about equal in length.

EXERCISE CIV.

MISCELLANEOUS EXAMPLES.

Every line of the work to be fully explained, in writing.

All £. s. d. results are to be given also in £. f. c. m.

1· Bought 58· gal. of wine at 13·s. per gal., and sold 40· gal. at such a rate as to clear the remainder. What was the selling price, per quart?

2· How much a day is £430· 3¾d. a year?

3· A has £84· 15·s. 9½d., and B has 7000· farthings. How much must A give to B so that they may have equal amounts?

Less + ½ Diff. = Greater — ½ Diff.

4· Out of an income of £500· a year, a man saves at the rate of 11·s. 4½d. a day. What are his weekly expenses?

5· How much does he spend in 31· days?

6· Sold 160· qrs. of wheat at 53·s. 9·d., and gained thereby £100·. What did it cost per qr.? *Three lines.*

Cost Price = Selling Price — Gain.

7· Buying cloth at 8·s. 3½d. per yard, how much must I sell, at 11·s. 6·d. per yard, to gain £15· 11·s.?

8· What is the content of a cask of cider, which, being full, costs £2· 12·s. at 4·d. per quart, the cask being worth 10·s.?

9· Supposing a furlong of silk thread to weigh 2· oz. Av., what would a similar thread weigh of sufficient length to encircle the globe. (See Table, Geographical Measure, p. 177·)

10· Find the value of 12· score and 10· lb. of pork at 4¾d. per lb.; also at 3¼d., at 5¼d., at 14· mils, and at 15· mils.

11· If 1· oz. Troy cost £4· 12·s. 6·d., what weight is worth £44· 12·s. 6·d.?

12· If one shilling weigh 3· dwts. 15$\frac{4}{11}$ grs., what will 7· oz. 5· dwts. 14$\frac{6}{11}$ gr. of silver be worth?

13· When 144· yds. of cloth cost £150·, how much is gained or lost per yard by selling it at a guinea a yard?

14· £10000· were bequeathed to a family consisting of four brothers and two sisters; and each of the latter was to receive £1000· less than each of the former. What were their respective shares?

15· A and B together have £18· 13·s. 4·d., but A has twice as much as B. What amount has each?

16· Divide £14· 3·s. 11·d. between two persons, so that the first may have 5·s. 8·d. more than the second.

17· If a sum of money be so divided that, for every £5· received by A, B receives £6·, and for every £5· received by B, C gets £4·, what is the sum divided, C's share being £103· 3·s.?

18· Divide £3421· 7·s. 3·d. among 3· persons, so that A shall have £284· 7·s. 2·d. more than B, and B £364· 9·s. 1·d. more than C.

19· Divide £143· 16·s. among 7· men, 5· women, and 4· boys, giving each man as much as 3· women, and each woman as much as 2· boys? (See page 162·)

20· Distribute £640· 7·s. 4·d. among 3· persons, so that B shall have £95· 13·s. 7·d. more than A, and £21· 5·s. 8·d. less than C.

21· In a small farm, the arable portion exceeds the pasture by 7· ac. 3· r. 15· p., and the pasture exceeds the orchard ground by 2· ac. 2· r. 28· p.; find the extent of each, that of the three being 23· ac. 1· r. 17· po.

22· Bought wine at 8·s. 8·d. per gal.; but some being lost by leakage, the remainder was sold at 2·s. 10·d. per quart, and the outlay exactly returned. What portion was lost?

Cost Price = $\frac{13}{17}$ *of Selling Price.*

∴ *Quantity which cost* 13·*d. was sold for* 17·*d.*

23· Bought 946· yds. of cloth at 3·s. 4¼d. per yd., and sold it so as to gain £51· 4·s. 10·d. on the whole. What was the Selling Price per yd.?

24· The owner of $\frac{3}{11}$ of 8¼ times $\frac{2}{9}$ of a ship, estimated as worth £4158·, sold a portion worth, at that rate, £346· 10·s. What Fraction did he still retain?

25· How many £. in 1000· dollars, each 5·s. 3·d.?

26· In 896328· half-guineas how many moidores?

27· In 784·67 Ells E. how many yards? *Two lines.*

28· In 748·75 Ells Fl. how many Ells E.? *One line.*

29· In 746944·55 yds. how many Ells E.? *Multiply by* ·8

30· What is the Diff. between 564· moidores and 745· guineas?

31· How many crowns must be added to 4263· half-guineas to make up half as many moidores? *Multiply by* ·6. *Why?*

32· What is the Diff. between £64· 13·s. 0·d. and 94368· mils.?
Work and Prove this Example in two ways:
I. *Reduce the* D. C. *to £. s. d. and Subtract.*
II. *Reduce £. s. d. to* D. C. *and Subtract.*
III. *Reduce First Remainder to* D. C. *and compare result with Second Remainder.*
IV. *Reduce Second Remainder to £. s. d. and compare with First Remainder.*

33· At $4\frac{1}{2}$d. per lb. how much beef could be bought for 13·s. $8\frac{1}{4}$d.?

34· How many lots each consisting of a half-sovereign, a crown, a half-crown, a florin, a groat, and a halfpenny, can be made from £13· 18·s. 3·d.?

35· Express 8436·25 guineas in £. s. d.; also in sixpences, in groats, in threepences, and in the Decimal Coinage.

36· Change £46· 8·f. 9·c. 7·m. to £. s. d.

37· Change £145· 7·f. 25·m. to pence.
Multiply £145·725 *by* 240·. (EXERCISES LXXXIII. and LXXXV.)

38· What weight Troy is equivalent to 32·lb. 14·oz. Avoir.?
1·*lb. Avoir.* = $\frac{7000}{5760}$ *or* $\frac{175}{144}$ *of* 1·*lb. Troy.* (Page 180·)
∴ 32·*lb.* 14·*oz.* (*or* $32\frac{7}{8}$ *lb.*) *Avoir.* = $32\frac{7}{8}$ *times* $\frac{175}{144}$ of 1·*lb. Troy.*

39· Change 1·cwt. 3·qrs. 9·lb. into Troy Weight.

40· What would 3·lb. 5·oz. 17·dwt. of gold weigh by Apothecaries' weight?

41· By how much are $7\frac{3}{4}$lb. of flour heavier than $7\frac{3}{4}$lb. of silver?

42· How much rhubarb* would just counterpoise two newly-coined shillings?

43· If shillings (of the present weight) were made of lead, how many would weigh 1·lb.?

44· Which is the heavier, 1·lb. of gold or 1·lb. of paper? How much silver must be added to the lighter to restore equilibrium? How much sugar would do the same? And how much of any drug?*

45· Add together $\frac{3}{5}$ of $\frac{8}{9}$ of a guinea, $\frac{6}{7}$ of £1·, and $3\frac{5}{11}$ of 14·s. 8·d.

46· If $\frac{1}{8}$ of a ship be worth £36· 10·s. $7\frac{1}{2}$d., what fraction will cost £125· 5·s.?
First. Find value of the whole ship.
Second. Find what part £125· 5·*s. are of that sum.* (P. 125· Ex. VII.) *Prove result.*

47· A father bequeathed £5000· to his six sons, the eldest's share to be five-fourths of that of each of the younger. What was the share of each?

48· £173· 5·s. are divided among 5· men and some women, each of the latter receiving $\frac{1}{3}$ of a man's share. How many women were there, each man receiving £24· 15·s.?

49· If the property in a parish be assessed at £43510·, how much in the pound must be paid to raise a poor rate amounting to £906· 9·s. 2·d.?

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* By Apothecaries' Weight.

0· A man gave £12· 18·s. 8·d. for a certain number of sheep; but if he had paid 2·s. 4·d. a-head more for them, they would have cost him £13· 17·s. 4·d. How many were there?

1· If a man earn £56· per annum, and expend at the rate of 25·s. per week, what will be the state of his affairs at the year's end?

2· A careful boy laid by 11·s. in a certain number of weeks; but he would have accumulated the same amount in 6· weeks, if he could have saved 5½d. per week more than he did. What did he save per week?

3· How long was he in saving the 11·s.?

4· What is the income tax on £254· 10·s. at 14·d. in the pound?

5· If a bankrupt's debts be £3465· 7·s. 6·d., and the clear proceeds of his estate £577· 11·s. 3·d., how much will he be able to pay in the pound?

Find what part £577· 11·s. 3·d. are of £3465· 7·s. 6·d. He will be able to pay only that fraction of each £1· that he owes.

6· How much will he pay *per centum* of his debts, that is to say, how much for every £100· of them?

7· How many times over can a half-sovereign, a crown, a half-crown, a florin, a groat, and a farthing, be paid out of £19· 17·s. 1·d.?

8· A father purchased for each of his children a cap costing 3·s. 4·d., a pair of gloves for 10½d., a neck-kerchief for 1·s. 3·d., and a pair of shoes for 6·s. 6·d. He afterwards deposited in the Savings' Bank 10·s. for each of them, and then found that he had only 14·s. 1½d. remaining out of £4·. How many children had he?

59· What is the value of 1·lb. of tea which is worth 50· oranges, 70· oranges being worth 42· lemons, at 18·d. per doz.?

60· Find the value of ·875 of £7· 7·s. 6·d.

Multiply £7·735 by ·875. Or, Multiply £7· 7·s. 6·d. by ⅞. Or, Change the unit, and Multiply £·875 by 7⅜ or 7·375. Or, Find the value of £·875 and Multiply that value by 7⅜ or 7·375. Or, Subtract ⅛ of itself from £7· 7·s. 6·d.

61· What is the value of ·584375 of $\frac{1}{11}$ of £1·?

Divide £·584375 by 11· and find value of result. Or, Find value of £·584375 and Divide that value by 11·

The latter method will apply to all such Examples. The former only to those in which Given Decimal is exactly divisible by Denominator of Multiplying Fraction.

62· What is the value of ·10625 of £18· 6·s. 8·d.?

Change unit, and find value of £·10625. Then Multiply by 18⅓; because ⅓ cannot be expressed as a Terminate Decimal. (Pages 139· and 140·)

Find the several values of

63· ·04583 of £18· 10·s.
64· ·071428 of 14·cwt. 1·qr. 14·lb.
65· ·9875 of 5·lb. 5·oz. Troy.
66· ·16875 of £5· 12·s. 6·d.
67· ·875 of 5·cwt.
68· ·275 of 9·crowns.
69· ·4375 of 7⅘cwt.
70· ·725 of 3·lin. yds.
71· ·0525 of 4·ho. 15·min.
72· ·83125 of 15·ac. 2·r.
73· ·625 of 9·moid.
74· ·4453125 of 1·fur. 24·po.
75· ·15625 of 11·da. 8·ho.
76· ·61328125 of 3·qr. 2·bu. 1·pe.

THE NEXT AND SIMILAR EXAMPLES ARE USUALLY PLACED UNDER A SPECIAL RULE, CALLED "ALLIGATION."

77· How much tea at 5·s. 4·d. per lb. must be added to 1·lb. at 3·s. 4·d. to make a mixture worth 5·s. per lb?

The value of the inferior tea is to be raised 20·d. per lb. Every lb. at 5·s. 4·d. adds 4·d. per lb. to value of inferior tea. Why? ∴ As many lb. at 5·s. 4·d. must be added as there are fourpences in 20·d.

78· By adding water to rum worth 13·s. 4·d. per gal. the value of the mixture was diminished to 12·s. 6·d. per gal. What portion of it was rum?

79· How much water in 342·gal. 3·qt. 1·pt. of such a mixture?

80· Seven lb. of tea at 5·s. 4·d. were mixed with 5·lb. at 4·s. 6·d. and 11·lb. at 3·s. 8·d. What was the mean or average price of the mixture?

Find value of the whole, and Divide by No. of lb. mixed.

81· 4½ gal. of brandy at 16·s. 3·d. are mixed with 8½ gal. at 15·s. and 12· at 17·s. 9·d. What is the mean value per gallon?

82· A goldsmith melts together 9·5 oz. of gold, 18· carats fine, with 5·25 oz., 22· carats fine, and 7·oz., 19·75 carats fine. What is the quality of the compound?

83· A mixture was compounded of 14·lb. 5·oz. of tea at 5·s. 4·d.; 7·lb. 11·oz. at 3·s. 8·d.; 8·lb. 7·oz. at 6·s.; and 15·lb. 9·oz. at 4·s. What is gained or lost by selling 35·lb. of the mixture at 4·s. per lb.?

FIRST EXAMPLE WORKED OUT. In what ratio must meal at 14·d. per doz. be combined with flour at 19·d. per doz. to make a compound of the mean value of 17·d. per doz.?

That at 14·d. must be enhanced 3·d. per doz. Why?
But, each doz. at 19·d. contributes 2·d. towards this enhancement.
∴ For each doz. meal, take as many doz. flour as there are twos in three.
But, No. of twos in three $= 3\cdot \div 2\cdot = \frac{3}{2}$. (PRIN. XV.)
∴ Ratio of Quantity of Flour to Quantity of Meal $= \frac{3}{2}$. (P. 145·)

Hence, For every doz. meal at 14·d., $\frac{3}{2}$ *doz. or* 1½ *doz. flour at 19·d.; or, for every 2· doz. at 14·d., 3· doz. at 19·d. must be taken.*

PROOF.

			s.	d.
2· doz.	meal, at 14·d. per doz.	cost	2·	4·
3· doz.	flour, at 19·d. „ „	cost	4·	9·
5· doz.	comp. at 17·d. „ „	cost	7·	1·

In this Example, we observe that

The Number of doz. at 14·d. = 2· doz. ∸ 19·d. — 17·d.
The Number of doz. at 19·d. = 3· doz. ∸ 17· — 14·

That is to say,

Quantity at lower rate ∸ Diff. of mean and higher rate.
Quantity at higher rate ∸ Diff. of mean and lower rate.

The same holds true in every similar Example. Hence we derive this simple and useful Rule.

When a Compound is formed of two Simples:

Quantity of each Simple $\doteq$ *Diff. of mean val. and that of the other Simple.*

Of course the prices, when compared, must be in the same denomination.

EXERCISE CIV. (*resumed.*)

84· What quantities of a commodity at 3·s. and at 6·s. per lb. will make a mixture worth 4·s. 8·d. per lb.?

85· A compound worth 16·s. per gal. was made by mixing spirit at 11·s. 6·d. per gal. with other worth 17·s. 9·d. per gal. What part of the mixture consisted of each sort?

When more than two Simples are to be combined, take them in couples, each consisting of one above the mean price and one below it. The Example will then become equivalent to as many Examples like No. 84· as there are couples. If there should be a greater number of Simples on one side of the mean value than on the other side, the same Simple will, of course, appear in more than one couple, for none may be omitted. No Simple must be twice employed except in this case.

SECOND EXAMPLE WORKED OUT. How many lb. of sugar, at the respective prices of 4½d., 5¼d., 7·d., 7½d., and 8·d., will constitute a mixture having the mean value of 6½d. per lb.?

Here are three prices above the mean, and only two below it. Consequently, either 4½d., or 5¼d. will appear in two couples. And, therefore, two lots at that price must be employed in the Compound.

Of First Couple,
d.
At 4½, take ½ lb. ∵ 7· — 6½ = ½
At 7·, take 2·lb. ∵ 6½ — 4½ = 2·

Of Second Couple,
At 4½, take 1·lb. ∵ 7½ — 6½ = 1·
At 7½, take 2·lb. ∵ 6½ — 4½ = 2·

Of Third Couple,
At 5¼, take 1½lb. ∵ 8· — 6½ = 1½
At 8·, take 1¼lb. ∵ 6½ — 5¼ = 1¼

PROOF.			s.	d.
1½ lb.	at 4½d.	cost	0·	6¾
2· lb.	at 7·d.	cost	1·	2·
2· lb.	at 7½d.	cost	1·	3·
1½ lb.	at 5¼d.	cost		7⅞
1¼ lb.	at 8·d.	cost		10·
8¼ lb.	at 6½d.	cost	4·	5⅝

It is plain that, instead of the quantities here found, their doubles, trebles, or any other equi-multiples of them may be employed.

To every question like the present, there are as many different answers as you can form separate sets of Couples of the Simples.*

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* Exclusive of the equi-multiples just mentioned.

86· Employing the same kinds of sugar as in the last Example, what quantities would make the required Compound, when coupled as follows. *Prove results.*

1st Set.	2nd Set.	3rd Set.	4th Set.	5th Set.	6th Set.
d. d.	d. d.	d. d.	d. d.	d. d.	d. d.
4½, 7·	4½, 7·	4½, 7·	5¼, 7·	5¼, 7·	5¼, 7·
4½, 7½	5¼, 7½	5¼, 7½	4½, 7½	5¼, 7½	4½, 7½
5¼, 8·	5¼, 8·	4½, 8·	5¼, 8·	4½, 8·	4½, 8·

When the Mixture is to be a limited Quantity.

First. Find, as before, any set of the Ingredients that would make such a Compound.

Second. Divide the whole Mixture into parts having the same ratios as the numbers thus found. (See pages 158· and 159·)

THIRD EXAMPLE WORKED OUT, What quantities of currants at the respective prices of 4·d., 6·d., 9·d., and 11·d., per lb., will compose 120·lb. at 8·d. per lb.?

Such a Mixture would be made by combining 1·lb. at 4·d., with 4·lb. at 9·d., 3·lb. at 6·d., and 2·lb. at 11·d.

So that, out of every 10·lb. there would be 1·lb. (or $\frac{1}{10}$ of the whole) at 4·d., &c. &c.

Hence, when the Compound weighs 120·lb.,

No. of lb. at 4·d. = $\frac{1}{10}$ of 120·lb. = 12·lb.
No. of lb. at 9·d. = $\frac{2}{5}$ of 120·lb. = 48·lb.
No. of lb. at 6·d. = $\frac{3}{10}$ of 120·lb. = 36·lb.
No. of lb. at 11·d. = $\frac{1}{5}$ of 120·lb. = 24·lb.

87· Prove the last Example, and then find another Result.

88· Two qualities of a commodity, respectively worth 5·s. 9·d. and 10·s. per lb., are mixed, and form a Compound of 1½ cwt. worth 8·s. 6·d. How much of each?

89· How much gold of 17·, 18·, 22·, and 24· carats fine must be melted together to form a mass weighing 2·lb. 1·oz. 5·4 grs. 21· carats fine? *Prove Result.*

90· A farm, consisting of 167· ac. 26⅔ po., is worth, on the average, a rent of £1· 2·s. 9·d. per acre. It is composed of two sorts of land, worth, respectively, 17·s. 6·d., and 34·s. 6·d. per acre. What extent is there of each quality?

FOURTH EXAMPLE WORKED OUT. How much wine at 5·s., 5·s. 6·d., and 6·s. per gal. must be mixed with 3· gal. at 4·s, to make a Compound worth 5·s. 4·d. per gal.?

Here, one of the ingredients is a limited quantity.

A Composition worth 5·s. 4·d. per gal. would be formed of 1· gal. at 4·s., 1· gal. at 5·s., 2· gal. at 5·s. 6·d., and 2· gal. at 6·s.

Quantity at 4·s. = $\frac{1}{6}$ of the whole. ∴ 3· gal. × 6· = Whole Quantity = 18· gals.

Quantity at 5·s. = $\frac{1}{6}$ of 18· gal. = 3· gal.
Quantity at 5·s. 6·d. = $\frac{1}{3}$ of 18· gal. = 6· gal.
Quantity at 6·s. = $\frac{1}{3}$ of 18· gal. = 6· gal.

91· How many women does a farmer employ at 8·d. per day, if, with 16· men receiving each 1·s. 9·d. per day, the average wages of the whole are 16·d. per day?

92· Giving to each of 32· men 2·s. each, and to 22· boys 6·d. each, to how many girls do I give 8·d. each, to average 14½d.?

The name "Alligation" (which means "*a binding to*,") was given by the old Arithmeticians to the Rule of which the foregoing are Examples, because they used to *link* the members of each couple together by a curved or crooked line.

They also divided Examples of this kind into four classes.

I. Alligation Medial. *When the price and Quantity of each Simple are given, to find Mean Price of Compound.*

Examples 80·, 81·, 82·, and 83·

II. Alligation Alternate. *Given, Mean Price of Compound and price of each Simple, to find Quantities of the latter.*

Examples 78·, 84·, 85·, and 86·

III. Alligation Total. *Given, as in last case, but the Compound to be a limited Quantity.*

Examples 87·, 88·, 89·, and 90·

IV. Alligation Partial. *Given, Mean Price of Compound, price of each Simple, and a limited Quantity of one, to find Quantities of the others.*

Examples 77·, 91·, and 92·

PRACTICE.

This is the name given to a process by which the value of a large number, at a comparatively small rate, may be more neatly and expeditiously computed than by direct Multiplication.

"Practice" is founded on Principle VI. and the change of unit.

Suppose it be required to find the cost of 5437· yards at 9¾d. per yard.

Cost of 5437· yards = 9¾d. × 5437·
But, 9¾ × 5437· = 5437· × 9¾. (PRIN. VI.)
∴ 9¾d. × 5437· = 5437·d. × 9¾

Where the Units of Value (9¾d.) are changed into Multiplying Units, and the Units of Commodity (5437· yards), previously employed as Multiplying Units, are changed into Units of Value. And we thus gain the advantage of working with a smaller Multiplier.

GENERAL RULE FOR PRACTICE.

First. Change Units of Commodity into any convenient Units of Price.

This will express the value of the whole at that price each.

Second. Multiply by the number of those Units of Assumed Price contained in Given Price, effecting the Multiplication, when it can be so done, by taking Aliquot Parts. (See pages 227· and 228·)

FIRST EXAMPLE WORKED OUT. What will 5437· yd. cost, at $9\frac{3}{4}$d. per yard?

As the 5437· yd. may be changed into "*Any convenient Units of Value*," this Example may be worked in a variety of ways. For, value of 5437· yd. at $9\frac{3}{4}$d. per yard. =

1st, 5437·fars. × 39·; ∵ As. Pr. 1·f.; Giv. Pr. 39·f.
2nd, 5437·pence × $9\frac{3}{4}$; ∵ As. Pr. 1·d.; Giv. Pr. $9\frac{3}{4}$d.
3rd, 5437·shgs. × $\frac{13}{16}$; ∵ As. Pr. 1·s.; Giv. Pr. $\frac{13}{16}$s.
4th, 5437·sovs. × $\frac{13}{320}$; ∵ As. Pr. £1·; Giv. Pr. £$\frac{13}{320}$
5th, 5437·threepences × $3\frac{1}{4}$. 6th, 5437·sixpences × $1\frac{5}{8}$.
&c. &c. &c. &c.

The first four of above schemes being the most simple and practically useful, we shall work out the Example by each of these only.

Example completed.

By 1*st Scheme. Assumed Price one farthing per yd.*

Farthings.
5437· = value of 5437· yd. at 1·far. or $\frac{1}{4}$d. per yd.
39·
48933· = value of 5437· yd. at 9· far. or $2\frac{1}{4}$d. per yd.
163110· = value of 5437· yd. at 30· far. or $7\frac{1}{2}$d. per yd.
4·)212043· = value of 5437· yd. at 39· far. or $9\frac{3}{4}$d. per yd.
12·)$53010\frac{3}{4}$d. = Ditto, in pence.
20·)4417·s. $6\frac{3}{4}$d. = Ditto, in shillings and pence.
£220· 17·s. $6\frac{3}{4}$d. = Ditto, in £. s. d.

By 2nd Scheme. Assumed Price one penny per yd.

d. d.
$\frac{1}{2}$ $\frac{1}{2}$ 5437· = cost of 5437·yd. at 1· d. per yd.
9·
48933· = cost of 5437·yd. at 9· d. per yd. = 9· times cost at 1·d.
$\frac{1}{4}$ $\frac{1}{2}$ $2718\frac{1}{2}$ = cost of 5437·yd. at $\frac{1}{2}$d. per yd. = $\frac{1}{2}$ of cost at 1·d.
$1359\frac{1}{4}$ = cost of 5437·yd. at $\frac{1}{4}$d. per yd. = $\frac{1}{2}$ of cost at $\frac{1}{2}$d.
$53010\frac{3}{4}$ = cost of 5437·yd. at $9\frac{3}{4}$d. per yd.
4417·s. $6\frac{3}{4}$d. = Ditto, in shillings and pence.
£220· 17·s. $6\frac{3}{4}$d. = Ditto, in £. s. d.

Or thus:

d.		d.	yd. d.
		5437·	cost of 5437· at 1·
		9·	
$\frac{3}{4}$	$\frac{1}{12}$	48933·	cost of 5437· at 9·
		$4077\frac{3}{4}$	cost of 5437· at $\frac{3}{4}$
		$53010\frac{3}{4}$†	cost of 5437· at $9\frac{3}{4}$

Or thus:

d.		d.	d.
$\frac{1}{4}$	$\frac{1}{40}$	54370·	cost of 5437· at 10·*
		$1359\frac{1}{4}$	cost of 5437· at $\frac{1}{4}$
		$53010\frac{3}{4}$†	cost of 5437· at $9\frac{3}{4}$

* PRINCIPLE VIII.

† The results are left in pence, to be reduced, as before, to £. s. d.

By 3rd Scheme. Assumed Price one shilling per yd.

d.		s.	d.	
6·	$\frac{1}{2}$	5437·	0·	cost of 5437·yd. at 1·s. per yd.
3·	$\frac{1}{2}$	2718·	6·	cost of 5437·yd. at 6·d. per yd.; being $\frac{1}{2}$ cost at 1·s.
$\frac{3}{4}$	$\frac{1}{4}$	1359·	3·	cost of 5437·yd. at 3·d. per yd.; being $\frac{1}{2}$ cost at 6·d.
		339·	$9\frac{3}{4}$	cost of 5437·yd. at $\frac{3}{4}$ d. per yd.; being $\frac{1}{4}$ cost at 3·d.
20·)		4417·	$6\frac{3}{4}$	cost of 5437·yd. at $9\frac{3}{4}$ d. per yd.; being $\frac{13}{16}$ cost at 1·s.
		£220·	17·s. $6\frac{3}{4}$d.	Ditto, in £. s. d.

Or thus:

d.		s.	d.	yd.
3·	$\frac{1}{4}$	5437·	0·	cost of 5437· at 1· s.
$\frac{3}{4}$	$\frac{1}{4}$	1359·	3·	cost of 5437· at 3· d.
		4077·	9·	cost of 5437· at 9· d.
		339·	$9\frac{3}{4}$	cost of 5437· at $\frac{3}{4}$d.
20·)		4417·	$6\frac{3}{4}$	cost of 5437· at $9\frac{3}{4}$d.
		£220·	17·s. $6\frac{3}{4}$d.	

Or thus:

d.		s.	d.	yd.
3·	$\frac{1}{4}$	5437·	0·	cost of 5437· at 1· s.
$\frac{3}{4}$	$\frac{1}{4}$	1359·	3·	cost of 5437· at 3· d.
			3·	
		4077·	9·	cost of 5437· at 9· d.
		339·	$9\frac{3}{4}$	cost of 5437· at $\frac{3}{4}$d.
		4417·	$6\frac{3}{4}$	cost of 5437· at $9\frac{3}{4}$d.

Or thus:

	s.	yd.
	5437·	c. of 5437· at 1·s.
	13·	
16·)	70681·	c. of 5437· at 13·s.
20·)	4417· $6\frac{3}{4}$d.	c. of 5437· at $\frac{13}{16}$ s.*
	£220· 17·s. $6\frac{3}{4}$d.	

* $\frac{1}{16}$ of 13·s. $=\frac{13}{16}$ of 1·s. PR. XVI.

Or thus:

d.		s.	d.	yd.
2·	$\frac{1}{6}$	5437·	0·	c. of 5437· at 1· s.
$\frac{1}{4}$	$\frac{1}{8}$	906·	2·	c. of 5437· at 2· d.
		4530·	10·	c. of 5437· at 10· d.
		113·	$3\frac{1}{4}$	c. of 5437· at $\frac{1}{4}$d.
20·)		4417·	$6\frac{3}{4}$	c. of 5437· at $9\frac{3}{4}$d.
		£220·	17·s. $6\frac{3}{4}$d.	

By 4th Scheme. Assumed Price £1· per yd.

d.		£.	s.	d.	yd.
6·	$\frac{1}{40}$	5437·	0·	0·	cost of 5437· at £1· per yd.
3·	$\frac{1}{2}$	135·	18·	6·	cost of 5437· at 6· d. per yd. $=\frac{1}{40}$ cost at £1·
$\frac{3}{4}$	$\frac{1}{4}$	67·	19·	3·	cost of 5437· at 3· d. per yd. $=\frac{1}{2}$ cost at 6·d.
		16·	19·	$9\frac{3}{4}$	cost of 5437· at $\frac{3}{4}$d. per yd. $=\frac{1}{4}$ cost at 3·d.
		£220·	17·	$6\frac{3}{4}$	cost of 5437· at $9\frac{3}{4}$d. per yd. $=\frac{13}{320}$ cost at £1·

Or thus:

d.		£.	s.	d.	yd.
3·	$\frac{1}{80}$	5437·	0·	0·	c. 5437· at £1·
$\frac{3}{4}$	$\frac{1}{4}$	67·	19·	3·	c. 5437· at 3· d.
				3·	
		203·	17·	9·	c. 5437· at 9· d.
		16·	19·	$9\frac{3}{4}$	c. 5437· at $\frac{3}{4}$d.
		220·	17·	$6\frac{3}{4}$	c. 5437· at $9\frac{3}{4}$d.

Or thus:

s.		£.	s.	d.	yd.
1·	$\frac{1}{20}$	5437·	0·	0·	c. 5437· at £1·
d. 3·	$\frac{1}{4}$	271·	17·	0·	c. 5437· at 1· s.
$\frac{3}{4}$	$\frac{1}{4}$	67·	19·	3·	c. 5437· at 3· d.
		203·	17·	9·	c. 5437· at 9· d.
		16·	19·	$9\frac{3}{4}$	c. 5437· at $\frac{3}{4}$d.
		220·	17·	$6\frac{3}{4}$	c. 5437· at $9\frac{3}{4}$d.

Or thus:

s.		£.	s.	d.	yd.
1·	$\frac{1}{20}$	5437·	0·	0·	c. 5437· at £1·
d. $\frac{3}{4}$	$\frac{1}{16}$	271·	17·	0·	c. 5437· at 1· s.
		16·	19·	$9\frac{3}{4}$	c. 5437· at $\frac{3}{4}$d.
			13·		
		220·	17·	$6\frac{3}{4}$	c. 5437· at $9\frac{3}{4}$d.

Or thus:

s.		£.	s.	d.	yd.
5·	$\frac{1}{4}$	5437·	0·	0·	c. 5437· at £1·
d. 10·	$\frac{1}{6}$	1359·	5·	0·	c. 5437· at 5· s.
$\frac{1}{4}$	$\frac{1}{40}$	226·	10·	10·	c. 5437· at 10·d.
		5·	13·	$3\frac{1}{4}$	c. 5437· at $\frac{1}{4}$d.
		220·	17·	$6\frac{3}{4}$	c. 5437· at $9\frac{3}{4}$d.

EXERCISE CV.

Every Example to be worked in, at least, two ways.
Every line of the work to be described in writing.

Find the value of

A.

	yd.	d.		lb.	d.		ft.	d.		yd.	d.
1·	7468·	at 1·	7·	9052·	at 7·	13·	5483·	at 13·	19·	1564·	at 19·
2·	9374·	at 2·	8·	3145·	at 8·	14·	9864·	at 14·	20·	3516·	at 20·
3·	6513·	at 3·	9·	8439·	at 9·	15·	6315·	at 15·	21·	7345·	at 21·
4·	8457·	at 4·	10·	7517·	at 10·	16·	9431·	at 16·	22·	9656·	at 22·
5·	3261·	at 5·	11·	5438·	at 11·	17·	3186·	at 17·	23·	8976·	at 23·
6·	4832·	at 6·	12·	7056·	at 12·	18·	5843·	at 18·	24·	6458·	at 24·

B.

	oz.	d.		lb.	d.		ft.	d.		yd.	d.
1·	7309·	at $\frac{1}{4}$	11·	7384·	at $5\frac{1}{2}$	21·	5931·	at $3\frac{3}{4}$	31·	6543·	at $9\frac{1}{2}$
2·	7213·	at $\frac{1}{2}$	12·	6162·	at $7\frac{1}{2}$	22·	8264·	at $4\frac{1}{4}$	32·	8765·	at $10\frac{1}{2}$
3·	7419·	at $\frac{3}{4}$	13·	7154·	at $2\frac{3}{4}$	23·	9254·	at $1\frac{1}{2}$	33·	2198·	at $9\frac{3}{4}$
4·	3416·	at $1\frac{1}{4}$	14·	9375·	at $4\frac{1}{2}$	24·	6273·	at $5\frac{1}{4}$	34·	9876·	at $8\frac{3}{4}$
5·	5478·	at $1\frac{1}{2}$	15·	8495·	at $5\frac{3}{4}$	25·	2728·	at $6\frac{1}{2}$	35·	4321·	at $10\frac{3}{4}$
6·	8376·	at $1\frac{3}{4}$	16·	1617·	at $6\frac{1}{4}$	26·	7273·	at $7\frac{3}{4}$	36·	7468·	at $11\frac{1}{2}$
7·	3710·	at $2\frac{1}{4}$	17·	5051·	at $7\frac{1}{4}$	27·	9495·	at $8\frac{1}{4}$	37·	5544·	at $11\frac{1}{4}$
8·	5932·	at $2\frac{1}{2}$	18·	1506·	at $8\frac{1}{2}$	28·	3219·	at $10\frac{1}{4}$	38·	4940·	at 7·
9·	1597·	at $3\frac{1}{4}$	19·	3839·	at $6\frac{3}{4}$	29·	6473·	at $11\frac{3}{4}$	39·	8384·	at 8·
10·	8486·	at $4\frac{3}{4}$	20·	3719·	at $3\frac{1}{2}$	30·	1987·	at $9\frac{1}{4}$	40·	9506·	at 6·

Second Example worked out. What will 9846·yd. cost, at $\frac{3}{8}$d. per yd.?

Example completed.

d.		d.	yd.
$\frac{2}{8}$	$\frac{1}{4}$	9846·	cost of 9846· at 1· d.
$\frac{1}{8}$	$\frac{1}{2}$	$2461\frac{1}{2}$	cost of 9846· at $\frac{1}{4}$d.
		$1230\frac{3}{4}$	cost of 9846· at $\frac{1}{8}$d.
	12·)	$3692\frac{1}{4}$	cost of 9846· at $\frac{3}{8}$d.
	20·)	307·s. $8\frac{1}{4}$d.	
		£15· 7·s. $8\frac{1}{4}$d.	

Or thus:

d.		d.	yd.
$\frac{1}{2}$	$\frac{1}{2}$	9846·	cost of 9846· at 1· d.
$\frac{1}{8}$	$\frac{1}{4}$	4923·	cost of 9846· at $\frac{1}{2}$d.*
		$1230\frac{3}{4}$	cost of 9846· at $\frac{1}{8}$d.
		$3692\frac{1}{4}$	cost of 9846· at $\frac{3}{8}$d.

Or thus:

	d.	yd.
	9846·	cost of 9846· at 1· d.
	3·	
8·)	29538·	cost of 9846· at 3· d.
	$3692\frac{1}{4}$	cost of 9846· at $\frac{3}{8}$d.†

* $\frac{1}{8}=\frac{1}{4}$ of $\frac{1}{2}$.

† $\frac{1}{8}$ of 3· $=\frac{3}{8}$ of 1· Prin. xvi.

EXERCISE CVI.

Work variously. Describe lines, Find the value of

A.

	yd.		d.		yd.		d.		yd.		d.		yd.		d.
1·	3416·	at	$\frac{1}{8}$	19·	7170·	at	$5\frac{5}{8}$	37·	4545·	at	$\frac{1}{32}$	55·	2148·	at	$1\frac{1}{32}$
2·	5478·	at	$\frac{3}{8}$	20·	8877·	at	$1\frac{5}{8}$	38·	6759·	at	$\frac{3}{32}$	56·	3259·	at	$2\frac{3}{32}$
3·	2376·	at	$\frac{5}{8}$	21·	9493·	at	$3\frac{1}{8}$	39·	9657·	at	$\frac{5}{32}$	57·	4362·	at	$3\frac{5}{32}$
4·	8345·	at	$\frac{7}{8}$	22·	3635·	at	$4\frac{1}{64}$	40·	7965·	at	$\frac{7}{32}$	58·	5473·	at	$4\frac{7}{32}$
5·	9413·	at	$\frac{1}{16}$	23·	3837·	at	$9\frac{3}{8}$	41·	5697·	at	$\frac{9}{32}$	59·	6584·	at	$5\frac{9}{32}$
6·	5361·	at	$\frac{3}{16}$	24·	9695·	at	$2\frac{7}{8}$	42·	6975·	at	$\frac{11}{32}$	60·	7695·	at	$6\frac{11}{32}$
7·	7435·	at	$\frac{5}{16}$	25·	4241·	at	$8\frac{7}{8}$	43·	5796·	at	$\frac{13}{32}$	61·	8764·	at	$10\frac{1}{8}$
8·	2195·	at	$\frac{7}{16}$	26·	6261·	at	$6\frac{7}{8}$	44·	9014·	at	$\frac{15}{32}$	62·	7653·	at	$10\frac{3}{8}$
9·	9436·	at	$\frac{9}{16}$	27·	7776·	at	$1\frac{1}{16}$	45·	8849·	at	$\frac{17}{32}$	63·	3846·	at	$7\frac{13}{32}$
10·	9895·	at	$\frac{11}{16}$	28·	7372·	at	$1\frac{5}{16}$	46·	2436·	at	$\frac{19}{32}$	64·	9368·	at	$10\frac{1}{3}$
11·	7384·	at	$\frac{13}{16}$	29·	5352·	at	$2\frac{7}{16}$	47·	7518·	at	$\frac{21}{32}$	65·	9592·	at	$11\frac{13}{16}$
12·	6543·	at	$\frac{15}{16}$	30·	4948·	at	$3\frac{9}{16}$	48·	4758·	at	$\frac{1}{64}$	66·	6438·	at	$10\frac{3}{10}$
13·	3322·	at	$1\frac{1}{8}$	31·	4746·	at	$4\frac{3}{16}$	49·	3764·	at	$\frac{3}{64}$	67·	2563·	at	$7\frac{6}{7}$
14·	9291·	at	$3\frac{3}{8}$	32·	4543·	at	$7\frac{7}{16}$	50·	5138·	at	$\frac{5}{64}$	68·	4359·	at	$5\frac{3}{5}$
15·	6059·	at	$7\frac{1}{8}$	33·	4039·	at	$5\frac{11}{16}$	51·	8437·	at	$\frac{7}{64}$	69·	9648·	at	$6\frac{1}{11}$
16·	7978·	at	$4\frac{5}{8}$	34·	3433·	at	$5\frac{5}{64}$	52·	7543·	at	$\frac{9}{64}$	70·	3287·	at	$8\frac{5}{9}$
17·	5150·	at	$7\frac{7}{8}$	35·	3231·	at	$8\frac{11}{16}$	53·	3864·	at	$\frac{11}{64}$	71·	8654·	at	$9\frac{1}{12}$
18·	6655·	at	$1\frac{7}{8}$	36·	3029·	at	$8\frac{2}{7}$	54·	5732·	at	$\frac{1}{5}$	72·	4359·	at	$11\frac{5}{12}$

B.

	lb.		s.		yd.		s.	d.		cwt.		£.	s.	d.
1·	984·	at	1·	20·	450·	at	2·	6·	39·	6148·	at	1·	1·	11·
2·	763·	at	2·	21·	325·	at	3·	4·	40·	5037·	at	1·	3·	$11\frac{1}{2}$
3·	472·	at	4·	22·	555·	at	1·	8·	41·	4259·	at	2·	6·	8·
4·	654·	at	5·	23·	986·	at	1·	4·	42·	7164·	at	2·	4·	$7\frac{1}{2}$
5·	871·	at	10·	24·	812·	at	1·	8·	43·	2859·	at	5·	12·	4·
6·*	358·	at	8·	25·	765·	at	6·	8·	44·	1564·	at	3·	6·	6·
7·*	593·	at	15·	26·*	493·	at	11·	3·	45·	9863·	at	1·	19·	9·
8·*	287·	at	3·	27·*	645·	at	3·	9·	46·	8143·	at	2·	5·	4·
9·*	945·	at	6·	28·*	540·	at	6·	3·	47·	6031·	at	2·	2·	$8\frac{3}{4}$
10·*	463·	at	7·	29·*	783·	at	12·	6·	48·	7104·	at	4·	11·	$8\frac{1}{2}$
11·*	759·	at	9·	30·*	900·	at	7·	6·	49·	3265·	at	2·	13·	$8\frac{1}{4}$
12·*	594·	at	11·	31·*	438·	at	13·	4·	50·	5654·	at	2·	5·	$10\frac{1}{4}$
13·	321·	at	13·	32.+	856·	at	17·	6·	51·	3721·	at	3·	11·	10·
14·*	644·	at	14·	33·+	512·	at	18·	9·	52·	1608·	at	3·	16·	5·
15·+	793·	at	16·	34·*	405·	at	2·	11·	53·	4321·	at	6·	11·	$7\frac{1}{4}$
16·	836·	at	17·	35·*	843·	at	2·	1·	54·	3458·	at	4·	7·	$3\frac{1}{2}$
17·+	289·	at	18·	36·*	285·	at	7·	4·	55·	2643·	at	5·	16·	$9\frac{3}{4}$
18·+	982·	at	19·	37·*	438·	at	9·	2·	56·	7154·	at	2·	2·	$2\frac{1}{2}$
19·*	405·	at	12·	38·*	593·	at	7·	4·	57·	6287·	at	1·	1·	$1\frac{1}{4}$

* The price in this Example is the Sum of *two* Aliquot Parts of £1·
+ The difference between this price and £1· is an Aliquot Part of £1·

C.

	Shares.		£		Shares.		£		Shares.		£		Shares.		£
1·	435·	at	$\frac{1}{8}$	7·	225·	at	$\frac{11}{16}$	13·	511·	at	$11\frac{1}{2}$	19·	375·	at	$42\frac{11}{16}$
2·	964·	at	$\frac{3}{8}$	8·	348·	at	$\frac{13}{16}$	14·	420·	at	$14\frac{7}{8}$	20·	484·	at	$31\frac{15}{16}$
3·	743·	at	$\frac{5}{8}$	9·	515·	at	$\frac{15}{16}$	15·	339·	at	$15\frac{5}{8}$	21·	593·	at	$75\frac{1}{3}$
4·	620·	at	$\frac{7}{8}$	10·	748·	at	$9\frac{1}{2}$	16·	248·	at	$13\frac{7}{16}$	22·	963·	at	$68\frac{2}{3}$
5·	400·	at	$\frac{5}{16}$	11·	432·	at	$4\frac{3}{4}$	17·	157·	at	$19\frac{5}{16}$	23·	694·	at	$95\frac{3}{8}$
6·	350·	at	$\frac{7}{16}$	12·	693·	at	$7\frac{5}{8}$	18·	266·	at	$25\frac{9}{16}$	24·	602·	at	$17\frac{11}{12}$

THIRD EXAMPLE WORKED OUT. What will $435\frac{5}{8}$ yd. cost, at $6\frac{3}{4}$d. yer yd.?

Example completed.

First Scheme.

d.		£.	s.	d.	yd.	£.
6·	$\frac{1}{40}$	435·	12·	6·	c. $435\frac{5}{8}$ at	1·
						d.
$\frac{3}{4}$	$\frac{1}{8}$	10·	17·	$9\frac{3}{4}$	c. $435\frac{5}{8}$ at	6·
		1·	7·	$2\frac{23}{32}$	c. $435\frac{5}{8}$ at	$\frac{3}{4}$
		12·	5·	$0\frac{15}{32}$	c. $435\frac{5}{8}$ at	$6\frac{3}{4}$

Second Scheme.

d.		s.	d.	yd.	s.
6·	$\frac{1}{2}$	435·	$7\frac{1}{2}$*	c. $435\frac{5}{8}$ at	1·
					d.
$\frac{3}{4}$	$\frac{1}{8}$	217·	$9\frac{3}{4}$	c. $435\frac{5}{8}$ at	6·
		27·	$2\frac{23}{32}$	c. $435\frac{5}{8}$ at	$\frac{3}{4}$
		£12· 5·	$0\frac{15}{32}$	c. $435\frac{5}{8}$ at	$6\frac{3}{4}$

* Here, we may Multiply by $\frac{9}{16}$ ∵ $6\frac{3}{4}$d. $= \frac{9}{16}$s.

The Assumed Price, and the Multiplier also, may, when convenient, be expressed in Decimals.

Third Scheme.

£.	yd.	£.
435·625	c. $435\frac{5}{8}$ at	1·
·028125*		£.
·002178125	c. $435\frac{5}{8}$ at	·000005
·052275	c. $435\frac{5}{8}$ at	·00012
3·485	c. $435\frac{5}{8}$ at	·008
8·7125	c. $435\frac{5}{8}$ at	·02
£12·251953125	c. $435\frac{5}{8}$ at	·028125*

= £12· 5·s. $0\frac{15}{32}$d. (Ex. LXXXV.)

* $6\frac{3}{4}$d. = £·028125 (Ex. LXXXVIII.)

Fourth Scheme.

s.	yd.	s.
435·625	c. $435\frac{5}{8}$ at	1·
·5625+		s.
·2178125	c. $435\frac{5}{8}$ at	·0005
·87125	c. $435\frac{5}{8}$ at	·002
26·1375	c. $435\frac{5}{8}$ at	·06
217·8155	c. $435\frac{5}{8}$ at	·5
s. 245·0390625	c. $435\frac{5}{8}$ at	·5625+

= £12· 5·s. $0\frac{15}{32}$d. (Ex. LXXXV.)

+ $6\frac{3}{4}$d. = ·5625 of 1·s. (Ex. LXXXVIII.)

Sometimes the Assumed Price *only* is taken in Decimals, the Multiplier retaining the form of a Vulgar Fraction or Mixed Number.

Fifth Scheme.

£.	yd.	£.
435·625	c. $435\frac{5}{8}$ at	1·
9·		
40·)3920·625	c. $435\frac{5}{8}$ at	£9·
8·)98·015625	c. $435\frac{5}{8}$ at	4/6
£12·251953125	c. $435\frac{5}{8}$ at	$6\frac{3}{4}$d.

* $6\frac{3}{4}$d. $= \frac{9}{320}$ of £1· 320· = 40· × 8·

Sixth Scheme.

s.	yd.	s.
435·625	c. $435\frac{5}{8}$ at	1·
9·*		
8·)3920·625	c. $435\frac{5}{8}$ at	9·s.
2·)490·078125	c. $435\frac{5}{8}$ at	$13\frac{1}{2}$d.
s. 245·0390625	c. $435\frac{5}{8}$ at	$6\frac{3}{4}$d.

* $6\frac{3}{4}$d. $= \frac{9}{16}$ of 1·s. 16· = 8· × 2·

Or, after the Change of Unit, the process may be carried out by Vulgar Fractions.

Seventh Scheme.

Cost of $435\frac{5}{8}$ yd. at $6\frac{3}{4}$d. $= 435\frac{5}{8}\text{d.} \times 6\frac{3}{4} = \frac{3485}{8}\text{d.} \times \frac{27}{4}$
$= \frac{94095}{32}\text{d.}$
$=$ £12· 5·s. $0\frac{15}{32}$d.

Eighth Scheme.

Cost of $435\frac{5}{8}$ yd. at $6\frac{3}{4}$d. $= 435\frac{5}{8}\text{s.} \times \frac{9}{16} = \frac{3485}{8}\text{s.} \times \frac{9}{16}$
$= \frac{31365}{128}\text{s.}$
$=$ £12· 5·s. $0\frac{15}{32}$d.

Ninth Scheme.

Cost of $435\frac{5}{8}$ yd. at $6\frac{3}{4}$d. $= £435\frac{5}{8} \times \frac{9}{320} = £\frac{3485}{8} \times \frac{9}{320}$
$= £\frac{6273}{512}$
$=$ £12· 5·s. $0\frac{15}{32}$d.

Compare the latter three schemes with Examples worked out on Pages 233· and 234·

PRACTICE DISTINGUISHED FROM COMPOUND MULTIPLICATION.

Aliquot Parts being employed in Compound Multiplication as well as in Practice, these two operations are frequently confounded.* The distinction between them is, however, broad, and should be carefully noted.

In Compound Multiplication we commence with given value of *one* article; and, then, each successive line shews real value of a different number of articles at the given rate each.

In Practice we start with value of the whole given number of articles at an assumed rate; and, then, each line of the work expresses the value of given number of articles at a different rate.

Multiplication shews true values of a succession of Numbers of things at a uniform given rate.

Practice exhibits a succession of values of the same Number of things, at various assumed rates.

Fourth Example worked out. Find the value of 819· cwt. 3· qr. 16· lb. at £2· 12·s. 6·d. per cwt.

Example completed.

I. *By Multiplication.*

£.	s.	d.		
2·	12·	6·	v.	1·cwt.
		10·		
26·	5·	0·	v.	10·cwt.
		4·		
105·	0·	0·	v.	40·cwt.
		8·		
840·	0·	0·	v.	320·cwt.
2·	12·	6·	v.	1·cwt.
837·	7·	6·	v.	319·cwt.
1·	6·	3·	v.	2·qr.
	13·	1½	v.	1·qr.
	6·	6¾	v.	14·lb.
		11¼	v.	2·lb.
839·	14·	4½	v.	319· cwt. 3· qr. 16· lb.

II. *By Practice.*

mr.		£.	s.	d.		cwt.	qr.	lb.	£.
		319·	0·	0·	v.	319·	0·	0·	at 1· per cwt.
			10·	0·	v.		2·	0·	at 1· per cwt.
			5·	0·	v.		1·	0·	at 1· per cwt.
			2·	6·	v.			14·	at 1· per cwt.
				$4\frac{2}{7}$	v.			2·	at 1· per cwt.
10/	½	319·	17·	$10\frac{2}{7}$	v.	319·	3·	16·	at 1· per cwt.
				2·					
		639·	15·	$8\frac{4}{7}$	v.	319·	3·	16·	at 2· per cwt.
2/6	¼	159·	18·	$11\frac{1}{7}$	v.	319·	3·	16·	at 10/ „
		39·	19·	$8\frac{11}{14}$	v.	319·	3·	16·	at 2/6 „
		839·	14·	4½	v.	319·	3·	16·	at £. 2· s. 12· d. 6·

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* This confusion occurs in the, perhaps, most popular of modern "School Arithmetics;" where the present example is to be found, worked out precisely as here, by Compound Multiplication, but described by the eminent author as an instance of *Compound Practice!*

III. *By Practice and Multiplication combined.*

This combination of processes is frequently adopted to evade Fractions.

	mr.	£.	s.	d.		cwt.	qr.	lb.		£.	s.	d.	
10/	$\frac{1}{2}$	319·	17·	6·	v.	319·	3·	14·	at	1·	0·	0·	per cwt.
				2·									
		639·	15·	0·	v.	319·	3·	14·	at	2·	0·	0·	per cwt.
2/6	$\frac{1}{4}$	159·	18·	9·	v.	319·	3·	14·	at		10·	0·	per cwt.
		39·	19·	$8\frac{1}{4}$	v.	319·	3·	14·	at		2·	6·	per cwt.
		839·	13·	$5\frac{1}{4}$	v.	319·	3·	14·	at	2·	12·	6·	per cwt.*
				$11\frac{1}{4}$	v.			2·	at	2·	12·	6·	per cwt.†
		839·	14·	$4\frac{1}{2}$	v.	319·	3·	16·	at	2·	12·	6·	per cwt.

* Practice thus far.

† This is found by Multiplication; $11\frac{1}{4}$d. being $\frac{1}{56}$ of £2· 12·s. 6·d. (PRIN. IX.)

EXERCISE CVII.

Work variously and describe lines. *Find the value of*

A.

	Articles.	d.		Articles.	d.		Articles.	d.
1·	$7148\frac{1}{8}$ at	6·	5·	$3050\frac{3}{8}$ at	4·	9·	$9847\frac{7}{8}$ at	9·
2·	$9376\frac{1}{4}$ at	9·	6·	$9165\frac{1}{2}$ at	$11\frac{1}{2}$	10·	$8963\frac{1}{12}$ at	9·
3·	$3148\frac{1}{3}$ at	$7\frac{1}{2}$	7·	$3064\frac{5}{8}$ at	10·	11·	$7346\frac{1}{6}$ at	$4\frac{1}{2}$
4·	$6482\frac{2}{3}$ at	$5\frac{1}{4}$	8·	$6298\frac{3}{4}$ at	$10\frac{1}{2}$	12·	$9345\frac{1}{3}$ at	$2\frac{1}{4}$

B.

		s. d.			s. d.			s. d.
1·	$9456\frac{2}{3}$ at	1· 3·	9·	$9843\frac{4}{5}$ at	2· $8\frac{1}{2}$	17·	$4321\frac{5}{12}$ at	3· 6·
2·	$4189\frac{1}{6}$ at	1· $7\frac{1}{2}$	10·	$3619\frac{1}{10}$ at	6· 8·	18·	$6495\frac{11}{12}$ at	5· 3·
3·	$7352\frac{5}{6}$ at	1· $4\frac{1}{2}$	11·	$9482\frac{3}{10}$ at	7· 6·	19·	$8326\frac{11}{16}$ at	3· 8·
4·	$8416\frac{1}{12}$ at	1· 6·	12·	$6845\frac{7}{10}$ at	12· $7\frac{1}{2}$	20·	$6000\frac{4}{5}$ at	3· $11\frac{1}{2}$
5·	$9643\frac{1}{8}$ at	2· 6·	13·	$5432\frac{9}{10}$ at	5· $2\frac{1}{2}$	21·	$2864\frac{3}{10}$ at	4· 7·
6·	$3042\frac{1}{5}$ at	3· 4·	14·	$3421\frac{7}{12}$ at	6· 3·	22·	$7642\frac{1}{5}$ at	4· $8\frac{1}{4}$
7·	$6984\frac{2}{5}$ at	3· $5\frac{1}{4}$	15·	$7149\frac{11}{16}$ at	2· 4·	23·	$2168\frac{3}{5}$ at	3· $1\frac{1}{2}$
8·	$7396\frac{3}{5}$ at	4· $3\frac{1}{4}$	16·	$9576\frac{3}{8}$ at	2· 6·	24·	$1463\frac{4}{15}$ at	15· $3\frac{3}{4}$

C.

	lb.	d.		yd.	d.		sq. ft.	d.		cwt.	s. d.
1·	$49\frac{1}{2}$ at	$6\frac{1}{4}$	14·	$48\frac{7}{16}$ at	$11\frac{1}{4}$	27·	$63\frac{1}{12}$ at	$5\frac{1}{4}$	40·	$93\frac{1}{7}$ at	5· 6·
2·	$34\frac{1}{4}$ at	$7\frac{1}{2}$	15·	$97\frac{9}{16}$ at	$14\frac{1}{4}$	28·	$91\frac{1}{6}$ at	$6\frac{1}{2}$	41·	$64\frac{2}{7}$ at	13· 4·
3·	$65\frac{3}{4}$ at	$6\frac{1}{2}$	16·	$84\frac{11}{16}$ at	21·	29·	$73\frac{5}{12}$ at	$7\frac{1}{4}$	42·	$85\frac{3}{7}$ at	21· 8·
4·	$98\frac{1}{8}$ at	$5\frac{1}{4}$	17·	$56\frac{13}{16}$ at	$19\frac{1}{4}$	30·	$86\frac{7}{12}$ at	$8\frac{1}{4}$	43·	$78\frac{6}{7}$ at	36· 6·
5·	$67\frac{3}{8}$ at	$8\frac{1}{4}$	18·	$68\frac{15}{16}$ at	$16\frac{3}{4}$	31·	$38\frac{2}{3}$ at	$5\frac{3}{4}$	44·	$59\frac{5}{7}$ at	25· 3·
6·	$59\frac{5}{8}$ at	$9\frac{1}{4}$	19·	$75\frac{5}{8}$ at	$4\frac{1}{8}$	32·	$56\frac{5}{6}$ at	$4\frac{1}{4}$	45·	$41\frac{3}{14}$ at	33· 10·
7·	$77\frac{7}{8}$ at	$4\frac{3}{4}$	20·	$39\frac{3}{8}$ at	$7\frac{3}{16}$	33·	$67\frac{11}{12}$ at	$3\frac{3}{4}$	46·	$68\frac{1}{28}$ at	42· 4·
8·	$83\frac{1}{16}$ at	$9\frac{1}{2}$	21·	$45\frac{7}{8}$ at	$5\frac{5}{16}$	34·	$98\frac{1}{3}$ at	$2\frac{1}{4}$	47·	$86\frac{1}{56}$ at	63· 8·
9·	$39\frac{3}{16}$ at	$3\frac{1}{4}$	22·	$82\frac{3}{16}$ at	$12\frac{5}{8}$	35·	$73\frac{3}{8}$ at	$6\frac{1}{4}$	48·	$94\frac{5}{28}$ at	91· 6·
10·	$61\frac{5}{16}$ at	$3\frac{1}{2}$	23·	$95\frac{11}{16}$ at	42·	36·	$21\frac{5}{8}$ at	$9\frac{1}{2}$	49·	$37\frac{11}{28}$ at	64· 8·
11·	$84\frac{2}{16}$ at	$4\frac{1}{2}$	24·	$67\frac{11}{32}$ at	84·	37·	$95\frac{3}{4}$ at	$6\frac{3}{4}$	50·	$82\frac{5}{14}$ at	28· 8·
12·	$93\frac{6}{16}$ at	$8\frac{1}{4}$	25·	$98\frac{7}{32}$ at	35·	38·	$66\frac{7}{8}$ at	$8\frac{1}{2}$	51·	$58\frac{11}{14}$ at	32· 3·
13·	$28\frac{12}{16}$ at	$7\frac{3}{4}$	26·	$61\frac{13}{32}$ at	28·	39·	$82\frac{5}{16}$ at	$7\frac{3}{4}$	52·	$83\frac{11}{56}$ at	98· 6·

D.

	lb.	oz. Av.		d.		sco.	lb.		d.		sco.	lb.		d.		st.	lb.		d.
1·	14·	7·	at	5½*	13·	5·	8·	at	3·	25·	18·	11·	at	4½	37·	19·	5·	at	6½
2·	21·	11·	at	6¼	14·	4·	9·	at	4·	26·	11·	15·	at	3¾	38·	11·	12·	at	8½
3·	16·	3·	at	4¾	15·	7·	6·	at	5·	27·	14·	9½	at	6½	39·	13·	4·	at	4¾
4·	19·	5·	at	7½	16·	3·	5·	at	6·	28·	12·	17·	at	5¼	40·	15	·10·	at	5½
5·	34·	6·	at	8·	17·	9·	7·	at	7·	29·	11·	16·	at	4¾	41·	21·	8·	at	7¼
6·	25·	9·	at	6½	18·	8·	3·	at	8·	30·	13·	18·	at	6¾	42·	32·	6·	at	3¾
7·	18·	2·	at	11·	19·	6·	8·	at	9·	31·	17·	14·	at	5½	43·	25·	9·	at	8¾
8·	15·	6·	at	7¼	20·	3·	4·	at	2·	32·	18·	13·	at	4¼	44·	54·	3·	at	11½
9·	16·	1·	at	10½	21·	4·	5·	at	5·	33·	16·	8½	at	7¼	45·	33·	12·	at	7½
10·	27·	13·	at	9½	22·	5·	8·	at	4·	34·	7·	19·	at	8¼	46·	16·	13·	at	7¾
11·	13·	12·	at	8½	23·	6·	7·	at	7·	35·	15·	12·	at	5¾	47·	18·	11·	at	5½
12·	17·	10·	at	11½	24·	9·	5·	at	8·	36·	9·	10·	at	6¼	48·	23·	13·	at	6¾

* Each price in this section is that of 1·lb.

E.

	ct.	qr.	lb.		£.	s.	d.		cwt.	lb.		£.	s.	d.		ct.	qr.	lb.		£.	s.
1·	5·	1·	14·	at	2·	18·	4·*	16·	14·	11·	at	37·	6·	8·+	31·	9·	3·	7·	at	4·	4·‡
2·	9·	2·	8·	at	3·	1·	6·	17·	32·	10·	at	46·	13·	4·	32·	7·	1·	8·	at	2·	18·
3·	7·	3·	21·	at	3·	5·	8·	18·	16·	13·	at	24·	0·	0·	33·	6·	3·	9·	at	3·	5·
4·	8·	3·	16·	at	1·	3·	4·	19·	19·	15·	at	25·	13·	4·	34·	4·	2·	11·	at	4·	3·
5·	4·	3·	7·	at	3·	12·	8·	20·	65·	43·	at	65·	13·	4·	35·	5·	0·	14·	at	2·	15·
6·	6·	1·	23·	at	4·	1·	8·	21·	87·	29·	at	10·	10·	0·	36·	6·	1·	26·	at	2·	18·
7·	8·	0·	22·	at	4·	6·	4·	22·	98·	37·	at	13·	5·	6·	37·	3·	2·	23·	at	2·	17·
8·	7·	2·	25·	at	7·	9·	4·	23·	25·	18·	at	9·	6·	4·	38·	4·	2·	17·	at	3·	8·
9·	5·	2·	17·	at	3·	4·	4·	24·	46·	9·	at	21·	10·	0·	39·	8·	3·	19·	at	4·	10·
10·	4·	3·	19·	at	3·	3·	8·	25·	38·	38·	at	3·	18·	6·	40·	7·	1·	16·	at	3·	10·
11·	8·	1·	27·	at	3·	17·	0·	26·	29·	40·	at	14·	10·	6·	41·	4·	2·	5·	at	6·	5·
12·	9·	1·	15·	at	3·	3·	0·	27·	54·	26·	at	32·	17·	6·	42·	8·	3·	6·	at	7·	10·
13·	6·	0·	8·	at	3·	10·	0·	28·	69·	85·	at	18·	12·	8·	43·	9·	1·	8·	at	6·	16·
14·	7·	1·	13·	at	4·	13·	4·	29·	42·	51·	at	7·	17·	0·	44·	8·	1·	9·	at	12·	10·
15·	9·	3·	6·	at	3·	19·	4·	30·	85·	10·	at	26·	13·	4·	45·	7·	2·	11·	at	15·	0·

* All at per cwt. + All at per ton. ‡ All at per cwt.

F.

	yd.	qr.	na.				sq. yd.	ft.	′				su. ft.	′	″		
1·	23·	3·	1·	at	14/6*	17·	342·	7·	5·	at	2/3+	33·	135·	5·	8·	at	3/6‡
2·	95·	2·	3·	at	17/3	18·	415·	3·	7·	at	4/6	34·	68·	3·	9·	at	14·d.
3·	62·	1·	2·	at	18/5	19·	298·	8·	10·	at	9·d.	35·	94·	6·	4·	at	6½d.
4·	84·	2·	2·	at	21/10	20·	654·	2·	9·	at	13½d.	36·	83·	5·	10·	at	3½d.
5·	56·	2·	1·	at	33/9	21·	150·	1·	8·	at	4½d.	37·	416·	10·	5·	at	2½d.
6·	34·	3·	2·	at	25/7	22·	361·	3·	6·	at	14·d.	38·	364·	9·	7·	at	9·d.
7·	78·	3·	3·	at	13/2	23·	543·	4·	5·	at	18·d.	39·	751·	8·	6·	at	21½d.
8·	49·	0·	1·	at	5/4½	24·	737·	6·	4·	at	2/9	40·	92·	11·	4·	at	13½d.
9·	35·	0·	3·	at	3/7¾	25·	312·	5·	3·	at	1/9	41·	36·	8·	2·	at	4¾d.
10·	81·	0·	2·	at	4/11½	26·	438·	0·	11·	at	6¾d.	42·	514·	6·	6·	at	7½d.
11·	97·	1·	3·	at	2/6¾	27·	356·	3·	1·	at	7½d.	43·	321·	7·	9·	at	8¼d.
12·	22·	1·	1·	at	1/8½	28·	863·	2·	7·	at	8¼d.	44·	84·	3·	11·	at	3¾d.
13·	46·	2·	3·	at	1/11	29·	185·	8·	6·	at	5/6	45·	150·	7·	3·	at	21·d.
14·	29·	1·	2·	at	4/5	30·	314·	7·	5·	at	8/3	46·	216·	9·	5·	at	32·d.
15·	54·	3·	3·	at	3/3	31·	263·	6·	9·	at	3/8	47·	345·	6·	3·	at	19·d.
16·	61·	2·	3·	at	6/4	32·	215·	4·	11·	at	5/10	48·	956·	11·	9·	at	15·d.

* All at per yard. + All at per sup. yard. ‡ All at per sup. ft.

G.

	ac.	r.	p.		£.	s.	d.
1·	314·	3·	20·	at	2·	12·	6·*
2·	415·	2·	35·	at	3·	10·	9·
3·	142·	1·	10·	at	2·	18·	6·
4·	236·	3·	5·	at	2·	15·	0·
5·	354·	2·	8·	at	4·	4·	0·
6·	95·	0·	32·	at	3·	13·	6·
7·	47·	1·	16·	at	3·	15·	0·
8·	212·	3·	21·	at	5·	10·	0·
9·	436·	0·	13·	at	7·	14·	6·
10·	545·	2·	31·	at	6·	15·	9·
11·	733·	1·	28·	at	5·	15·	6·
12·	363·	3·	14·	at	4·	18·	4·

* All at per acre.

	su. po.	ft.*		
13·	64·	20·	at	1/†
14·	57·	15·	at	1/1
15·	38·	11·	at	1/2
16·	20·	17·	at	1/3
17·	32·	215·	at	1/4
18·	40·	200·	at	1/5
19·	19·	95·	at	1/6
20·	84·	84·	at	/6
21·	26·	260·	at	/10
22·	54·	95·	at	/9
23·	73·	54·	at	/8
24·	65·	77·	at	/7

† All at pr. cust. s. p.

	cu. yd.	ft.	′		£.	s.	d.
25·	314·	25·	6·	at	1·	7·	6·‡
26·	45·	9·	3·	at	1·	12·	6·
27·	256·	13·	6·	at	2·	5·	9·
28·	144·	3·	0·	at		17·	6·
29·	436·	20·	9·	at		18·	9·
30·	65·	14·	3·	at	1·	13·	4·
31·	256·	5·	4·	at	2·	1·	8·
32·	134·	6·	6·	at		15·	6·
33·	38·	3·	9·	at		13·	4·
34·	42·	15·	6·	at	5·	4·	6·
35·	155·	13·	8·	at	3·	13·	6·
36·	74·	16·	6·	at	2·	12·	6·

‡ All at per cubic yard.

H.

	oz.	dwt.	gr.		£.	s.	d.	
1·	8·	13·	15·	at	3·	12·	6·	per oz.
2·	10·	15·	22·	at	3·	17·	6·	per oz.
3·	9·	10·	11·	at	44·	10·	0·	per lb.
4·	11·	7·	18·	at	4·	4·	0·	per oz.
5·	5·	12·	23·	at	32·	16·	8·	per lb.
6·	7·	9·	21·	at	28·	12·	6·	per lb.
7·	3·	18·	17·	at	2·	11·	8·	per oz.
8·	6·	16·	13·	at	2·	17·	4·	per oz.
9·	2·	14·	19·	at	25·	15·	6·	per lb.
10·	4·	11·	20·	at	3·	5·	6·	per oz.

	℥	ʒ	℈	gr.		£.	s.	d.
11·	11·	5·	2·	16·	at	1·	3·	8·*
12·	9·	7·	2·	18·	at	2·	12·	6·
13·	8·	6·	0·	19·	at	1·	5·	9·
14·	4·	3·	1·	15·	at		17·	6·
15·	10·	4·	0·	14·	at	4·	11·	6·
16·	5·	5·	2·	13·	at	3·	5·	9·
17·	6·	2·	0·	12·	at	2·	17·	9·
18·	3·	6·	2·	8·	at	3·	6·	4·
19·	1·	7·	1·	7·	at	1·	7·	8·
20·	7·	4·	0·	12·	at	1·	16·	4·

* All at per lb.

What is the cost

I.

1· Of 8·gal. 3·qt. 1·pt. of wine, at 15/3 per gallon?
2· Of 32·qr. 7·bu. 3·pe. of wheat, at 47/6 per qr.?
3· Of four score sheep, at 33/9 each?
4· Of 4·tons 17·cwt. of hay, at five guineas per ton?
5· Of a pipe of wine, at 18/4 per gallon?
6· Of repairing 3·mi. 7·fur. 21·po. of roadway, at 3/6 per pole?
7· Of 49· men's work for 17·we. 5·da. at 4/6 per day for each?
8· Of 72· doz. 7· bottles of wine, at 57/ per doz.?
9· Of a bullock weighing 13·cwt. 1·qr. 11·lb., at £3· 1·s. 6·d. per cwt.?
10· Of a sheep weighing 1·cwt. 8·lb., at 4½d. per lb.; also at 3¾d.?
11· Of a pig weighing 17·score 13·lb., at 3¾d. per lb., and at 4½d. per lb.?
12· Of 100· eggs, at 7· for 6·d.; and of 7· doz. at 5· for 2·d.?
13· Of keeping a horse for 2·yrs. 3·mo. 5·da., at 17/6 per week?
14· Of 45· packs of wool, ea. 240·lb., at 4½d., at 7¼d., and at 13½d. per lb.?
15· Of travelling expenses for 7·we. 5·da., at half-a-guinea a day?
16· Of 9·hhd. 48·gal. of wine, at 56/9 per hhd.?
17· Of 3·r. 28·p. 18·yd. of building land, at £280· per acre?
18· Of 30· bags of potatoes, each weighing 6·score 15·lb., at £4· 11·s. 8·d. per cwt.?
19· Of mowing 17·ac. 3·r. 15·po., at 4/6 per acre?
20· Of a labourer's wages for the year 1860·, at 2·s. a day?
21· Of printing 2000· copies of a book, consisting of 600· pages, 24· to a sheet, at £7· 17·s. 6·d. per sheet?

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* The old or customary sup. po. = 324· su. ft.

J.

1· What will the income-tax amount to on each of the following sums of money. I. At 5·d. in the £1· II. At 4·d. III. At 6·d. IV. At 7·d. V. At 7½d. VI. At 8·d. VII. At 8½d. VIII. At 13·d.?

	£.	s.		£.	s.		£.		£.		£.	s.
1·	643·	10·	4·	9864·	5·	7·	43168·	10·	35463·	13·	112·	15·
2·	985·	15·	5·	3186·	10·	8·	84305·	11·	28549·	14·	145·	18·
3·	369·	18·	6·	8432·	16·	9·	21864·	12·	96312·	15·	132·	6·

2· What will the poor rate amount to on each of the following rateable values. I. At 3·d. in the £1· II. At 4½d. III. At 5·d. IV. At 5¼d. V. At 5½d. VI. At 6½d. VII. At 8·d. VIII. At 6·d. IX. At 10·d. X. At 1·s. 3·d. XI. At 2·s. 1·d. XII. At 3·s. 4·d.?

	£.	s.		£.	s.		£.	s.		£.	s.	d.		£.
1·	64·	5·	6·	319·	10·	11·	7184·	5·	16·	4·	11·	6·	21·	76432·
2·	39·	7·	7·	454·	13·	12·	3642·	10·	17·	3·	2·	9·	22·	34365·
3·	81·	11·	8·	368·	15·	13·	1986·	5·	18·	8·	14·	6·	23·	21849·
4·	35·	10·	9·	755·	5·	14·	2184·	15·	19·	7·	13·	0·	24·	71687·
5·	23·	15·	10·	436·	10·	15·	3019·	8·	20·	5·	10·	0·	25·	54321·

EXTENDED APPLICATION OF PRACTICE.

Practice is usually employed only in the computation of money-values, but it is just as convenient for calculating lengths, weights, capacities, &c.

Fifth Example worked out. What is the united length of 347· pieces of ribbon, each 2·yd. 2·qr. 1·na.?

Example completed.

qr.		yd.	qr.	na	pieces.	y.	q.	n.
2·	$\frac{1}{2}$	347·	0·	0·	=L. of 347· ea.	1·	0·	0·
				2·				
		694·	0·	0·	=L. of 347· ea.	2·	0·	0·
na. 1·	$\frac{1}{8}$	173·	2·	0·	=L. of 347· ea.		2·	0·
		21·	2·	3·	=L. of 347· ea.			1·
		889·	0·	3·	=L. of 347· ea.	2·	2·	1·

Proof. By Comp. Multn.

yd.	qr.	na.	
2·	2·	1·	=L. of 1·piece.
		10·	
25·	2·	2·	=L. of 10·pieces.
		10·	
256·	1·	0·	=L. of 100·pieces.
		3·	
768·	3·	0·	=L. of 300·pieces.
102·	2·	0·	=L. of 40·pieces.
17·	3·	3·	=L. of 7·pieces.
889·	0·	3·	=L. of 347·pieces.

EXERCISE CVIII.

Explain every line. Prove each Example by Compound Multiplication.

A.

1· What is the gross weight of 735· Dutch cheeses, each 2·lb. 11·oz.?

2· How much wine will fill 40·doz. bottles, each containing 1·pt. 3·gi.?

3· What will 795· boxes of soap weigh, each being 2·cwt. 3·qr.?

4· If 1· piece of paper, contain $7\frac{3}{4}$ su. yd., what extent of wall will be covered by 355· such pieces?

5· Find the united length of 500· dress pieces, each $13\frac{1}{9}$ yd.?

6· How much cloth is required to make great coats for an army of 40000· men, each coat requiring 3·yd. 2·qr. 2·na.?

7· What is the gross weight of 42· days' rations for 50000· men, at 4·lb. $7\frac{1}{2}$oz. Av. per day for each man?

8· An Irish mile being $1\frac{3}{11}$ of an English mile, how many English are equal to $135\frac{5}{8}$ Irish?

9· How many Irish miles are equal to 1000· English?

10· Supposing a single copy of a book to weigh 13·oz. 5·dr., what would an edition, consisting of 2050· copies, weigh?

11· How much glass is contained in 35· sashes, of 15· panes each, a pane being 80· su. ins.?

12· What extent of ground will be required to make 320· allotments, of 1·r. 15·po. each?

13· How much hay will 32· horses consume in 50· days, at the rate of 1·qr. 2·lb. per day for each?

14· What extent of floor will be covered by 3500· tiles, each having a surface of 2· su. pr. 32· su. in.?

15· What is the weight of 1543· new sovereigns?

16· By how much would their weight be exceeded by that of their value in new shillings?

17· How much time is gained in seven years by rising daily at 6· o'clock instead of 20· minutes later?

18· What is the produce of 540·ac. 3·r. 20·po., at the average of 9·qr. 5·bu. per acre?

19· How much glass will make 1456· panes, each 1· su. ft. 3·′?

20· Each man's allowance of rum being $1\frac{1}{4}$ gill, what quantity will supply a ship's crew of 947· men for 180· days?

Find the united length of B.

	pieces.		yd.		pieces.		yd.		pieces.		yd.	qr.	n.
1·	969·	each	$2\frac{1}{4}$	8·	4168·	each	$5\frac{1}{16}$	15·	$9364\frac{1}{2}$	each	11·	2·	3·
2·	798·	each	$5\frac{5}{8}$	9·	3245·	each	$8\frac{3}{8}$	16·	$3216\frac{1}{4}$	each	13·	2·	3·
3·	345·	each	$3\frac{3}{8}$	10·	9653·	each	$7\frac{3}{16}$	17·	$7364\frac{1}{2}$	each	2·	2·	1·
4·	618·	each	$1\frac{1}{8}$	11·	7342·	each	$11\frac{5}{16}$	18·	$4936\frac{3}{4}$	each	6·	0·	2·
5·	591·	each	$2\frac{3}{4}$	12·	3856·	each	$4\frac{9}{16}$	19·	$2136\frac{1}{4}$	each	15·	1·	1·
6·	837·	each	$6\frac{3}{8}$	13·	5942·	each	$11\frac{5}{16}$	20·	$5943\frac{3}{4}$	each	23·	3·	2·
7·	498·	each	$7\frac{7}{8}$	14·	8365·	each	$4\frac{13}{16}$	21·	$3648\frac{1}{2}$	each	19·	3·	1·

Find the weight of C.

	parcels.		lb. Av.		hhd.		cwt.	qr.	lb.		bars.		lb.	oz.	dwt.	gr.
1·	450·	each	$7\frac{1}{2}$	8·	845·	each	3·	2·	7·	15·	152·	each	1·	10·	11·	6·
2·	398·	each	$6\frac{3}{4}$	9·	916·	each	4·	1·	8·	16·	495·	each	1·	5·	4·	10·
3·	943·	each	$9\frac{1}{8}$	10·	334·	each	3·	3·	5·	17·	368·	each	3·	9·	10·	11·
4·	658·	each	$3\frac{1}{4}$	11·	758·	each	2·	2·	12·	18·	298·	each	2·	7·	16·	18·
5·	484·	each	$5\frac{3}{8}$	12·	693·	each	5·	1·	16·	19·	734·	each	4·	5·	14·	16·
6·	739·	each	$7\frac{5}{16}$	13·	586·	each	3·	3·	24·	20·	473·	each	3·	1·	0·	22·
7·	936·	each	$9\frac{11}{16}$	14·	729·	each	1·	1·	21·	21·	645·	each	2·	3·	18·	18·

MENTAL CALCULATIONS.

In actual business the habit of mental computation is very generally acquired. One commonly employed mental method is to divide the given number of articles into dozens. Then *each penny in the price of one article becomes a shilling in the price of a dozen.* And the value of one dozen thus found is multiplied by the number of dozens given. For this kind of computation the subjoined Tables will be useful.

TABLES OF VALUES OF 1· DOZ. ARTICLES.

1· doz.		1· doz.		1· doz.		1· doz.		1· doz.	
d.	s.	d.	d.	d.	d.	d.	d.	d.	d.
At 1· costs	1·	At ¼ costs	3·	At ⅛ costs	1½	At $\frac{1}{16}$ c.	¾	At $\frac{13}{16}$ c.	9¾
At 2· ,,	2·	At ½ ,,	6·	At ⅜ ,,	4½	At $\frac{3}{16}$ c.	2¼	At $\frac{15}{16}$ c.	10¼
At 3· ,,	3·	At ¾ ,,	9·	At ⅝ ,,	7½	At $\frac{6}{16}$ c.	3¾	At $5\frac{1}{16}$ c.	5/0¾
At 4· ,,	4·	At 1¼ ,,	1/3	At ⅞ ,,	10½	At $\frac{7}{16}$ c.	5¼	At $8\frac{3}{16}$ c.	8/2¼
At 12· ,,	12·	At 2½ ,,	2/6	At 1⅛ ,,	1/1½	At $\frac{9}{16}$ c.	6¾	At $7\frac{6}{16}$ c.	7/3¾
At 15· ,,	15·	At 9¾ ,,	9/9	At 4⅝ ,,	4/7½	At $\frac{11}{16}$ c.	8¼	At $6\frac{7}{16}$ c.	6/5¼
&c.		&c.		&c.		&c.		&c.	

By extending and combining these Tables, the value of any number of articles (whether an exact number of dozens or not) may be easily calculated mentally.

EXAMPLES WORKED OUT, Find the value of

yds. d.	yds. d.	yds. d.	d.
1· 48· at 5¼	4· 120· at ⅛	7· 1000· at 1⅛	10· 12· doz. at 3⅛ each
2· 38· at 6¾	5· 300· at ⅜	8· 350· at $3\frac{3}{16}$	11· 24· doz· at $5\frac{3}{16}$ each
3· 57· at 4⅛	6· 643· at ⅝	8· 7000· at $1\frac{5}{16}$	12· 36· doz. at 1⅜ each

I.

48· = 4· doz. 1· doz. at 5¼d. cost 5·s. 3·d.

∴ 48· yd. at 5¼d. cost 4· times 5·s. 3·d. or £1· 1·s. 3·d.

II.

38· = 3· doz. + 2· 1· doz. at 6¾d. cost 6·s. 9·d.

2· at 6¾d. cost 1·s. 1½d.

∴ 38·yd. at 6¾d. c. 3· times 6·s. 9·d.+1·s. 1½d.=£1· 1·s. 4½d.

Or thus:

38· yd. at 6¾d. cost 38· sixpences + 38· halfpence + 38· farthings = 19·s. + 1·s. 7·d. + 9½d. = £1· 1·s. 4½d.

III.

57·=4· doz. + 9·=4¾ doz. 1· doz. at 4⅛d. cost 4·s. 1½d.

∴ 57· yd. at 4⅛d. cost 4¾ times 4·s. 1½d. = 19·s. 7⅛d.

Or thus:

57· yd. at 4⅛d. cost 4· times 4·s. 1½d. + 9· times 4⅛d. = 16·s. 6·d. + 3·s. 1⅛d. = 19·s. 7⅛d.

Or thus:

57·=5· doz.—3·; 5· doz. at 4⅛d. cost 20·s. 7½d. 3· cost 12⅜d.

∴ 57· yd. at 4⅛d. cost 20·s. 7½d, — 1·s. 0⅜d. = 19·s. 7⅛d.

IV.

120· yd. at ⅛d. cost 10· times 1½d. or 1 s. 3·d.

Or thus: 120· yd. at ⅛d. cost ⅛ of 10·s. or 1·s. 3·d.

V.

300· = 25· doz. ∴ 300· yd. at $\frac{3}{8}$d. cost 25· times $4\frac{1}{2}$d.

25· = 2· doz. + 1· 1· doz. at $4\frac{1}{2}$d. cost 4·s. 6·d.

∴ 25· at $4\frac{1}{2}$d. cost 9·s. $4\frac{1}{2}$d.

∴ 300· at $\frac{3}{8}$d. cost 9·s. $4\frac{1}{2}$d.

Or thus:

300· yd. at 1·d. cost £1· 5·s. ∴ 300· yd. at $\frac{1}{8}$d. cost 3·s. $1\frac{1}{2}$d.

∴ 300· yd. at $\frac{3}{8}$d. cost 9·s. $4\frac{1}{2}$d.

Or thus:

100· yd. at 1·d. cost 8·s. 4·d. 100· yd. at $\frac{1}{8}$d. cost 1·s. $0\frac{1}{2}$d.

100· yd. at $\frac{3}{8}$d. cost 3·s. $1\frac{1}{2}$d. 300· yd. at $\frac{3}{8}$d. cost 9·s. $4\frac{1}{2}$d.

VI.

643·=53·doz.+7·=$53\frac{1}{2}$doz.+1· And 1·doz. at $\frac{5}{8}$d. c. $7\frac{1}{2}$d.

But, $53\frac{1}{2}$ at $7\frac{1}{2}$ = 4· doz. + $5\frac{1}{2}$ at $7\frac{1}{2}$d.

4· doz. + $5\frac{1}{2}$ at $7\frac{1}{2}$d. cost 7·s. 6·d. × 4· + $7\frac{1}{2}$d. × $5\frac{1}{2}$

= £1· 10·s. + 3·s. $5\frac{1}{4}$d. = £1· 13·s. $5\frac{1}{4}$d.

∴ 643·yd. at $\frac{5}{8}$d. cost £1· 13·s. $5\frac{1}{4}$d.+$\frac{5}{8}$d.=£1· 13·s. $5\frac{7}{8}$d.

VII.

1000·=83·doz.+4·=$83\frac{1}{3}$doz. And 1·doz. at $1\frac{1}{8}$d. c. 1·s. $1\frac{1}{2}$d.

Now 83·doz.=7·doz.-1· And 7·doz. at $13\frac{1}{2}$d. c. £4· 14·s. 6·d.

∴ 83· at $13\frac{1}{2}$d. c. £4. 14·s. 6·d.—1·s. $1\frac{1}{2}$d.=£4· 13·s. $4\frac{1}{2}$d.

∴ 1000·yd. at $1\frac{1}{8}$d. c. £4· 13·s. $4\frac{1}{2}$d.+$4\frac{1}{2}$d.=£4· 13·s. 9·d.

Or thus:

100·yd. at 1·d. cost 8·s. 4·d. 100·yd. at $\frac{1}{8}$d. cost 1·s. $0\frac{1}{2}$d.

∴ 100· yd. at $1\frac{1}{8}$d. cost 9·s. $4\frac{1}{2}$d.

∴ 1000·yd. at $1\frac{1}{8}$d. c. 10· times 9·s. $4\frac{1}{2}$d. or £4· 13·s. 9·d.

VIII.

350·=30·doz.—10· 30·doz. at $3\frac{3}{16}$d. cost £4· 15·s. $7\frac{1}{2}$d.

∴ 350·yd. at $3\frac{3}{16}$d. c. £4·15·s. $7\frac{1}{2}$d.–2·s. $7\frac{7}{8}$d.=£4·12·s. $11\frac{5}{8}$d.

Or, 350·=7· times 50· And 50· at $3\frac{3}{16}$d. cost 13·s. $3\frac{3}{8}$d.

∴ 350·yd. at $3\frac{3}{16}$d. c. 7· times 13·s. $3\frac{3}{8}$d. or £4· 12·s. $11\frac{5}{8}$d.

IX.

1000· yd. at $1\frac{5}{16}$ cost £5· 9·s. $4\frac{1}{2}$d. (See Ex. VII. above.)

∴ 7000·yd. at $1\frac{5}{16}$ c. 7· times £5· 9·s. $4\frac{1}{2}$d.=£38· 5·s. $7\frac{1}{2}$d.

			d.		d.		£.	s.	d.
Or,	1000· yd.	at	1·	cost	1000·	or	4·	3·	4·
	1000· yd.	at	$\frac{1}{4}$	cost	250·	or	1·	0·	10·
	1000· yd·	at	$\frac{1}{16}$	cost	$62\frac{1}{2}$	or		5·	$2\frac{1}{2}$
∴	1000· yd.	at	$1\frac{5}{16}$	cost	$1312\frac{1}{2}$	or	5·	9·	$4\frac{1}{2}$
					7·				7·
∴	7000· yd.	at	$1\frac{5}{16}$	cost	$9187\frac{1}{2}$	or	38·	5·	$7\frac{1}{2}$

X.

12·doz. or 1·gross=1·doz. of dozs. 1·doz. at $3\frac{1}{8}$d. c. 3·s. $1\frac{1}{2}$d.
And 1· doz. at 3·s. $1\frac{1}{2}$d. costs £1· 17·s. 6·d.
∴ 12· doz. or 1· gross at $3\frac{1}{8}$d. cost £1· 17·s. 6·d.

XI.

24·doz.=2·gross or 2·doz. of dozs. 1·doz. at $5\frac{3}{16}$d. c. 5·s. $2\frac{1}{4}$d.
1· doz. at 5·s. $2\frac{1}{4}$d. costs £3· 2·s. 3·d.
∴ 24· doz. at $5\frac{3}{16}$ cost twice £3· 2·s. 3·d. or £6· 4·s. 6·d.

XII.

36·doz.=3·gross or 3·doz. doz. 1·doz. at $1\frac{3}{8}$d. c. 1·s. $7\frac{1}{2}$d.
1· gross at $1\frac{3}{8}$d. (= 1· doz. at 1·s. $7\frac{1}{2}$d.) costs 19·s. 6·d.
∴ 36·doz. or 3·gross at $1\frac{3}{8}$d. cost £2· 18·s. 6·d.

EXERCISE CIX.

EXAMPLES TO BE COMPUTED MENTALLY, BY THE DOZEN METHOD.

Nothing to be written but the Results. Find the value of

A.

doz. d.	doz. d.	doz. s. d.	gro. d.	doz. d.
1· 1· at $5\frac{1}{4}$	13· 3· at $\frac{1}{8}$	25· 1· at 1· 5·	37· 1· at $\frac{1}{8}$	49· 13· at $4\frac{1}{16}$
2· 1· at $3\frac{3}{4}$	14· 2· at $\frac{5}{8}$	26· 1· at 3· $2\frac{1}{4}$	38· 1· at $\frac{1}{16}$	50· 14· at $3\frac{1}{8}$
3· 1· at $1\frac{3}{8}$	15· 5· at $\frac{7}{8}$	27· 1· at 2· $6\frac{3}{4}$	39· 1· at $\frac{3}{8}$	51· 15· at $2\frac{1}{4}$
4· 2· at $2\frac{1}{4}$	16· 7· at $\frac{3}{8}$	28· 1· at 1· $1\frac{7}{8}$	40· 1· at $\frac{5}{8}$	52· 16· at $5\frac{3}{4}$
5· 3· at $2\frac{1}{8}$	17· 8· at $\frac{3}{4}$	29· 1· at 1· $10\frac{7}{8}$	41· 1· at $\frac{7}{8}$	53· 17· at $1\frac{3}{8}$
6· 4· at $5\frac{1}{16}$	18· 9· at $\frac{1}{4}$	30· 1· at 4· $9\frac{1}{4}$	42· 1· at $\frac{5}{16}$	54· 18· at $1\frac{1}{8}$
7· 3· at $2\frac{3}{16}$	19· 6· at $\frac{1}{2}$	31· 2· at 3· $6\frac{1}{8}$	43· 1· at $\frac{9}{16}$	55· 19· at $3\frac{5}{8}$
8· 1· at $7\frac{11}{16}$	20· 5· at $\frac{5}{16}$	32· 3· at 4· $1\frac{1}{4}$	44· 1· at $\frac{7}{16}$	56· 20· at $1\frac{7}{8}$
9· 1· at $5\frac{7}{8}$	21· 4· at $\frac{7}{16}$	33· 4· at 2· $2\frac{1}{8}$	45· 1· at $\frac{3}{4}$	57· 11· at $2\frac{3}{16}$
10· 5· at $2\frac{1}{8}$	22· 3· at $\frac{9}{16}$	34· 5· at 3· $1\frac{3}{4}$	46· 2· at $\frac{1}{2}$	58· 10· at $4\frac{9}{16}$
11· 3· at $4\frac{5}{8}$	23· 8· at $\frac{11}{16}$	35· 6· at 7· $10\frac{1}{2}$	47· 3· at $\frac{1}{4}$	59· 21· at $7\frac{11}{16}$
12· 6· at $7\frac{7}{8}$	24· 9· at $\frac{13}{16}$	36· 7· at 1·· $5\frac{3}{16}$	48· 4· at $\frac{11}{16}$	60· 24· at $8\frac{15}{16}$

B.

doz. d.	doz. art. d.	gro. doz d.	art. d.
1· $1\frac{1}{2}$ at $\frac{1}{4}$	13· 10· 4· at $6\frac{1}{8}$	25· 1· 7· at $8\frac{1}{2}$	37· 480· at $1\frac{5}{8}$
2· $2\frac{1}{4}$ at $\frac{1}{8}$	14· 13· 8· at $4\frac{1}{4}$	26· 2· 3· at $6\frac{3}{4}$	38· 395· at $\frac{1}{16}$
3· $3\frac{3}{4}$ at $\frac{3}{4}$	15· 9· 6· at $1\frac{3}{4}$	27· 2· 5· at $5\frac{1}{4}$	39· 260· at $\frac{1}{8}$
4· $4\frac{1}{6}$ at $\frac{1}{8}$	16· 7· 5· at $3\frac{1}{8}$	28· 6· 7· at $1\frac{7}{8}$	40· 550· at $\frac{3}{8}$
5· $3\frac{2}{3}$ at $\frac{1}{16}$	17· 11· 10· at $4\frac{5}{8}$	29· 3· 4· at $2\frac{7}{8}$	41· 450· at $2\frac{5}{8}$
6· $2\frac{1}{12}$ at $\frac{3}{8}$	18· 5· 7· at $2\frac{7}{8}$	30· 5· 2· at $3\frac{5}{8}$	42· 756· at $1\frac{3}{16}$
7· $3\frac{5}{12}$ at $\frac{5}{16}$	19· 4· 11· at $6\frac{1}{16}$	31· 4· 11· at $9\frac{3}{4}$	43· 950· at $9\frac{3}{4}$
8· $4\frac{7}{12}$ at $\frac{7}{16}$	20· 8· 1· at $1\frac{3}{16}$	32· 7· 9· at $5\frac{3}{16}$	44· 720· at $4\frac{1}{2}$
9· $5\frac{11}{12}$ at $\frac{9}{16}$	21· 9· 5· at $2\frac{5}{16}$	33· 2· 10· at $3\frac{11}{16}$	45· 136· at $5\frac{3}{8}$
10· $6\frac{1}{8}$ at $\frac{11}{16}$	22· 12· 3· at $1\frac{3}{8}$	34· 1· 8· at $4\frac{7}{8}$	46· 348· at $6\frac{1}{4}$
11· $4\frac{3}{8}$ at $\frac{7}{8}$	23· 6· 9· at $1\frac{11}{16}$	35· 1· 11· at $8\frac{3}{4}$	47· 843· at $7\frac{7}{8}$
12· $9\frac{5}{8}$ at $\frac{13}{16}$	24· 15· 4· at $1\frac{15}{16}$	36· 1· 4· at $9\frac{1}{4}$	48· 770· at $8\frac{1}{8}$

ALLOWANCES ON GOODS, OR TARE AND TRET.

Gross weight is that of goods and packages together.
Tare is allowed for the weight of the package.

Tare Suttle remains after deducting Tare from Gross.

Tret is an allowance of 4·lb. in every 104·lb. (that is $\frac{1}{26}$) of Tare Suttle, on goods subject to waste.

Tret Suttle remains after deducting Tret.

Cloff is an allowance of 2·lb. in every 3·cwt. (that is $\frac{1}{168}$) of Tret Suttle, for turning the scale in retailing.

Net weight remains after all the stipulated allowances are deducted.

EXAMPLE WORKED OUT. TARE PER PACKAGE GIVEN. Find the net weight of 9·hhd. of tobacco, each 18·cwt. 3·qr. 15·lb., allowing Tare 35·lb. per hhd., Tret and Cloff.

Example completed: by three methods.

The first is the best method, for it employs fewer figures than the third, and encounters fewer fractions than the second.

I.	cwt.	qr.	lb.	
	18·	3·	15·	Gross per hhd.
		1·	7·	Tare per hhd.
	18·	2·	8·	Tare Sut., per hhd.
			9·	
26·)	167·	0·	16·	Tare Suttle, 9·hhd.
	6·	1·	20·	Tret, 9·hhd.
168·)	160·	2·	24·	Tret Suttle, 9·hhd.
		3·	23$\frac{1}{7}$	Cloff,* 9·hhd.
	159·	3·	0$\frac{6}{7}$	Net weight, 9·hhd.

II.	cwt.	qr.	lb.	
	18·	3·	15·	Gross per hhd.
		1·	7·	Tare per hhd.
26·)	18·	2·	8·	Tare S. per hhd.
		2·	24·	Tret per hhd.
168·)	17·	3·	12·	Tret S· per hhd.
			11$\frac{19}{21}$	Cloff† per hhd.
	17·	3·	0$\frac{2}{21}$	Nt. w. per hhd.
			9·	
	159·	3·	0$\frac{6}{7}$	Net wt. 9·hhd.

III.	cwt.	qr.	lb.	
	18·	3·	15·	Gross per hhd.
			9·	
	169·	3·	23·	Gross, 9·hhd.
	2·	3·	7·	Tare, 9·hhd.
26·)	167·	0·	16·	Tare Suttle, 9·hhd.
	6·	1·	20·	Tret, 9·hhd.
168·)	160·	2·	24·	Tret Suttle, 9·hhd.
		3·	23$\frac{1}{7}$	Cloff,* 9·hhd.
	159·	3·	0$\frac{6}{7}$	Net weight, 9·hhd.

* In actual business these fractions would not be exactly worked out, but the nearest pound or half pound would be taken as sufficiently near the truth. Thus, in the present instance, the Cloff on 9·hhd. would be reckoned as 3·qr. 23$\frac{1}{4}$ lb., instead of 3·qr. 23$\frac{1}{7}$ lb.

† But, when the allowances are deducted from each package separately, then any error admitted in computing a fraction will, of course, be repeated as many times as there are packages. Thus, 11$\frac{19}{21}$ lb. being reckoned as 12·lb., would be $\frac{2}{21}$ lb. too much out of each hhd., or $\frac{6}{7}$ lb. on the 9·hhd. ‡ Being 9· times 35·lb.

When Tare is given at per cwt., &c., it is found by taking aliquot parts of gross.

EXERCISE CX.

Work each example in two ways. Explain every line in writing. Find the net weight of

1· 314·cwt. 3·qr. 18·lb., allowing Tare 4·lb. per cwt., Tret and Cloff.

2· Fifty-four chests of tea, each 3·qr. 19·lb., Tare 18·lb. per chest, and Cloff.

3· 48·bags of coffee, each 3·cwt. 22·lb., Tare 15·lb. per cwt.

4· 35·cwt. 2·qr., Tare 11·lb. per cwt., Tret and Cloff.
5· 43·cwt. 3·qr. 25·lb., Tare 9·lb. per cwt., Tret and Cloff.
6· 85·cwt. 1·qr. 21·lb., Tare 15·lb. per cwt., Tret and Cloff.
7· 72·cwt. 2·qr. 16·lb., Tare 10½ lb. per cwt., Tret and Cloff.
8· Bought 7665·lb of metal, at £1· 5·s. 8·d. per cwt., but was allowed one pound gratis on every score; what did it cost me?
9· What is the net value of 14· casks of oil, each weighing 3·cwt. 2·qr. gross, allowing Tare at 15·lb. per cwt., and that a gallon weighs 7½ lb., at 13·s. 6·d. per gallon?
10· What is the net value of 54·bags of cotton wool, each 2·cwt., 3·qr. 5·lb., allowing Draft at 1·lb. per bag, Tare at 2½ lb. per 100·lb., and reckoning it was worth 14·d. per lb.?

SUPERFICIAL OR SQUARE MEASURE.

A Figure (in Geometry,) is that which is enclosed by one or more boundaries.

The space thus enclosed is called the Surface of the Figure; and the number of superficial units, of any given kind, which will exactly cover the surface of the figure is its **Area.** (P. 173·)

No. of sup. inches,	which will exactly cover the surface of the figure, expresses its	Area, in sup. inches,
No. of sup. feet,		Area, in sup. feet,
No. of sup. yards,		Area, in sup. yards,
No. of sup. links,		Area, in sup. links,
No. of sup. chains,		Area, in sup. chains,
No. of acres,		Area, in acres.

A Parallelogram is a foursided Figure, whose opposite sides are parallel.

A Rectangle is a parallelogram having four right angles.* (P. 177·)

A Square is an equilateral rectangle: that is to say, a figure having four equal sides and four right angles.

Diagram 5·

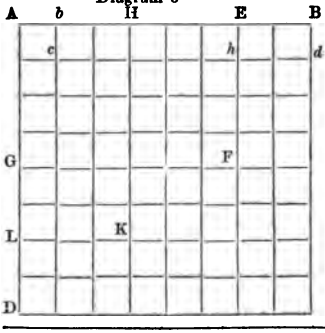

Diagram 5· is a square; and, supposing each of the twelve units in either side, like A*b*, to be a lineal chain, then each of the smaller squares, like A*c*, will represent a superficial chain. And the number of these covering the surface of the Figure, (that is its Area in superficial chains,) *coincides with* the

---

* Every parallelogram which has one right angle, has all its angles right angles. (Euclid Bk. I. Prop. XLVI. Cor.)

No. of lin. ch. in the length AB, repeated as many times as there are lin. ch. in the breadth AD.

For, in the strip A*d*., which is as long as the Figure, (namely, 12· chains,) and one chain wide, there are just as many (12·) *superficial* chains, like A*c*, as there are *lineal* chains in AB, the *length* of the Figure.

And the Surface of the Figure comprises just as many such strips as there are lineal chains in AD, the *breadth* of the Figure.

Length of Figure = 12· Lineal Chains. Breadth = 12· Lineal Chains.

Area of Figure = 12· times 12· or 144· Superficial Chains.

Similarly, of the Rectangle AEFG,
Length AE = 6· Lineal Chains. Breadth AG = 4· Lineal Chains.
∴ Area = 4· times 6· or 24· Superficial Chains.

For the strip A*h* contains 6· sup. ch., and the Rectangle AEFG comprises four such strips.

Also, of the Rectangle AHKL,
Length AL = 6· Lineal Chains. Breadth AH = 3· Lineal Chains.
∴ Area = 3· times 6· or 18· Superficial Chains.

In all which Examples, as in that of any Rectangle,

Area ≐ No. Lin. Un. in Length × No. Lin. Un. in Breadth.

In this last expression, we do not, of course, mean that we Multiply by Lineal Units, for this is impossible from the nature of Multiplication, which is simply *repetition of the Multiplicand.* And, the Multiplier merely indicating the number of such repetitions, or, in other words, *how many times* the Multiplicand must be taken to yield the required product, it becomes evident that

No Multiplier can be a Concrete Quantity. (See p. 149·)

We can understand the repetition of a given Multiplicand *any number of times*, but what idea can be conceived of

12· Lineal Chains repeated 12· *chains times?*
Or, £19· 19·s. 11$\frac{3}{4}$d. repeated £19· 19·s. 11$\frac{3}{4}$d. *times?*
Or, 18· ft. 11· in. repeated 7· *inches times?*

Such expressions are utterly meaningless. What is meant, then, by Area ≐ No. Lin. Un. in Length × No. Lin. Un. in Breadth is, that the *Number* which expresses the Length, being Multiplied by the *Number* which expresses the Breadth, will produce the *Number* expressing the Superficial Units in the Area.

This is also the only intelligible interpretation of the rule as ordinarily given; namely,

Area = Length × Breadth,

a statement which, although conveniently concise, is yet, for two reasons, objectionable. For, as already shewn,

I. Breadth, being a Concrete Quantity, cannot be a Multiplier.

II. Area, consisting of *Superficial Units*, cannot be produced by the repetition of *Lineal Units.* See Implicit change of Unit. Page 84·

The latter of these two objections is removed by employing the Sign of Coincidence instead of the Sign of Equality.

Area $\doteq$ *Length* × *Breadth*,

a formula which may be advantageously retained provided the two Changes of Unit be not forgotten; namely,

I. Units of Length, into Superficial Units in one Strip.

II. Units of Breadth, into Multiplier, being the No. of such Strips.

And, taken in this sense, it is true of any Parallelogram; for, Euclid* having demonstrated that all Parallelograms of the same length and breadth are equal, the area of any Parallelogram is known when that of the Rectangle of like length and breadth is ascertained.

Hence, we always work for the Area of the Rectangle.

To find the Area of any Parallelogram.

Multiply base by perpendicular height.

FIRST EXAMPLE WORKED OUT. What is the area of a field, having the form of a Rectangle, (or other Parallelogram,) the length being 1975· links, and the breadth 725· links?

Example completed.

lin. ch. supl. ch.
Length 19·75 $\doteq$ 19·75 = Area, if 1· ch., or 100· links wide.
Breadth 7·25 $\doteq$ 7·25 = No. of strips, each 1· chain wide.
·9875 = Area, if ·05 ch., or 5· links wide.
3·95 = Area, if ·2 ch., or 20· links wide.
acre. su. ch 138·25 = Area, if 7· ch., or 700· links wide.
1· = 10·)143·1875 = Area, if 7·25 ch., or 725· links wide.
14·31875 = No. of acres in 143·1875 supl. ch.

acres. ac. r. po.
∴ Area of field = 14·31875 = 14· 1· 11· (EXERCISE LXXXV.)

Or thus:

su. ch. acres. ac. r. po.
19·75 = 1·975 = 1· 3· 36· = Area, if 1· ch. wide.
7¼ = No. of strips, each 1· ch. wide.
13· 3· 12· = Area, if 7· ch. wide.
1· 39· = Area, if ¼ ch. wide.
14· 1· 11· = Area, if 7¼ ch. wide.

The two foregoing processes may be respectively proved by exchanging Multiplicand for Multiplier in each instance; thus:

lin. ch. sup. ch. ac. r. po.
B = 7·25 $\doteq$ 7·25 or 2· 36· being Area of strip 1· ch. wide.
L = 19·75 $\doteq$ 19·75 or 19¾ being No. of strips 1· ch. wide.
143·1875 or 14· 1· 11· being Area when 19·75 ch. wide.

Or thus: By Vulgar Fractions.

c. su. ch. su. ch. su. ch.
Area of strip 1· wide = 19·75 = 19¾ = $\frac{79}{4}$
No. of such strips = 7·25 = 7¼ = $\frac{29}{4}$

su. ch. su. ch. ac.
∴ Area of field = $\frac{79}{4} \times \frac{29}{4} = \frac{2291}{16} = \frac{2291}{160}$
= 14·ac. 1·r. 11·po. (Ex. LXXXIV.)

---

* Book I. Prop. XXXVI.

The divisions in the sides of Diagram 5· may, of course, be taken to represent *any lineal* units; as, inches, feet, yards, poles, miles, &c. And then the squares within the Figure will represent *corresponding superficial* units; each superficial unit in the Area having the same name as the lineal unit employed to measure the length and breadth.

SECOND EXAMPLE WORKED OUT. What is the superficial extent of a floor 14·ft. 9·in. long, by 10·ft. 7·in. broad?

Example completed.

I. Taking the length and breadth in lineal inches.

$$\begin{array}{lll}
L = \overset{\text{ft.}}{14\cdot}\ \overset{\text{in.}}{9\cdot} = \overset{\text{lin. in.}}{177\cdot} \doteq \overset{\text{sup. in.}}{177\cdot} & = & \text{Area of strip 1· in. wide.} \\
B = 10\cdot\ 7\cdot = 127\cdot \doteq \underline{127\cdot} & = & \text{No. of strips each 1· in. wide.} \\
\qquad\qquad\qquad\qquad 1239\cdot & = & \text{Area of floor, if}\ \ \ 7\cdot\ \text{in. wide.} \\
\qquad\qquad\qquad\quad \underline{21240\cdot} & = & \text{Area of floor, if 120· in. wide.} \\
1\cdot\ \text{sup. prime} = \overset{\text{su. in.}}{12\cdot})\underline{22479\cdot} & = & \text{Area of floor, if 127· in. wide.} \\
1\cdot\ \text{sup. foot} = \overset{\text{su. pr.}}{12\cdot})\ \underline{1873\cdot} & & \text{sup. pr. 3· sup. in.} \\
\therefore\ \textit{Area of Floor} = \underline{\underline{156\cdot\ \text{sup. ft. 1· sup. pr. 3· sup. in.}}} & &
\end{array}$$

II. By Vulgar Fractions.

$$\begin{array}{ll}
\text{Area of strip 1· ft. wide} & = \overset{\text{su. ft.}}{14\tfrac{3}{4}} = \overset{\text{su. ft.}}{\tfrac{59}{4}} \\
\text{No. of such strips} & = 10\tfrac{7}{12} = \tfrac{127}{12} \\
\therefore\ \text{Area of floor} & = \overset{\text{su. ft.}}{\tfrac{59}{4}} \times \tfrac{127}{12} = \overset{\text{su. ft.}}{\tfrac{7493}{48}} = \overset{\text{su. ft.}}{156\tfrac{5}{48}} \\
 & = \overset{\text{su. ft.}}{156\cdot}\ 1\cdot'\ 3.''
\end{array}$$

It will be noticed that all the methods employed in working the above two Examples proceed on the same principle as Practice. For, in all, we first change one of the given quantities into Units, &c., of the same kind as the answer sought; and, secondly, change the other given quantity into a Multiplier.

For other methods, see Duodecimal Multiplication.

A Triangle is a Figure contained by three straight lines.

The Area of any Triangle is one half the Area of a Parallelogram having the same length and breadth. (EUCLID, BOOK I. PROP. XLI.)

For "length" we often say "base," and for "breadth," "perpendicular height."

To find the area of a Triangle, Base and Perpendicular Height being given.

There are three ways of doing this.

I. Find Area of Parallelogram of same length and breadth, and divide by 2· Or,

II. Find Area of Parallelogram of same length and half the breadth. Or,

III. Find Area of Parallelogram of same breadth and half the length.

The same three methods may be thus described.

I. Multiply base by perp. height and Divide by 2·

II. Multiply base by half perpendicular height.

III. Multiply half base by perpendicular height.

THIRD EXAMPLE WORKED OUT. Required the Area of a Triangle, of which the Base measures 1945· links, and the Perpendicular height 864· links.

Example completed.

I.

$$\begin{aligned}\text{Area of Triangle} &= (\overset{\text{sup. ch.}}{19{\cdot}45} \times 8{\cdot}64) \div 2{\cdot} = \overset{\text{sup. ch.}}{168{\cdot}048} \div 2{\cdot}\\ &= 84{\cdot}024{\cdot}\ \text{sup. ch.} = 8{\cdot}4024\ \text{ac.}\\ &= 8{\cdot}\text{ac.}\ \ 1{\cdot}\text{r.}\ \ 24{\cdot}\text{po.}\ \ 11{\cdot}616\ \text{sq. yd.}\end{aligned}$$

II.

$$\begin{aligned}\text{Area of Triangle} &= \overset{\text{sup. ch.}}{19{\cdot}45} \times 4{\cdot}32 = \overset{\text{sup. ch.}}{84{\cdot}024} = \overset{\text{ac.}}{8{\cdot}4024}\\ &= 8{\cdot}\text{ac.}\ \ 1{\cdot}\text{r.}\ \ 24{\cdot}\text{po.}\ \ 11{\cdot}616\ \text{sq. yd.}\end{aligned}$$

III.

$$\begin{aligned}\text{Area of Triangle} &= \overset{\text{sup. ch.}}{9{\cdot}725} \times 8{\cdot}64 = \overset{\text{sup. ch.}}{84{\cdot}024} = \overset{\text{ac.}}{8{\cdot}4024}\\ &= 8{\cdot}\text{ac.}\ \ 1{\cdot}\text{r.}\ \ 24{\cdot}\text{po.}\ \ 11{\cdot}616\ \text{sq. yd.}\end{aligned}$$

DUODECIMALS.

As a Decimal is any Fraction which has for its Denominator a power of 10·, so a Duodecimal Fraction (or Duodecimal) is one whose Denominator is any power of 12·

Thus: $\frac{7}{12}, \frac{5}{144}, \frac{11}{1728}$ (being $\frac{7}{12^1}\ \frac{5}{12^2}\ \frac{11}{12^3}$) &c., are Duodecimal Fractions.

The word "Duodecimal" is derived from the Latin "duodecim," *twelve.*

Lineal, Superficial, and Solid Feet are duodecimally divided and subdivided. See Tables. In Superficial Measure, for example, the

Su. Prime (= Surf. of Rect., 1·ft. by 1·in.) $= \overset{\text{sup. ft.}}{\frac{1}{12}}$

Su. Second $\left\{\begin{array}{l}=\text{Surf. of Rect., 1·ft. by 1·line}\\ =\text{Surf. of Sqre, 1·in by 1·in.}\end{array}\right\} = \frac{1}{144} = \overset{\text{pr.}}{\frac{1}{12}}$

Su. Third (= Surf. of Rect., 1·in. by 1·line) $= \frac{1}{1728} = \frac{1}{144} = \overset{\text{sec.}}{\frac{1}{12}}$

Su. Fourth (= Surf. of Sq., 1·line by 1·line) $= \frac{1}{20736} = \frac{1}{1728} = \frac{1}{144} = \overset{\text{thd.}}{\frac{1}{12}}$

DUODECIMALS WITH DENOMINATORS SUPPRESSED.

We have no Duodecimal Point. But, just as Decimals are written without their Denominators in successive *places* to the right of Units, so also Duodecimals may be expressed by their Numerators only, (their Denominators being understood,) in *columns* decreasing in value, in duodecimal gradation, from right to left. Hence, Nos. representing

12ths, stand in 1st Col. of Duo.; being 1st Col. to right of Units.
144ths, „ in 2nd Col. of Duo.; „ 2nd Col. to right of Units.
&c. &c. &c.

Hence, in the expression $19\cdot$ sup. ft. $11\cdot' \ 5\cdot'' \ 3\cdot''' \ 6\cdot''''$ Unit being $1\cdot$ sup. ft.

The 19· (for 19· sup. ft.) occupies Unit's Column.
The 11· (for 11· primes,) „ 1st Col. of Duodecimals.
The 5· (for 5· seconds,) „ 2nd Col. of Duodecimals.
The 3· (for 3· thirds,) „ 3rd Col. of Duodecimals.
The 6· (for 6· fourths,) „ 4th Col. of Duodecimals.

And it is now evident why Primes, Seconds, Thirds, &c. are so named.

And, when we have to Multiply feet, primes, inches, &c. by a Multiplier containing Duodecimal Fractions, we write these latter without Denominators, under the corresponding Duodecimals of Multiplicand.

Thus, in preparing to Multiply $\overset{\text{sup. ft.}}{19\cdot} \ \overset{\text{pr.}}{11\cdot} \ 5\cdot'' \ 3\cdot''' \ 6\cdot''''$ by $16\cdot + \frac{7}{12} + \frac{5}{144} + \frac{11}{1728}$, we write down

The Multiplicand $19\cdot \ 11\cdot' \ 5\cdot'' \ 3\cdot''' \ 6\cdot''''$
The Multiplier $16\cdot \ \ 7\cdot \ \ 5\cdot 11\cdot = 16\cdot + \frac{7}{12} + \frac{5}{144} + \frac{11}{1728}$

A very convenient practice, but too generally misunderstood to imply that we Multiply by Feet, Inches, &c., which is absurd and impossible.

EXERCISE CXI.

1· Draw a superficial prime, second, (in both forms,) third and fourth.

2· Draw, on the scale of an inch to the foot, six Diagrams, of the following dimensions respectively. Carry a line from each division of each of the four sides quite across the Figure to the corresponding division of the opposite side.

I. A Rectangle 5· ft. by 3· ft., shewing that Area=15· sup. feet.
II. A Rectangle 4· ft. by 7· in., „ „ Area=28· sup. primes.
III. A Rectangle 3· ft. by 5· lines, „ „ Area=15·sup.sec.or in.
IV. A Rectangle 8· in. by 5· in., „ „ Area=40·sup.sec.or in.
V. A Rectangle 11· in. by 7· lines, „ „ Area=77· sup. thirds.
VI. A Rectangle 7· li. by 6· lines, „ „ Area=42· sup. fourths.

DUODECIMAL, OR CROSS MULTIPLICATION.

When the Length and Breadth of a Rectangle, or other Parallelogram, are taken in Feet, and Inches, &c., the Area may be at once found in Superficial Feet, Primes, &c.,

by Duodecimal, or (as it is sometimes called,) Cross Multiplication, which we now proceed to explain.

The examples in Exercise CXI. prove that

I. L in ft. with B in ft. gives A. in sup. feet
II. L in ft. „ B in in. „ A. in sup. primes
III. L in ft. „ B in lines „ A. in sup. seconds
IV. L in in. „ B in in. „ A. in sup. in. or sec.
V. L in in. „ B in lines „ A. in sup. thirds
VI. L in lines „ B in lines „ A. in sup. fourths.

Whence it is usual, erroneously to say that

I. Feet × Feet give Feet.
II. Feet × Inches give Inches.*
III. Feet × Lines give Seconds.
IV. Inches × Inches give Seconds.
V. Inches × Lines give Thirds.
VI. Lines × Lines give Fourths.

The correct statement to be employed instead of the foregoing, is

I. Units × Units produce Units.
II. Units × 12ths produce 12ths.
III. Units × 144ths pro. 144ths.
IV. 12ths × 12ths pro. 144ths.
V. 12ths × 144ths pro. 1728ths.
VI. 144ths × 144ths pro. 20736ths.

GENERAL PRINCIPLE FOR MULTIPLICATION.

PRINCIPLE XXVII. *Multiplying by a Number standing in any column of Duodecimals will remove the Product-Figures as many columns to the right as Multiplier is to the right of Units Column.*

Compare this with Principle XII.

To Multiply Feet, Primes, Inches, &c. by any Duodecimal Multiplier.

First. Place Multiplier under Multiplicand; place for place, that is to say, Units under Units, 12ths under 12ths, &c., column for column.

Second. Multiply successively by each separate part of Multiplier; placing Product-Figures in their proper columns, according to Principle XXVII., and carrying *one* for every *twelve* from each column to next on the left.

FIRST EXAMPLE WORKED OUT. Find the area of a wall 14·ft. 11¼ in. by 9·ft. 6¾ in.

Example completed.

	ft.	in.	li.		su. ft.	pr.	in.				
Length	14·	11·	3·	≐	14·	11·	3·			=	Ar. of strip 1·ft. wide.
Breadth	9·	6·	9·	≐	9·	6·′	9·″			=	No. of such strips.
						11·	2·	5·‴	3·⁗	=	Ar. if 9·li. wide.
					7·	5·	7·	6·		=	Ar. if 6·in. wide.
					134·	5·	3·			=	Ar. if 9·ft. wide.
Area of wall				=	142·	10·	0·	11·	3·	=	9·ft. 6¾ wide.

---

* Primes are here meant; but Artificers generally call the twelfth part of a sup. ft. an inch, at the same time aware that "144· sq. in. = 1· sq. ft."

First Line.

a. 3· sup. in. × $\frac{9}{144}$ = $\frac{9}{144}$ of 3· sup. in. (Pr. ix.) = 27· sup. 4ths. = $\overset{\text{su.}'''}{2\cdot}$ $\overset{''''}{3\cdot}$

Set down 3·'''' *and carry* 2·'''

b. 11· sup. pr. × $\frac{9}{144}$ = $\frac{9}{144}$ of 11· sup. pr. (Pr. ix.) = 99· sup. 3rds., making, with 2·''' carried, 101· sup. 3rds., = 8·'' 5·'''

Set down 5·''' *and carry* 8·''

c. 14· sup. ft. × $\frac{9}{144}$ = $\frac{9}{144}$ of 14· sup. ft. (Pr. ix.) = 126· sup. 2nds., or in., making, with 8·in. carried, 134· sup. in. = 11·' 2·''

Set down 2·'' *and carry* 11·'

d. *Set down* 11· *primes.*

Second Line.	Third Line.
a. 3·su. in. × 6·' = $\frac{6}{12}$ of 3·su. in. = 18·'''	a. 3·su. in. × 9· = 27·su. in.
b. 11·su. pr. × 6·' = $\frac{6}{12}$ of 11·su. pr. = 66·''	b. 11·su. pr. × 9· = 99·su. pr.
c. 14·su. ft. × 6·' = $\frac{6}{12}$ of 14·su. ft. = 14·'	c. 14·su. ft. × 9· = 126·su. ft.

The work may be proved in a variety of ways.

I.

By exchanging Md. for Mr. (Pr. vi.)

```
       su.ft.  ,  "        ft.
B ≑     9·  6· 9· = A. if 1· lg.
L ≑    14· 11· 3· strips 1· lg.
       ------------
          2· 4· 8¼''' = A. 3·'' lg.
     8·   9· 2· 3·    = A. 11·' lg.
   133·  10· 6·       = A. 14· f. lg.
A. = 142· 10· 0· 11¼ if 14·ft. 11¼' lg.
     ================
```

II.

By Practice.

```
              su.ft.  ,  "              ft.
6·'  | ½ |    14· 11· 3·          = A. if 1· lg.
     |   |             9·
     |   |   ------------
     |   |   134·  5· 3·          = A. if 9· lg.
9·'' | ⅛ |     7·  5· 7· 6·'''    = A. if 6·' lg.
     |   |        11· 2· 5¼       = A. if 9·''' lg.
             ------------------
A. = 142· 10· 0· 11¼ if 14·ft. 11¼' lg.
     ==================
```

III. *By Vulgar Fractions.*

Area of strip $\overset{\text{ft.}}{1\cdot}$ wide ≑ $\overset{\text{su. ft.}}{14\cdot}$ $\overset{'}{11\cdot}$ $\overset{''}{3\cdot}$ = $14\frac{15}{16}$ sup. ft.

No. of such strips = 9· 6· 9· = $9\frac{9}{16}$

∴ Area of wall = $\overset{\text{sup. ft.}}{14\frac{15}{16}} \times 9\frac{9}{16} = \frac{36567}{256}$ sup. ft.

= $\overset{\text{sup. ft.}}{142\cdot}$ $\overset{'}{10\cdot}$ $\overset{''}{0\cdot}$ $\overset{'''}{11\cdot}$ $\overset{''''}{3\cdot}$

IV. *By Decimal Fractions* (*sometimes*).

Area of wall = $\overset{\text{sup. ft.}}{14\cdot9375} \times 9\cdot5625$ or = $\overset{\text{sup. ft.}}{9\cdot5625} \times 14\cdot9375$

= $\overset{\text{sup. ft.}}{142\cdot83984375}$

= $\overset{\text{sup. ft.}}{142\cdot}$ $\overset{'}{10\cdot}$ $\overset{''}{0\cdot}$ $\overset{'''}{11\cdot}$ $\overset{'''}{3\cdot}$

V. *By Reduction and Multiplication.* See Second Example worked out, page 234·

CUSTOMARY AND STATUTE LINEAL MEASURES.

The old or customary lineal pole or landyard = 18·ft.
The statute lineal rod, pole, or perch = 16½ ft.
∴ Ratio of Customary to Statute pole = $\frac{18}{16\frac{1}{2}} = \frac{36}{33} = \frac{12}{11}$
Ratio of Statute to Customary pole = $\frac{16\frac{1}{2}}{18} = \frac{33}{36} = \frac{11}{12}$

To Reduce Statute Lineal Units to Customary.

Multiply by 11· and Divide by 12·

To Reduce Customary Lineal Units to Statute.

Multiply by 12· and Divide by 11·

CUSTOMARY AND STATUTE SUPERFICIAL MEASURES.

An acre of old or customary measure contains the same number of roods and poles as the statute acre.

But, Cust. Sup. po. = 18· times 18· sup. ft. = 324· sup. ft.
Whilst, Stat. Sup. po. = 16½ times 16½ sup. ft. = 272¼ sup. ft.

Hence, Ratio of Cust. Sup. po. to Stat. Sup. po. $= \frac{324}{272\frac{1}{4}} = \frac{144}{121}$

And, Ratio of Stat. Sup. po. to Cust. Sup. po. $= \frac{272\frac{1}{4}}{324} = \frac{121}{144}$

To Reduce Statute Area to Customary.

Multiply by 121· and Divide by 144·

To Reduce Customary Area to Statute.

Multiply by 144· and Divide by 121·

To find customary area of a small piece of land, take the dimensions in customary poles or landyards of 18· lin. ft. or 6· paces, each. The Area will then come out in customary superficial poles or landyards.

Or, take dimensions in feet. Find Area in sup.ft. Divide by 324·

EXERCISE CXII.

EXAMPLES IN THE MENSURATION OF PARALLELOGRAMS AND TRIANGLES.

Explain every line, fully, in writing; specially noting the change of Unit. Work each Example in, at least, two ways; Vulgar Fractions to be one.

A.

Find the Area

1· Of a square field, measuring 948· links in the side.*
2· Of a parallelogram, 3450· links long, by 640· links broad.*
3· Of a rectangular estate, 19348· links long, by 8495· links wide.*
4· Of an oblong garden, 98· yards long, by 15· yards broad.*
5· Of a floor, a rectangle in shape, 18·ft. 11·in. by 14·ft. 3·in.
6· Of a quadrangular park, two adjacent sides of which measure, respectively, 1·mi. 5·fur. 25·po., and 45· chains.*
Reduce both dimensions to miles, or to poles, or to chains.
7· Of a triangular plantation, which is 945· links in length, and 240· links in perpendicular breadth.*

---

* Examples marked thus to be worked out: I. In Statute Measure. II. In Old or Customary Measure.

8· Of a three-sided orchard, 28· po. long, and 15· po. wide.

9· Of a trilateral pond, 245· ft. long, by 64· ft. broad.

10· A certain road has a uniform width of 17· ft.; how much land do 2½ miles of it occupy?

11· At £2· 17·s. 6·d. per acre, Statute Measure, what is the rent of a rectangular field, 209· paces long, and 121· paces wide?

12· What is the area of an oblong court-yard, 189· ft. long, and 88· ft. wide; and what would it cost to pave, at 3·s. 10½d. per square yard?

13· Round the inner sides of the four walls enclosing a garden runs a foot-path 3· ft. wide. The garden is a rectangle, 11· po. long, by 5· po. wide. Required the extent of the road, and of the garden-ground within it.

1· *Draw the Figure.*
2· *Find Area, including path.*
3· *Find Area, within path.*
4· *Subtract.*

14· What would it cost to plaster a chamber, 15· ft. long, 10½ ft. broad, and 9· ft. high, at 14½d. per square yard?

See Second Example worked out, Page 279·

15· If a person rent a meadow, of rectangular form, 3245· links long, and 1775· links wide, at £3· 15·s. per acre, customary measure, and sub-let it at the same rate per statute acre, how much will he gain?

16· If the rent of a triangular plat of ground, 19· po. by 22· po. be £6· 10·s. 7½d., what is the rate per statute acre, and per customary acre?

17· A rectangular garden is intersected by two paths, having the uniform width of 4· ft. 6· in., and crossing each other, at right angles, in the middle of the garden. Ascertain the extent of each of the four rectangular plats into which the garden is thus divided, the paths being, respectively, 50· ft. and 40· ft. long. Find, also, how much space the paths occupy.

Draw the Figure.

18· How much land is occupied by all the turnpike roads in England, taking their united length at 24,000· miles, and their average breadth 30· ft.?

19· If this land were in tillage, what rent would it produce at £2· 15·s. per statute acre?

20· Find the two similar results with reference to the cross roads of England, said to amount, in united length, to 100,000· miles; allowing their average breadth to be one pole.

21· A rectangular field, measuring 18·75 chains, by 11·3 chains, is bounded by a ditch of the uniform width of 15· links. What space does the top of the ditch occupy?

See Example 13·

22· What area of gilding is there on a picture-frame 21½ in. by 15¾ in., outside measure, and 2⅜ in. wide?

Draw the Figure.

B.

Find the respective Areas of the Rectangles having the following dimensions. Explain every line of the work. Solve each in two ways, Vulgar Fractions to be one.

	ft.	in.		ft.	in.		ft.	′	″		ft.	in.	li.
1·	8·	11$\frac{3}{4}$	by	4·	3$\frac{1}{4}$	11·	7·	3·	6·	by	2·	1·	3·
2·	17·	6$\frac{1}{4}$	by	5·	9$\frac{3}{4}$	12·	4·	2·	8·	by	1·	4·	9·
3·	19·	6$\frac{1}{2}$	by	7·	5$\frac{1}{4}$	13·	8·	5·	9·	by	3·	6·	1·
4·	15·	4$\frac{1}{4}$	by	9·	10·	14·	7·	6·	4·	by	1·	5·	8·
5·	38·	7·	by	8·	2$\frac{1}{4}$	15·	3·	5·	7·	by	4·	6·	11·
6·	95·	8·	by	9·	3$\frac{1}{2}$	16·	9·	2·	11·	by	9·	7·	5·
7·	64·	10·	by	12·	7·	17·	5·	6·	1·	by	5·	9·	7·
8·	44·	8·	by	33·	8·	18·	2·	4·	10·	by	2·	4·	10·
9·	85·	6·	by	21·	4·	19·	6·	7·	7·	by	6·	7·	7·
10·	57·	5·	by	36·	3·	20·	7·	5·	4·	by	7·	5·	4·

The cost of superficial measures may be found by Practice. EXERCISE CVII. F. and G., pages 261· and 262·, but fractional methods are also frequently used.

FIRST EXAMPLE WORKED OUT. Find the cost of flooring a rectangular room, 32· ft. 8· in. long, by 18· ft. 6· in. broad, at 21·s. per square of 100· superficial feet.

Example completed.

$$\text{Area in squares} = 32\tfrac{2}{3} \times 18\tfrac{1}{2} \div 100\cdot = \frac{98\cdot \times 37\cdot}{3\cdot \times 2\cdot \times 100\cdot}$$

$$\text{Cost of flooring} = \frac{21\cdot\text{s.} \times 98\cdot \times 37\cdot}{100\cdot \times 3\cdot \times 2\cdot} = 126\cdot 91\text{s.} = £6\cdot\ 6\cdot\text{s. } 10\tfrac{3}{4}\text{d. } \tfrac{17}{25}\text{q.}$$

SECOND EXAMPLE WORKED OUT. What would be the cost of painting a wainscotted room, 12· ft. 3· in. high, 18· ft. 6· in. long, and 14 ft. 9· in. wide, at 7½d. per square yard?

Example completed.

I.

Here, the breadth of the rectangle is the height of the room. And

Whole length of wall = twice length and breadth of room.

$$= (\overset{\text{ft.}}{18\cdot}\ \overset{\text{in.}}{6\cdot} + \overset{\text{ft.}}{14\cdot}\ \overset{\text{in.}}{9\cdot}) \times 2\cdot = \overset{\text{ft.}}{66\cdot}\ \overset{\text{in.}}{6\cdot}$$

$$\therefore \text{Area painted} = (66\tfrac{1}{2} \times 14\tfrac{3}{4})\text{ su. ft.} = 980\tfrac{7}{8}\text{ su. ft.} = 108\tfrac{71}{72}\text{ su. yd.}$$

Now, 109· (or 9· doz. + 1·) at 7½d. cost (7/6 × 9·) + 7½d.

∴ Cost of painting = £3· 8·s. 1½d. very nearly.

The error is only $\frac{1}{72}$ of 7½d., or $\frac{15}{36}$ of a farthing.

II.

But, in computing sup. yds., much time and trouble will be saved by turning the lineal dimensions at once into yards, and multiplying, if necessary, by fractions.

Thus, in foregoing example, Length $= 66\frac{1}{2}$ ft. $= 22\frac{1}{6}$ yd. Breadth $= 14\frac{3}{4}$ ft, $= 4\frac{11}{12}$ yd.

$$\therefore \text{Area} = (22\tfrac{1}{6} \times 4\tfrac{11}{12}) \text{ sup. yd.} = \left(\frac{133}{6} \times \frac{59}{12}\right) \text{ sup. yd.}$$

$$= \frac{7847}{72} \text{ sup. yd.} = 108\tfrac{71}{72} \text{ sup. yd.}$$

III.

Here is another method, partly duodecimal, but of no particular value, except as an exercise. The pupil will endeavour to explain the process.

	yd.	ft.	in.		
L. =	22·	0·	6·		≐ Area, if 1· yd. wide.
B. =	4·	2·	9·		
	88·	6·	0·		= Area, if 4· yd. wide.
	14·	7·	0·		= Area, if 2· ft. wide.
	5·	4·	10·	6·″	= Area, if 9· in. wide.
	108·	8·	10·	6·″	= Area, if 4· yd. 2· ft. 9· in. wide.

$= 108\cdot$ su. yd. $8\frac{7}{8}$ ft. $= 108\frac{71}{72}$ su. yd., as before.

THIRD EXAMPLE WORKED OUT. A rectangular plat of garden-ground measures 76· ft. 6· in. by 51· ft. 9· in. Ascertain Area and Rent, at 8·d. per customary superficial pole or landyard.

Example completed.

I. Reducing lin. dimensions to cust. lin. po. of 18· ft.

Length in C. po. $= 76\frac{1}{2}$ ft. $\div$ 18· ft. $= 4\frac{4\frac{1}{2}}{18} = 4\frac{9}{36} = 4\frac{1}{4}$ C. lin. po.

Breadth in C. lin. po. $= 51\frac{3}{4}$ ft. $\div$ 18· ft. $= 2\frac{15\frac{3}{4}}{18}$ or $2\frac{7}{8}$ C. lin. po.

$$\therefore \text{Area} = (4\tfrac{1}{4} \times 2\tfrac{7}{8}) = \tfrac{391}{32} = 12\tfrac{7}{32} \text{ C. su. po.}$$

$$\text{And, rent} = 8\text{d.} \times \tfrac{391}{32} = \tfrac{391}{4}\text{d.} = 8\cdot\text{s. } 1\tfrac{3}{4}\text{d.}$$

II. Or thus: less conveniently,

$$\text{Area} = (76\tfrac{1}{2} \times 51\tfrac{3}{4}) \text{ su. ft.} = 3958\tfrac{7}{8} \text{ su. ft.}$$

$$= (3958\tfrac{7}{8} \div 324\cdot) \text{ cus. sup. po.}$$

$$= 12\tfrac{70\frac{7}{8}}{324} \text{ or } 12\tfrac{7}{32} \text{ cus. sup. po.}$$

$$\therefore \text{Rent} = 8\cdot\text{d.} \times 12\tfrac{7}{32} = 8\cdot\text{s. } 1\tfrac{3}{4}\text{d.}$$

Area of same plat in Statute Measure, found by rule given on page 277· $= (\frac{391}{32} \times \frac{144}{121})$ statute poles.

$= \frac{3519}{242}$ or $14\frac{1}{2}$ st. po. nearly.

The error is $\frac{5}{121}$ of a statute pole, or $\frac{1}{24}$ st. po. nearly

EXERCISE CXII. C.

Find Area and cost of the following. Explain every line. Solve each by two methods, Vulgar Fractions to be one.

Flooring rectangles, at per square of 100· sup. ft.

	ft.	in.		ft.	in.	
1·	35·	9·	by	10·	6·	at 21/
2·	64·	3·	by	25·	3·	at 31/6
3·	28·	6·	by	17·	9·	at 24/6
4·	96·	3·	by	21·	4·	at 18/9

	ft.	in.		ft.	in.	
5·	108·	9·	by	31·	6·	at 21/6
6·	36·	8·	by	36·	8·	at 33/
7·	29·	2·	by	29·	2·	at 35/
8·	47·	11·	by	25·	10·	at 27/6

Plastering rectangles, at per square yard.

	ft.		ft.	
9·	35·	by	13·	at 2/3
10·	47·	by	21·	at 1/6
11·	63·	by	17·	at 2/7
12·	59·	by	23·	at 3/9
13·	53·	by	19·	at 1/10

	ft.	in.		ft.	in.		d.
14·	28·	6·	by	5·	6·	at	10½
15·	42·	9·	by	2·	3·	at	11·
16·	94·	6·	by	17·	9·	at	6½
17·	38·	3·	by	1·	9·	at	11·
18·	59·	7½	by	1·	10½	at	13·

Painting rectangles, at per superficial yard.

	ft.	in.		ft.	in.		d.
19·	65·	9·	by	1·	1½	at	3½
20·	34·	10½	by	2·	3·	at	5½
21·	123·	4½	by	1·	8¼	at	4½
22·	94·	9·	by	1·	9·	at	3¾
23·	52·	1½	by	2·	7½	at	6½

	ft.	in.		in.		d.
24·	198·	9·	by	1½	at	3¾
25·	283·	9·	by	4½	at	2½
26·	114·	6·	by	11¼	at	7·
27·	153·	3·	by	13½	at	8½
28·	400·	0·	by	6·	at	6¼

29· At 10·d. per sup. yd., what would be the cost of plastering the walls and ceiling of a room 24· ft. 11· in. long, by 15· ft. 9· in. wide, and 12· ft. 3· in. high, deducting for two doors, each 3· ft. 10· in. by 7· ft. 1· in., and a fireplace 4· ft. 2· in. by 5· ft. 8· in.?

30· An oblong cistern measures in the clear 4· ft. 2· in. long, 3· ft. 6· in. wide, and 17· in. deep. What weight of lead will line it at 9· lb. per sup. ft.?

31· What would the leaden lining cost at 12·s. per cwt., and 7· lb. per sup. ft.?

32· Supposing that 5· cwt. 2· qr. 2· lb. of lead, uniform in thickness, will line a cistern 5¼ ft. long, by 4½ ft. wide, and 2¾ ft. deep, what does a sup. ft. of the lead weigh?

D.

Find Area, Customary Measure, of the following rectangular allotments of garden-ground, and compute rent of each at given rate per cust. sup. pole or landyard.

	ft.		ft.		d.
1·	144·	by	94½	at	8·
2·	99·	by	81·	at	9·
3·	119¼	by	60¾	at	7·
4·	103½	by	85½	at	10·
5·	67½	by	67½	at	10·
6·	168¾	by	85½	at	9·
7·	139½	by	96¾	at	11·
8·	173¼	by	49½	at	7·
9·	121½	by	78¾	at	10·
10·	85½	by	24¾	at	8·

	c. po.	ft.		c. po.	ft.	
11·	4·	6¾	by	4·	6¾	at 1/.
12·	7·	13½	by	2·	9·	at 1/4
13·	5·	11¼	by	3·	4½	at 1/2
14·	6·	15¾	by	6·	15¾	at 1/3
15·	13·	9·	by	5·	13½	at 1/8
16·	4·	6·	by	4·	6·	at 1/6
17·	18·	4½	by	18·	4½	at 1/9
18·	9·	10½	by	2·	15·	at 2/.
19·	16·	15·	by	9·	10½	at 2/3
20·	11·	12·	by	4·	11·	at 2/6

In measuring small allotments by the customary superficial pole, the length and breadth need not be taken more accurately than to the nearest eighth of a cust. lin. pole, of which measure it will be useful to remember the following fractions:

ft.	c. po.	ft.	c. po.	ft.	c. po.	ft.	c. po.
9·	= ½.	13½	= ¾	6¾	= ⅜	15¾	= ⅞
4½	= ¼	2¼	= ⅛	11¼	= ⅝		

COMPUTING BRICKLAYERS' WORK.

A square rod, pole, or perch = 272¼ sup. ft.; but bricklayers reckon, for a rod of their work, 272· sup. ft. of a *brick and a half* (or 13½ in.) *thick*, a brick being 9· inches long. And all walls not of this thickness are to be reduced to it: to do which, we multiply by the number of half bricks in the thickness, and divide by 3·

A neat practical method of performing this reduction is exhibited in the subjoined example, taken from Skyring's Builders' Price Book, 1854·

Note, that a course of bricks is a row of them, and is 3· inches in height; ∴ 4· courses = 1· foot high.

"Suppose the front of a house to be 18· ft. long; the footings, two course in three and a half bricks, one course in three bricks, and one course in two and a half bricks; the basement" (8· ft. high), "two bricks; the ground floor" (10· ft. 6· in. high), "one and a half brick, and from thence, (17· ft. 6· in.) one brick thick, to the underside of the coping." Take the dimensions as under, and form the same into the following abstract, viz.:

Dimensions.			Abstract.		
	ft. in.	su. ft. brk.	su. ft. ′ brk.		su. ft. ′ brk.
L. 2· c.	18· 0· 6·	9· of 3½ =	4· 6· of 1·	+	18· 0· of 1½
L. 1· c.	18· 0· 3·	4½* of 3· =	0· 0· of 1·	+	9·*0· of 1½
L. 1· c.	18· 0· 3·	4½ of 2½ =	4· 6· of 1·	+	4· 6· of 1½
L. bst.	18· 0· 8· 0·	144· of 2· =	288· 0· of 1·	+	0· 0· of 1½
L. gd. f.	18· 0· 10· 6·	189· of 1½ =	0· 0· of 1·	+	189· 0· of 1½
L. to. cp.	18· 0· 17· 6.	315· of 1· =	315· 0· of 1·	+	0· 0· of 1½
			612·		220· 6·
			2· br.		408·
			3·)1224· of ½		272·)628½
			408· of 1½		2· rods 84½ ft."

Area of wall at 1½ brick thick = 2⅓ rods, nearly.

---

* To a little attention, the foregoing work will explain itself. Any Area of wall 3· bricks thick evidently = twice that Area 1½ brick thick.

† Any Area 3½ brk. th. = twice that Area 1½ brk. th. + half that Area 1· brk. th.; and so on.

The same may be done thus:

su. ft. brk. su. ft. ′ brk.
18· ft. by 6· in. = 9· (of 3½) = $\frac{9\cdot \times 7\cdot^{*}}{3\cdot}$ or 21· 0· of 1½
18· ft. by 3· in. = 4½ (of 3·) = $\frac{4\frac{1}{2} \times 6\cdot^{*}}{3\cdot}$ or 9· 0· of 1½
18· ft. by 3· in. = 4½ (of 2½) = $\frac{4\frac{1}{2} \times 5\cdot^{*}}{3\cdot}$ or 7· 6· of 1½
18· ft. by 8· ft. = 144· (of 2·) = $\frac{144\cdot \times 4\cdot^{*}}{3\cdot}$ or 192· 0· of 1½
18· ft. by 10½ ft. = 189· (of 1½) = 189· 0· of 1½
18· ft. by 17½ ft. = 315· (of 1·) = $\frac{315\cdot \times 2\cdot^{*}}{3\cdot}$ or 210· 0· of 1½
Area of Wall = 628· 6· of 1½

Or still more concisely, thus; reducing each separate portion of the wall to the thickness of half a brick, adding, and dividing by 3· once for all. (PRINCIPLE XXVI.)

su. ft. su. ft.
1st footing = 9· × 7·* = 63· of ½ brick.
2nd footing = 4½ × 6·* = 27· " "
3rd footing = 4½ × 5·* = 22½ " "
Basement = 144· × 4·* = 576· " "
Ground floor = 189· × 3·* = 567· " "
To top of work = 315· × 2·* = 630. " "
3·)1885½
628½ of 1½ brick.

* *Where do these Multipliers come from?*

In the West of England, and in other districts, freestone walls are computed by the superficial perch or rod, of 2· square yards (18· ft. long, by 1· ft. high); the price per perch being regulated by the thickness agreed on.

EXERCISE CXII. E.

Find cost of undermentioned rectangular pieces of brickwork, at stated prices per rod of 272· su. ft., 1½ brick thick.

ft. ′ ft. ′
1· 44· 9· by 18· 6· of 4· bricks, at £13·
2· 13· 6· by 9· 3· of 2· bricks, at £12· 5·s.
3· 245· 6· by 14· 6· of 2½ bricks, at £14· 15·s.
4· 148· 3· by 12· 8· of 3· bricks, at £12· 15·s.
5· 97· 6· by 15· 6· of 3½ bricks, at £14· 10·s.
6· 45· 6· long, 14· courses high, 3· br. at £15· 5·s.
7· 85· 9· long, 24· courses, 2· br. th. at £2· 4·s.*
8· 125· 6· long, 32· courses, 3½ br. th. at £2· 2·s.*

* These prices are for labour only.

ESTIMATING WEIGHT OF LIVE CATTLE.

EXAMPLE WORKED OUT. A grazier, wishing to estimate roughly the weight of an ox, considers the superficial extent of the hide as equivalent to that of a rectangle having the girth and length of the animal for its two dimensions. He then fixes on 22· lb. as the probable average weight under a superficial foot of the hide. What is the weight of the beast on these suppositions?

Example completed.

Area of rectangle 6· ft. 5· in. by 4· ft. 2· in. $= \overset{\text{s. ft.}}{26\cdot}\ \overset{\prime}{8\cdot}\ \overset{\prime\prime}{10\cdot} = \overset{\text{s. ft.}}{26\tfrac{53}{72}}$

∴ Estimated weight $= 22\cdot$ lb. $\times\ 26\tfrac{53}{72} = 5\cdot$ cwt. 1· qr. $0\tfrac{7}{36}$ lb.

But, in actual business, the $\tfrac{53}{72}$ would be taken as $\tfrac{3}{4}$, which would occasion an error of only $\tfrac{1}{72}$, for $\tfrac{3}{4} = \tfrac{54}{72}$.

∴ *Estimated weight* = (*nearly*) 22· lb. $\times\ 26\tfrac{3}{4} = 5\cdot$ cwt. 1· qr. $0\tfrac{1}{2}$ lb.

EXERCISE CXII. F.

Find weight, when dimensions and estimated weight per superficial foot are as follow.

		ft.	in.		ft.	in.		lb.			ft.	in.		ft.	in.		lb.
1·	G.	6·	9·,	L.	4·	7·	at	23·	7·	G.	7·	8·,	L.	4·	10·,	at	27·
2·	G.	5·	11·,	L.	4·	3·	at	24·	8·	G.	6·	4·,	L.	4·	9·,	at	22·
3·	G.	7·	3·,	L.	4·	8·	at	26·	9·	G.	6·	10·,	L.	4·	11·,	at	24·
4·	G.	7·	1·,	L.	4·	10·	at	28·	10·	G.	5·	8·,	L.	3·	10·,	at	25·
5·	G.	6·	6·,	L.	4·	9·	at	25·	11·	G.	6·	1·,	L.	4·	0·,	at	26·
6·	G.	8·	0·,	L.	5·	1·	at	28·	12·	G.	6·	3·,	L.	4·	6·,	at	23·

This method of estimating the weight of cattle is constantly employed by graziers and butchers. Its accuracy, in any instance, depends mainly on the precision with which the weight per foot is fixed; a matter in which experience must guide.

The girth is measured immediately behind the shoulder. The length is taken from the point of the shoulder to the root of the tail, or nearly, according to the form of the animal.

AREA AND ONE DIMENSION GIVEN TO FIND THE OTHER DIMENSION.

Calculations in Superficial Measure deal with Area, Length, and Breadth. When any two of these quantities are given, the third may be found. For, in the same sense in which we say that

Area = Length × Breadth, (Page 270·)

we say also that

Length = Area ÷ Breadth, (PRIN. XX.) and
Breadth = Area ÷ Length, (PRIN. XX.)

We must here repeat that these formulæ are only *convenient*, and not *accurate*. For, Area, consisting of superficial Units, cannot be divided by (that is, distributed into lots of) Lineal Units. Hence, Area cannot be divided by Length or Breadth. (See Pages 105· and 218·) The subjoined examples worked out will exhibit the true meaning of the foregoing formulæ.

First Example worked out. How wide is a rectangle whose Area is 64·75 super. chains, and Length 12·95 lin. chains?

The Breadth, which we are to discover, will be equal to as many lineal chains as there are, in the rectangle, longitudinal strips each 1· chain wide. And each such strip contains 12·95 super. chains. (Why?) The question, therefore, resolves itself into this: How many rectangular strips, each containing 12·05 super. ch. can be formed out of a rectangle whose Area is 64·75 super. chains?

$$\left.\begin{array}{l}\text{Ar. strip 1·ch. w.}\\ \doteq\text{L. of rect.}\end{array}\right\} = \begin{array}{r}\overset{\text{sup. ch.}}{12{\cdot}95}\big)\underline{\overset{\text{sup. ch.}}{64{\cdot}75}} = \text{Area of rectangle.}\\ \underline{\underline{5{\cdot}}}\ \ = \text{No. of strips 1·ch. wide.}\end{array}$$

∴ *Breadth of Rectangle = 5· lineal chains.*

If Area and Breadth be given to find Length, then, Length = as many lineal chains as there are *transverse strips*, one chain long. And each such strip contains 5· sup. chains, because the rectangle is 5· chains wide.

$$\left.\begin{array}{l}\text{Ar. tr. strip, 1· ch. lg.}\\ \doteq\text{B. of rect.}\end{array}\right\} = \begin{array}{r}\overset{\text{su. ch.}}{5{\cdot}}\big)\underline{\overset{\text{sup. ch.}}{64{\cdot}75}} = \text{Area of rectangle.}\\ \underline{\underline{12{\cdot}95}} = \text{No. of tr. strips 1· ch. long.}\end{array}$$

∴ *Length of Rectangle = 12·95 lineal chains.*

Second Example worked out. The Area of a rectangular court is 68· sup. ft. 3·′ 3·″, and its length 9· ft. 5· in. Required its Breadth.

Example completed.

Area = 68· s. ft. 3·′ 3·″ = 9831· su. in. Length = 113· lin. in.

$$\left.\begin{array}{l}\text{Area of strip}\\ \text{1· in. wide.}\end{array}\right\} \begin{array}{r}\overset{\text{su. in.}}{113{\cdot}}\ \big)\underline{\overset{\text{su. in.}}{9831{\cdot}}}\ \text{Area of rectangle.}\\ \underline{\underline{87{\cdot}}}\ \text{strips, long as rect. and 1 in. wide.}\end{array}$$

∴ *Breadth of rectangle = 87· lin. in. = 7·ft. 3· in.*

If Area and Breadth be given to find Length, then

$$\left.\begin{array}{l}\text{Area of transv.}\\ \text{stp. 1· in. long.}\end{array}\right\} = \begin{array}{r}\overset{\text{su. in.}}{87{\cdot}}\big)\underline{\overset{\text{su. in.}}{9831{\cdot}}} = \text{Area of rectangle.}\\ \underline{\underline{113{\cdot}}}\ \text{transv. strips, ea. 1· in. long.}\end{array}$$

∴ *Length of Rectangle = 113· lin. in. = 9·ft. 5· in.*

For other Parallelograms the process is precisely the same as for rectangles.

Third Example worked out. How long is the

Base of a Parallelogram whose Area is 47· ac. 1· r. 30· po., and Perpendicular Height 16·5 chains?

Example completed.

Area of Parallelogram = 47·4375 ac. = 474·375 sup. ch.
Area of transv. strip, 1· ch. long (≐ perp. ht. of Par.) = 16·5 su. ch.
∴ No. of such strips = 474·375 ÷ 16·5 = 28·75
∴ *Length of Parallelogram* = 28·75 *lineal chains.*

FOURTH EXAMPLE WORKED OUT. What is the Perpendicular height of a Triangle whose Area is 3· ac. 3· r. 3½ po., and Base 17¾ chains?

Example completed.

$$\overset{\text{ac.}}{3\cdot}\ \overset{\text{r.}}{3\cdot}\ \overset{\text{po.}}{3\tfrac{1}{2}} = \overset{\text{sup. ch.}}{37{\cdot}71875} = \text{Area of Triangle.}$$

$$\begin{array}{rl} & \underline{\quad 2\cdot \quad} \\ \text{Ar. of strip} = \overset{\text{su. ch.}}{17{\cdot}75})\,75{\cdot}4375 & = \text{Ar. of Par. of same dimensions.} \\ \underline{\underline{\quad 4{\cdot}25 \quad}} & = \text{No. of strips}, \doteq \text{Perp. H}^{\text{t.}} \text{ of Paral}^{\text{m.}} \end{array}$$

But, Perpend. Height of Triangle is the same as that of Parallelogram, for they are supposed to be on the same base, and the Parallelogram double of the Triangle. (Page 272·)

∴ *Perpendicular Height of Triangle* = 4¼ *lineal chains.*

Or thus:

$$\begin{array}{ll} {}^*\overset{\text{su. ch.}}{17{\cdot}75})\overset{\text{su. ch.}}{37{\cdot}71875} & \text{Ar. of Tri., or of Par, same height, but half as long.} \\ \quad 2{\cdot}125 & \text{strips}^*, \doteq \text{Perp. Height of that Parallelogram.} \\ \quad \underline{2\cdot} & \\ \quad \underline{\underline{4{\cdot}25}} & \text{strips}^* \text{ in Par. which is double of Triangle} \doteq \text{Perp. Ht.} \end{array}$$

∴ *Perpendicular Height of Triangle* = 4¼ *lineal chains.*

EXERCISE CXIII.

Describe, accurately, in writing, each Divisor, Dividend, and Quotient. Prove each Example by working out the Area, with the Dimension found.

A.

Given Area

1· Of a Parallelogram, 22· ac. 12·8 po., and Perp. Ht. 640· links. Find Base.
2· Of a Rectangle, 1643· ac. 2· r. 18·016 po., and Length 19348 links. Find Breadth.
3· Of a triangular meadow, 5· r. 10· po., and Base 28· po. Find Perp. Height.
4· Of an oblong room, 269· su. ft. 6·′ 9·″, and length 18· ft. 11· in. Find Breadth.

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* The strip here spoken of is a rectangle on same base as the Triangle, and one chain in Perpendicular Height.

5· Of a triangle, 28· su. po. 24· yd. 1· ft., and Breadth 64· feet. Find Length.

6· How wide is an oblong passage of which 17· ft. are exactly covered by 20· ft. of canvas 4· ft. 3·′ wide?

7· In exchange for a rectangular strip of land, (required for road-making,) 6241· links long, and 72· links wide, a triangular plat is offered which may be 474· links in perpendicular breadth. What length of Base should it have?

B.

1· How many yards of carpet 20· inches wide will cover the floor of a rectangular room 16· ft. 8· in. long, by 13· ft. 4·′ wide, making no allowance for matching the pattern, in the several lengths?

2· Supposing the room referred to in last example to be 9· ft. 3· in. high, how many pieces of paper 21· inches wide will cover the walls, each piece containing 11½ lineal yards?

3· What length of rectangular board 8½ in. wide will be equal to a superficial foot?

4· How much planking 10· in. wide will floor a room 14· ft. 11· in. long, by 12· ft. 4· in. wide?

5· How many yards of velvet trimming $\frac{1}{16}$ yd. wide can be cut from a yard of velvet ¾ yd. wide?

6· How much silk velvet ⅝ yd. wide will furnish 21½ yd. of trimming ¾ in. wide?

C.

1· The length of a rectangle being 64· ft. 10· in., and its Area 815· su. ft. 9·′ 10·″, what is its Breadth?

2· From a rectangular piece of building-ground, which is 36¼ ft. wide, what Length must be cut off to make an Area of 2081· su. ft. 4·′ 3·″?

3· Given Area of Rect. 888· su. ft. 10·′ 10·″, and L. 95⅔ ft. find B.

4· The space occupied by a certain oblong piece of common is known to be 613· ac. 20· po., and its uniform width 45· chains. Required its length.

5· If a triangle 15· po. in perp. height, have an Area of 5· r. 10· po., what is its Base?

6· What is the average width of a road, of which 2½ miles have an Area of 5· ac. 4· po. 28· yd. 3· ft.?

7· The walls of a rectangular chamber cover an area of 51· sup. yd. The apartment is 15· feet long, and 9· ft. high. How wide is it?

8· Find Breadth of a rectangle of which Area is 1824· su. ft., and Length 85½ ft.

9· From a strip of land 81· ft. in perpendicular width, what length must be cut off to be worth 18·s. 6¾d. rent, at 9·d. per customary superficial pole?

10· A rectangular field, 1125· links by 500· links, is to be divided into two equal longitudinal strips, and these again subdivided into rectangular allotments of 30· superficial statute poles each. How many allotments will there be, and what will be the length and breadth of each?

D.

1· At what rate per square yard will the plastering of a wall 35· ft. by 13· ft. cost £5· 13·s. 9·d.?

2· A parallelogram having an area of 5· su. ft. 9·′ 3·″ 4·‴ 4·⁗, is 2· ft. 4·′ 10·″ long. Required its perp. width.

3· An oblong of garden-ground 24· ft. 9· in. wide is worth a rent of 4·s. 4¼d. at 8·d. per customary sup. perch. What is its length?

4· At what rate per square will a floor 36· ft. 8· in. square cost £22· 3·s. 8·d.?

5· The paving of a court-yard cost £28· 5·s. 5·d. at 3/9 per square yard. What was its breadth, its length being 59· ft.?

6· What is the perpendicular height of a triangle, which, being 945· links in the base, has an Area of 1· ac. 21· po. 13·31 su. yd.?

7· At 14½d. per sup. yd. the plastering of the walls of an apartment costs £3· 1·s. 7½d. Now the room is a rectangle 10½ ft. wide, and the walls are 9· ft. in height. How long is it?

8· How wide a rectangle or parallelogram, 144· ft. long, will be worth 28/. a year, at 8·d. per cust. sup. pole or landyard?

9· At 2/7 per square yard, a rectangular floor cost £15· 7·s. 5·d., and it was 63· ft. long. How wide was it?

CUBIC OR SOLID MEASURE.

The **Solidity** or Solid Content of a body is expressed by the number of solid units which it contains. And the **Capacity** of a hollow vessel is expressed by the number of solid units which would exactly fill it.

A Prism is a solid having three or more parallelograms for its sides, and any two equal, similar, and parallel figures for its two ends.

If the ends are triangles, it is a triangular prism, or wedge.
If the ends are rectangles, it is a rectangular prism.

A Rectangular Prism, or **Parallelopipedon,** is a solid, having six rectangular sides, the opposite ones being equal and parallel.

The ends are also called sides in this definition.
If the ends are squares, it is a square prism.
Hence, a cube is a square prism.

COMPUTATION OF SOLIDITY AND CAPACITY.

In computing the solidity of a prism, we imagine it to be cut, either into

I. As many *horizontal* slices, of equal *thickness*, as there are units in the thickness of the prism. Or into

II. As many *Vertical Transverse* slices,* of equal

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* **Vertical, Transverse; or, Upright, cross, slice.**

length, as there are units in the length of the prism. Or into

III. As many *Vertical Longitudinal*[†] slices, of equal width, as there are units in the *Breadth* of the prism.

Then, in the first case:
∵ Solidity of Horizontal Slice ≑ Area of *Top* ≑ L × B.[‡]
And, No. of Horizontal Slices = No. of units in thickness of Psm.
∴ Solidity of Prism ≑ Area of Top × Thickness.[‡] ≑ (L × B) × T.[‡]

In the second case:
∵ Solidity of Vertical Transverse Slice ≑ Area of *End* ≑ B × T.[‡]
And No. of such slices = No. of units in length of Prism.
∴ Solidity of Prism ≑ Area of End × Length ≑ (B × T) × L.[‡]

In the third case:
∵ Solidity of Vertical Longitudinal slice ≑ Area of *Side* ≑ L × T.[‡]
And, No. of such slices = No. of units in Breadth of Prism.
∴ Solidity of Prism ≑ Area of Side × Breadth ≑ (L × T) × B.[‡]

Solidity of Prism ≑ L × B × T. (PRINCIPLE VI.)

FIRST EXAMPLE WORKED OUT. What is the solid content of a rectangular block of stone, 6· ft. 7· in. long, 5· ft. 11· in. wide, and 3· ft. 2· in. thick?

ft.	in.			Example completed.
6·	7·			= L ≑ Ar. of top or bottom, if 1· ft. wide (1·).
5·	11·			= B ≑ No. of strips, ea. 1· ft. wide, in top or bottom (2·).
32·	11·			= Ar. of top or bottom, if 5· ft. wide.
6·	0·	5·″		= Ar. of top or bottom, if 11· in. wide.
38·	11·	5·		= Ar. of top or bottom, ≑ Solidity, if 1· ft. thick (3·).
3·	2·			= T. ≑ No. horizontal slices, ea. 1· ft. thick (4·).
116·	10·	3·		= Solidity of block, if 3· ft. thick.
6·	5·	10·	10·‴	= Solidity of block, if 2· in. thick.
123·	4·	1·	10·	= Solidity of block, of given dimensions.

IMPLICIT CHANGES OF UNIT.

In the work of the foregoing example four implicit changes of unit are made, to which the figures at the ends of the lines of explanation refer.

The first and second of these changes of unit occur in the computation of superficial Area, and have been already explained. (See pages 270·, &c.)

In the third, the *superficial units* expressing the *Area of the top or bottom* are implicitly changed into *solid units*, expressing the *solidity of a slice* 1·*ft. thick.* For, as soon as we begin to multiply the 38· su. ft. 11·′ 5·″, we must consider them as solid ft., primes, and seconds; since the repetition of nothing but solid units (or fractions of them) can yield a product of solid units.

[†] Vertical Longitudinal; or, Upright, longwise, slice.

[‡] See preceding remarks on the true meaning of these and similar formulæ.

The fourth implicit change is that of the 3· ft. 2· in. thickness, into 3⅙ horizontal slices, ea. 1· ft. thick.

These implicit changes of unit are of necessity made whenever we work out Solidity from Lineal dimensions. They are, however, most commonly entirely lost sight of, and, consequently, the process in which they occur, is not clearly understood.

Proof. We have found the solidity of the prism by the first of the three processes described above, that is to say, by multiplying solidity of a horizontal slice by the No. of units in thickness. The correctness of the result may be tested by also working out solidity by either of the other two modes, namely:

II. By multiplying Solidity of *Vertical Transverse* slice, by the No. of units in *Length*.

∵ Solidity of Prism = (B. × T.) × L. (PRIN. VI.)

III. By multiplying Solidity of *Vertical Longitudinal* slice, by the No. of units in *Breadth*.

∵ Solidity of Prism = (L. × T.) × B. (PRIN. VI.)

Example proved.

The pupil will find it a good exercise to copy out the subjoined work, writing against each line its true and its coincident significations.

II.

ft.	in.		
5·	11·		
3·	2·		
	11·	10·″	
17·	9·		
18·	8·	10·	
6·	7·		
10·	11·	1·	10·‴
112·	5·		
123·	4·	1·	10·

III.

ft.	in.		
6·	7·		
3·	2·		
1·	1·	2·″	
19·	9·		
20·	10·	2·	
5·	11·		
19·	1·	3·	10·‴
104·	2·	10·	
123·	4·	1·	10·

Before multiplying the dimensions together they may be reduced to one denomination, which, in the present instance, would be lineal inches. The result would then come out in cubic or solid inches. Thus:

Solidity of Prism=(79· × 38· × 71·) *cu. in.* = 213142· *cu. in.*
= 123· cu. ft. 4·′ 1·″ 10·‴

EXERCISE CXIV.

Explain every line. Work each Example in two ways.

A.

Find the Solidity of each of the ten rectangular prisms or parallelopipedons, which have the following dimensions, respectively.

	Length.		Breadth.		Thickness.			Length.		Breadth.		Thickness.	
1·	14·	3·;	7·	6·;	4·	2·	6·	9·	$8\frac{1}{2}$;	5·	$4\frac{1}{2}$;	1·	$3\frac{1}{4}$
2·	17·	8·;	8·	11·;	5·	7·	7·	7·	$6\frac{3}{4}$;	4·	$2\frac{1}{4}$;	2·	$9\frac{1}{2}$
3·	13·	9·;	9·	5·;	4·	8·	8·	8·	$10\frac{1}{4}$;	1·	$6\frac{1}{4}$;		$10\frac{1}{4}$
4·	54·	10·;	16·	4·;	9·	2·	9·	1·	$2\frac{1}{8}$;		$11\frac{3}{8}$;		$1\frac{1}{16}$
5·	75·	4·;	18·	10·;	8·	8·	10·		$10\frac{3}{8}$;		$7\frac{1}{16}$		$\frac{3}{8}$

11· What is the solid content of a triangular prism, or wedge, the length of its base (a rectangle) being 14· in., breadth of same 4· in., and perpendicular height, from middle of base to edge, 12· in.?

Solidity $\doteq$ *Area of End* × *Length.*

B.

Find solid content and cost

1· Of a squared log of timber, 32· ft. 6· in. long, by 21· in. wide, and 18· in. thick, at 1·s. 4·d. per cu. ft.

2· Of a pavement of Aberdeen granite, 40· yd. long, 8· ft. wide, and 10· in. deep, at 6/6 per cubic foot, including labour in hoisting and setting.

3· Of 100· blocks of Yorkshire stone, each 6· ft. 2· in. by 2· ft. 3· in. by 1· ft. 9· in., at 4/6 per cubic foot.

4· Of 500· inch-and-a-half planks of Memel fir, each 21· ft. 6· in. by 22· in., at £4· 15·s. per load.

5· Of a wall, 40· ft. long, $18\frac{1}{2}$ ft. high, and 30· in. thick, at 7·s. $10\frac{1}{2}$d. per cubic yard.

6· How much marble is contained in a squared block which is 1· ft. $11\frac{1}{25}$ in. long, 15· in. wide, and 5· in. thick?

Work, first by Vulgar Fractions, and then by Duodecimals.

7· What is the approximate weight of a hay-rick, the base of which is a rectangle, the sides and ends sloping regularly outward from the bottom, and the top forming a roof-ridge; the mean length being 14· ft. 3· in., mean breadth 6· ft. 9· in., and mean height 9· ft. 6· in.; reckoning 4· lb. to the cubic foot?

The mean dimensions are those which the rick would have were its four sides vertical, and its top flat. Mean length and breadth are taken midway between the bottom and eaves; and mean height in a vertical line from bottom to a point half-way between eaves and apex.

The rick will, therefore, be computed as a square prism, of given dimensions.

Find weight and value of similar ricks, of following dimensions, estimated density, and rate.

	M. L.		M. B.		M. H.		Wt. cu. ft.	Price per Ton.		
	ft.	in.	ft.	in.	ft.	in.	lb.	£.	s.	d.
8·	7·	8·;	3·	4·;	5·	1·;	7·;	4·	0·	0·
9·	8·	6·;	4·	2·;	4·	8·:	6·;	3·	18·	0·
10.	14·	10·;	8·	1·;	6·	10·;	8·;	5·	5·	0·
11.	13·	9·;	7·	3·;	7·	4·;	4·;	3·	3·	0·
12.	11·	5·;	6·	10·;	6·	0·:	9·;	7·	0·	0·
13.	16·	8·;	7·	9·;	6·	6·;	5·;	3·	10·	0·
14·	25·	9·;	12·	3·;	8·	5·;	10·;	6·	8·	4·
15·	33·	4·;	10·	5·;	9·	3·;	7·;	4·	13·	4·

In thus calculating the weight of a rick, as of an animal, the most important element is the safe estimation of the weight per foot.

Second Example worked out. How many gallons of water will a rectangular-sided cistern contain, which is 15· ft. long, $4\frac{1}{2}$ ft. deep, and $7\frac{1}{2}$ ft. wide?

A cubic foot of water weighs $62\frac{1}{2}$ lb. ⎫ 18· and 19· Vic.
A gallon weighs 10· lb. ⎭ cap. 72·
$\therefore$ A cubic foot contains $62\frac{1}{2} \div 10\cdot = 6\frac{1}{4}$ gal. or 25· qts.
$\therefore$ Capacity of cistern, in qts. $= 25\cdot \times \text{L.} \times \text{B.} \times \text{D.}$
$\therefore$ No. of gal. cistern will contain $= 25\cdot \times 15\cdot \times 4\frac{1}{2} \times 7\frac{1}{2} \div 4\cdot$
$= 25\cdot$ qts. $\times\, 506\frac{1}{4} \div 4\cdot$
$= 3164\cdot$ gal. 2· gills.

From this example we derive a neat and simple rule.

To find the quantity contained in a cistern.

Multiply mean length, breadth, and depth, together; then multiply by 25· and you have the quantity in quarts.

EXERCISE CXIV.

C.

1· How many hogsheads will a cubical vessel contain, measuring 5· ft. 6· in. in the clear?

2· What is the weight of an inch deep of rain over an acre of land?

3· How many children can be properly accommodated in a school-room, 32· ft. 6· in. long, 24· ft. wide, and 16· ft. 8· in. high, allowing sixty-five cubic feet for each child?

4· It is said that the mass of water passing per minute over the falls of Niagara is equal to that contained in one-fifth of a mile of the Thames, at a part where its mean depth is 20· feet, and mean width 1000· feet. Compute the quantity in gallons.

5· In what time will a leet 4· feet wide, through which a stream, 15· in. deep, flows at the uniform rate of 8· ft. per second, fill a reservoir whose mean dimensions are 50· ft., 30· ft., and 18· ft.?

Quantity passing per second $= (4\cdot \times 1\frac{1}{4} \times 8\cdot)$ *cu. ft.*

Find cost of excavating ditches, of undermentioned mean dimensions, at per cubic yard.

	Length.		Breadth.		Depth.		Price.
	ft.	in.	ft.	in.	ft.	in.	d.
6·	16·	6·;	4·	6·;	1·	6·;	8·
7·	29·	3·;	6·	9·;	2·	3·;	16·
8·	46·	6·;	7·	6·;	4·	$10\frac{1}{2}$;	7·
9·	95·	3·;	6·	9·;	6·	0·;	10·
10·	86·	3·;	11·	3·;	7·	6·;	9·

Also of making hedges or embankments, at per cubic yard.

	Length.	Breadth.	Height.	Price.
11·	32· ch.;	4½ ft.;	4½ ft.;	6· d.
12·	344· ft.;	4· ft.;	6· ft.;	9· d.
13·	100· pa.;	5· ft.;	5· ft.;	7· d.
14·	15· ch.;	5½ ft.;	5½ ft.;	4½d.
15·	473· ft.;	6· ft.;	6· ft.;	5½d.

16· What quantity of wood is contained in a cubical box which measures 15· in., outside measure, and whose sides and bottom are 1· in. thick?

I. *Find quantity, supposing box to be solid.*
II. *Find capacity of the hollow box.*
III. *Subtract.*

(See Exercise cxii. a. 13·, 17·, 21·, and 22·)

17· A rectangular granite trough measures, *outside*, 6· ft. 6· in. long, 2· ft. 6· in. wide, and 1· ft. 3· in. deep. The bottom is 9· in. thick, the sides and ends are 6· in. thick, and the stone weighs 1· cwt. 2· qr. per cubic foot. What is the weight of the trough?

18· How much granite was excavated in making the above-mentioned trough out of the solid square prism?

19· How many gallons will it contain?

MEASURING ROUGH TIMBER.

A log of rough timber is computed as equivalent to a square prism, of like length, having the sum of its four sides equal to the mean circumference of the log.

Hence, side of supposed prism = ¼ circumference of log.

If the log taper uniformly to a point, its mean circumference is its middle girth.

If the log taper uniformly, but not to a point, its mean circumference is half the sum of its two end girths.

If it be irregular, several girths must be taken, at equal intervals, these girths added, and their sum divided by the number of them. The result will be the mean circumference of the irregular log.

To compute approximate solidity of a log of rough timber.

First. Find mean circumference of the log. (See above.)
Second. Take one-fourth of this and square it. (P. 43·)
Third. Multiply result by length of log.

Describe a circle. Place a square in it, so that each of the four angles may rest on circumference of the circle.

It is evident that each side of the square is less than one-fourth of the circumference of circle. Therefore, the prism of which w have been speaking is greater than could be hewn out of the log equal to it in circumference.

The excess of bulk assigned to the prism is to make up for the wood removed in squaring. This, however, it only does in part, the labour of squaring makes up for the remainder of wood removed.

EXERCISE CXIV. D.

Find approximate solidity of rough logs of following dimensions.

1· Length 15· ft. 6· in., middle girth 32· inches.
2· Length 33· ft. 4· in., end girths 11· in. and 45· in.
3· Length 41· ft., girths at equal intervals 4· ft., 3· ft. 9· in., 4· ft. 3· in., and 2· ft. 6· in.

It is frequently advisable to compute the content of a rough log from two sets of girths, as a test.

SPECIFIC GRAVITIES.

The **Specific Gravity** of any material is the multiple or part which its weight is of the weight of distilled water, equal bulks of each being compared.

Thus: 1· cu. in. of gold weighs as much as $19\frac{1}{5}$ cu. in. of water.

$\therefore$ Specific Gravity of gold $= 19·2$.

This, of course, supposes the weight of water to be called unity.

Now, Weight of 1· cu. ft. of water = 62·5 lb. av. = 1000· oz. av.
$\therefore$ Weight of 1· cu. ft. of gold = 19200· oz. av.

Sp. gr. × 1000· *gives Weight of* 1· *cu. ft. in oz. avoir.*

Hence, we may compute the weight of any given mass of matter when we know its Sp. gr.

The Sp. gr. of every kind of matter has been ascertained, and recorded in tables; some of which, however, give the weight of a cubic foot, in oz. avoir., instead of the true Sp. gr. It is easily discovered of which kind the table is, by referring to "water." In the True Specific Gravity tables we find "Water 1·," in the others, "Water 1000·"

The methods of ascertaining the Specific Gravities of bodies are very clearly explained in Moseley's "Mechanics applied to the Arts." From this work the subjoined are selected.

	sp. gr.		sp. gr.		sp. gr.
Chalk	2·252	Ash	·845	Portland stone	2·496
Coal	1·3	Beech	·85	Platinum	21·47*
Sea Water	1·028	Oak-heart	1·17	Hydrogen gas	·069*
Granite	2·956	Walnut	·681	Hammered gold	19·35
Cork	·24	Mahogany	1·063	Ivory	1·825

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* Platinum is the heaviest, and Hydrogen gas the lightest of known bodies.

CALCULATION OF WEIGHT FROM SPECIFIC GRAVITY.

First Example worked out. What is the weight of a squared block of granite, 8· ft. 3· in. long, 4· ft. 6· in. broad, and 2· ft. 8· in. thick, Sp. gr. being 2·956?

Example completed.

Solidity of block = $(8\frac{1}{4} \times 4\frac{1}{2} \times 2\frac{2}{3})$ cu. ft. = 99· cu. ft.

Weight of a cubic foot = 2956· oz. avoir.

∴ *Weight of block* = 2956· *oz.* × 99· = 8· T. 3· *cwt.* 1· *qr.* 6· *lb.* 4· *oz.*

The Sp. gr. of granite varies from 2·613 to 2·956. Stonemasons usually compute 1· cu. ft. as $1\frac{1}{2}$ cwt., which is correct, taking Sp. gr. at 2·688; for $1\frac{1}{2}$ cwt. = 2688· oz.

Second Example worked out. What is the weight of a gallon of vinegar, Sp. gr. 1·08?

Example completed.

∵ 1· cu. ft. contains 25· qts., and 1· gal. = 4· qts.,

∴ 1· *gal.* = $\frac{4}{25}$ *or* ·16 *of a cubic foot.**

Now, Weight of 1· cu. ft. of vinegar = 1080· oz. avoir.

∴ *Weight of* 1· *gal. of vinegar* = 1080· *oz.* × ·16 = 10· *lb.* 12·8 *oz.*

To reduce gallons to cubic feet.

Multiply by ·16.

To reduce cubic feet to gallons.

Divide by ·16.

SOLIDITY AND TWO DIMENSIONS OF A PARALLELOPIPEDON GIVEN, TO FIND THE OTHER DIMENSION.

Since in any parallelopipedon (page 288·),
Solidity $\doteq$ L × B × T. ∴ By Principle XX.

Length $\doteq$ Solidity ÷ (B × T)
Breadth $\doteq$ Solidity ÷ (L × T)
Thickness $\doteq$ Solidity ÷ (L × B)

For hollow vessels of the above form we merely substitute the word "Capacity" for "Solidity," and the rule, applicable to both cases is, as shewn above.

Either dimension $\doteq$ *Solid^{y}· or Cap^{y}·* ÷ *Prod. of other two dimen.*

Similar changes of unit are implied in this formula to those explained in the process for finding one dimension of a parallelogram from Area and the other dimension given. For, just as in that process, our real Divisor, to find Length, is the Area of a transverse strip (page 285·), so here, when we say

Length $\doteq$ Solidity ÷ (Breadth × Thickness),

our real Divisor is the *solidity of a vertical transverse slice, one foot long* (see page 288·). For, every time we cut off such a slice, we diminish the length of the prism by one foot. Hence, there are as many such slices in the prism as it is feet long.

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* This is a fact worth remembering.

In other words, No. of feet in length of prism = No. of vertical transverse slices, each 1· foot long.

And Solidity of such a slice $\doteq$ Area of end $\doteq B \times T$. (Page 289·)

$$\therefore \text{Length of Prism} \doteq \text{Solidity} \div (B \times T.)$$

To find Breadth, our true divisor is *solidity of a vertical longitudinal slice, one foot wide*, the solidity of which slice $\doteq$ Area of side $\doteq L \times T$. (Page 289·)

To find Thickness, our true divisor is *solidity of a horizontal slice, one foot thick* $\doteq$ Area of top or bottom $\doteq L \times B$. (Page 289·)

THIRD EXAMPLE WORKED OUT. What length of squared timber which is 1· ft. 8· in. wide, by 10· in. thick, will make $18\frac{3}{4}$ cubic feet?

Example completed.

Here, solidity of prism $= 18\frac{3}{4}$ cubic feet.

Solidity of vert. tr. slice $\doteq$ Ar. of end $\doteq (B \times T) = (1\frac{2}{3} \times \frac{5}{6})$ cu. ft·

$$\therefore \textit{Length of parallelopipedon} = \left\{18\tfrac{3}{4} \div (1\tfrac{2}{3} \times \tfrac{5}{6})\right\} \textit{lin. feet.}$$
$$= (\tfrac{75}{4} \times \tfrac{3}{5} \times \tfrac{6}{5})\ \textit{lin. ft.}$$
$$= 13\cdot \textit{ft. } 6\cdot'$$

Or thus:

Solidity of prism $= 18\cdot\ 9\cdot' = 2700\cdot$ cubic seconds.

Solidity of v. t. slice $= (1\cdot$ c. ft. $8\cdot' \times 10\cdot') = 1\cdot$ c. ft. $4\cdot'\ 8\cdot''$
$= 200\cdot$ cubic seconds.

Length of prism $= (2700\cdot \div 200\cdot) = 13\frac{1}{2}$ *lin. ft.*

EXERCISE CXV.

A.

Find weight of squared block

1· Of beech, 17· ft. by 3· ft. 3· in. by 18· in. (See table p. 204·)
2· Of oak-heart, 20· ft. long, and 15· in. square.
3· Of walnut, $15\frac{1}{2}$ ft. by $2\frac{1}{4}$ ft. by $1\frac{1}{8}$ ft.
4· Of mahogany, 11· ft 3·′ by 2· ft. 9·′ by 2· ft. 6·′
5· Of Portland stone, 9· ft. 6· in. by 4· ft. 8·′ by 3· ft. 6·′

Find weight

6· Of sea-water, filling a cistern (whose sides are rectangles) 14· ft. long, 7· ft. wide, and $3\frac{1}{2}$ ft. deep.
7· Of a mass of chalk, mean length 40· ft., mean breadth, 21· ft., mean thickness 16· ft. 9· in.
8· Of a log of cork, 13· ft. by 3· ft. by 1· ft. 3· in.
9· Of the cargo of coals contained by a barge whose mean inner dimensions are 32· ft., 9· ft., and 7· ft.
10· Of a bar of platinum, (squared,) 3· ft. long, 4· in. wide, and $1\frac{1}{4}$ in. thick.

B.

1· If a rectangular-sided trench be $14\frac{1}{4}$ ft. long, and $7\frac{1}{2}$ ft. wide, how deep must it be to contain 445· c. ft. 540· c. in.?

2· What length of squared timber, which is 8· ft. 11· in. wide, and 5· ft. 7· in. thick, will contain 879· cu. ft. 6·′ 4·″ 4·‴?

3· How much in length that is 5· in. wide and 15· in. thick will be equal to a cubic foot?

4· What length of excavation can be done for £1· 13·s., the mean breadth and depth being 4· ft. 6· in. and 1· ft. 6· in. respectively, and the labour worth 8·d. per cubic yard?

5· The excavation of a trench cost £4· 7·s. 9·d. at 16·d. per cubic yard. It was 58· ft. 6· in. long, and 6· ft. 9· in. wide. What was its depth?

6· At what rate per hour must a stream of water flow, its breadth being 3· ft., and depth 20· in., to fill, in three quarters of an hour, a reservoir of which the mean dimensions are 100· ft., 60· ft., and 36· ft.?

7· How high is a wall which, being 40· ft. long, and 2· ft. 6· in. thick, costs £26· 19·s. 7·d., at 7·s. 10½d. per cubic yard?

8· How thick is a pavement which, at 6/6 per cubic foot, costs £260·, its length being 40· yards, and breadth 8· ft.?

9· A cistern, whose sides are rectangles, is 11· ft. long, 5· ft. 6· in. wide, and contains 16· hhds. 31· gal. 3· qt. 3· gi. How deep is it?

Compute 1· *cu. ft. to contain* 25· *qts.*

∴ *Quantity contained (in qts.)* ÷ 25· ≐ *Capacity in cu. ft.*

Or, Content in gals. × ·16 ≐ *Cap*^{y}· *in cu. ft.*

10· A cistern 9· ft. long, 6· ft. wide, and 3· ft. deep, being quite full, is discharged into a cubical tank, which measures 8· ft. in the clear. How deep will the water stand in the tank?

11· Out of a reservoir, 50· ft. long, 35· ft. wide, and 15· ft. deep, four cisterns are filled, each 14· ft. long, 5· ft. wide, and 2· ft. 6· in. deep. How far will this depress the surface of the water in the reservoir?

12. What is the weight of a cistern, (whose sides are rectangles,) 8· ft. 3· in. long, 3· ft. wide, and 2· ft. deep, outside measure; the sides and ends being 3· in., and the bottom 6· in. thick; the material being granite, Sp. gr. 2·8?

13· What weight of sea-water will it hold, Sp. gr. 1·028?

A body floating in a fluid displaces its own weight of that fluid.

FOURTH EXAMPLE WORKED OUT. To what depth will a squared log of beech sink in water, the thickness of the log being 2· ft. 1· in., its density uniform, and the Specific Gravity of the wood ·85?

The density or closeness of grain of the wood is given *uniform* that we many understand it to float evenly, and thus to displace a mass of water a parallelopipedon in form.

There are, then, two such figures in question.

I. The parallelopipedon of beech, supported by the water.
II. The parallelopipedon of water, displaced by the beech.

These two solids are of equal weight. They are also of equal length and breadth. Hence, length and breadth of the log are not taken into account.

But, they are of unequal depths. For, beech (Sp. gr. ·85), being lighter than water (Sp. gr. 1·), will obtain sufficient support, that is, will float, by displacing less than its own bulk of water; and length and breadth being the same in both, the *depth* of the water displaced will be less than the whole *thickness* of the log of beech.

The Sp. gr. of the wood being ·85, any bulk of it will weigh ·85 of the weight of the same bulk of water.

Hence, the log in question must displace ·85 of its own bulk of water; and this it will do by sinking ·85 of its whole thickness.

∴ *Depth sunk* = ·85 of 2· ft. 1· in.
= 25· in. × $\frac{85}{100}$ = $\frac{85}{4}$ lin. in.
= 1· ft. $9\frac{1}{4}$ in.

PROOF.

Suppose any length and breadth for the piece of timber, say 40· ft. long, and 4· ft. wide. Then,

∵ Specific gravity of beech is ·85,
∴ Weight of 1· cu. ft. of beech is 850· oz. avoir.
∴ *Weight of log* = 850· oz × 30· × 4· × $2\frac{1}{12}$
= 212500· oz. avoir.

And, *Weight of water disp.* = 1000· oz. × 30· × 4· × $\frac{21\frac{1}{4}}{12}$
= 212500· oz. avoir.

The weight of displaced water being, then, equal to that of the log, the latter floats when immerged to the depth named, 1· ft. $9\frac{1}{4}$ in.

FIFTH EXAMPLE WORKED OUT. How far would the said log sink in Vinegar, Sp. gr. 1·02?

Weight of 1· cu. ft. of vinegar = 1020· oz. avoir.
Weight of 1· cu. ft. of beech = 850· oz. avoir.
Weight of 1· cu. ft. of beech = $\frac{850}{1020}$ or $\frac{5}{6}$ wt. of cu. ft. of vinegar.

∴ A cu. ft. or any mass of beech (Sp. gr. ·85) will float by displacing $\frac{5}{6}$ of its own bulk of vinegar (Sp. gr. 1·02).

∴ Parallelopipedon of such timber, floating evenly, displaces $\frac{5}{6}$ of its own depth of vinegar; or, in present example,

$\frac{5}{6}$ of 2· ft. 1· in. = $1\frac{53}{72}$ ft.
= 1· ft. $8\frac{5}{6}$ in.

∴ *Depth log will sink* = 1· ft. 8· in. 10· li.

PROOF.

Suppose the timber to be 1· ft. long and 1· ft. wide.

Then, *Wt. of block* = 850· oz. × $2\frac{1}{12}$ = $1770\frac{5}{6}$ oz.
And, *Wt. of vinegar* = 1020· oz. × $1\frac{53}{72}$ = $1770\frac{5}{6}$ oz.

∴ Block, by sinking in Vinegar to a depth of 1· ft. $8\frac{5}{6}$ in. displaces its own weight of the liquid, and is consequently supported by, or floats in it.

SIXTH EXAMPLE WORKED OUT. What additional weight, placed *on the top* of a squared log of beech, 19· ft. by 3· ft. by 15· in., will just sink it to the water's edge; Specific Gravity of the wood being ·85?

Example completed.

The log, when immersed to the water's edge, displaces its own bulk or (19· × 3· × 1¼) cubic feet = 71¼ cu. ft.

But its own weight is supported by displacing only ·85 of its bulk.

Hence, Bulk of water supporting added weight } = ·15 of bulk of Log.
= ·15 of 71·25 cubic feet.
= 10·6875 cubic feet.

And, added weight = weight of water displaced to support it.
= 1000· oz. × 10·6875 = 10687·5 oz. PRIN. VIII.

∴ Added weight = 5· *cwt.* 3· *qr.* 23· *lb.* 15· *oz.* 8· *dr.*

PROOF.

		oz.		oz.
Weight of beech	=	850· × 71¼	=	60562·5
Added weight			=	10687·5
Weight of water disp.	=	1000· × 71¼	=	71250·

The combined bodies are, together, of the same weight as the water displaced, which therefore supports them.

From this Example we derive a simple rule.

To find what weight, not immersed, a body lighter than a liquid will support when fully immersed in that liquid.

Subtract weight of the supporting body from weight of its own bulk of the liquid.

The difference is the weight which may be added unimmersed.

SEVENTH EXAMPLE WORKED OUT. What weight of iron, Sp. gr. 7·25, *attached to the bottom* of above-mentioned log, will just sink it to the water's edge?

Example completed.

Here, the iron, being also immersed, is partly supported by displacing its own bulk of water.

A cubic foot of iron weighs 7250· oz.
But displaces only 1000· oz. of water.
Hence, of each cubic foot of iron, 6250· oz. derive support from the log of wood.

Now, the loaded log displaces 10687·5 oz. of water beyond what is requisite for its own support. (See last Example.)

∴ No. of cu. ft. of iron = 10687·5 ÷ 6250· = 1·71 cu. ft.

Weight of iron = 7250· oz. × 1·71 = 6· *cwt.* 3· *qr.* 18· *lb.* 13· *oz.* 8· *dr.*

PROOF.

Whole *bulk* of water displaced = United *bulk* of wood and iron.

= 71·25 c. ft. + 1·71 c. ft. = 72·96 cu. ft.

∴ *Weight of water displaced* = 1000· oz. × 72·96 = 72960· oz.

Now, this should be equal to joint weight of the wood and iron. And it is so, for

Weight of wood = 850· oz. × 71·25 = 60562·5 oz.
Weight of iron = 7250· oz. × 1·71 = 12397·5 oz.
∴ *United weights supported* = (*Wt. of water*) = 72960· oz.

From the examination of this example we may also deduce a useful rule. For, suppose the question to be confined to a single cubic foot of beech, Sp. gr. ·85; then:

∵ Weight of 1· cubic ft. of water = 1000· oz.
And, Weight of 1· cubic ft. of beech = 850· oz.
∴ 1· *cu. ft. of beech will support* 150· *oz. addl.*

Again:

∵ Weight of 1· cubic ft. of iron = 7250· oz.
And, Weight of 1· cubic ft. of water = 1000· oz.
∴ *One cu. ft. of iron requires* 6250· oz. *additional support.*

Hence:

One cu. ft. of beech will support $\Big\}$ $\frac{150}{6250}$ or $\frac{3}{125}$ of a cu. ft. of iron.

And this fraction is brought out thus:

$$\frac{1000\cdot - 850\cdot}{7250\cdot - 1000\cdot} \text{ that is } \frac{\text{Water} - \text{Beech}}{\text{Iron} - \text{Water}} \text{ or } \frac{\text{Wr.} - \text{Lighter B.}}{\text{Heavier B.} - \text{Wr.}}$$

Any bulk of beech, Sp. gr. ·85, will, then, support $\frac{3}{125}$ of that bulk of iron, Sp. gr. 7·25.

In other words

125· cu. ft. of beech will support 3· cu. ft. of iron, both being wholly immersed.

∴ 1· cu. ft. of iron requires the support of $\frac{125}{3}$ cu. ft., or $41\frac{2}{3}$ cu. ft. of beech.

And this latter fraction is the reciprocal of the former.

To find what bulks of any two materials, one specifically heavier, and the other specifically lighter than a given liquid, must be combined, so that the united bodies may just sink to the surface of that liquid.

First. Find the difference of weight of a cubic foot of the liquid and of the same bulk of the lighter body.

Second. Find the difference of weight of a cubic foot of the heavier body and of a cubic foot of the liquid.

Third. Divide the first of those differences by the second. The result will express the fraction or multiple of its own bulk which the lighter body can support of the heavier.

Fourth. Divide the second of the differences by the first, and the result will express the multiple or fraction of its own bulk which the heavier body will require of the lighter to support it.

PROOF OF LAST EXAMPLE.

$$\begin{array}{rlrcr} & & \text{oz.} & & \text{oz.}\\ 125\cdot & \text{cu. ft. of beech weigh,} & 850\cdot \times 125\cdot & = & 106250\cdot \\ \underline{3\cdot} & \text{cu. ft. of iron weigh,} & 7250\cdot \times 3\cdot & = & \underline{21750\cdot} \\ \underline{\underline{128\cdot}} & \text{cu. ft. of water weigh,} & 1000\cdot \times 128\cdot & = & \underline{\underline{128000\cdot}} \end{array}$$

Arithmetical Symbol. The sign $\sim$ placed between two quantities, shews that their difference is to be taken.

Hence, if F represent one quantity, and B represent another, $F \sim B$ means that the less of the two, whether placed first or second, is to be taken from the other.

$$\therefore \text{ If F be the greater, } F \sim B = F - B;$$
$$\text{If B be the greater, } F \sim B = B - F.$$

Let F be weight of a certain bulk of any fluid, and B be weight of the same bulk of any solid, whether specifically heavier or lighter than the fluid.

Suppose the solid to be put into a vessel containing a sufficient quantity of the said fluid;

Then the fraction $\dfrac{F \sim B}{B}$ will coincide with

I. Ratio of unimmersed bulk to immersed.

If the solid be specifically lighter than the fluid.

II. Ratio to lighter body of additional unimmersed weight which it will sustain.

III. Ratio of unsupported weight to that supported by fluid.

If the solid be specifically heavier than the fluid.

IV. Ratio to heavier body of additional support requisite to sustain it.

V. Fraction of heavier body which must be excavated to make it float in the fluid. (See EXERCISE CXVI., Example 7·)

Let the fluid be distilled water, Sp. gr. 1·; the heavier solid, iron, Sp. gr. 7·25; and the lighter solid, cork, Sp. gr. ·24. Then the following instances will illustrate and prove the foregoing statements.

I.

Since a cu. ft. of cork weighs 240· oz., and a cu. ft. of water weighs 1000· oz.; and the cork will float when it displaces its own weight of water,

∴ Any mass of cork will float by displacing $\frac{240}{1000}$ of its bulk of Water, leaving unimmersed $\frac{1000\cdot-240\cdot}{1000\cdot}$ of its whole bulk.

But $\frac{1}{1000}$ of the whole bulk $=\frac{1}{240}$ of immersed bulk.

$$\therefore \textit{Unimmersed bulk} = \frac{1000\cdot-240\cdot}{240\cdot}\ \textit{of imm}^{d.}\ \textit{bk.} \doteq \frac{F-B}{B} \quad \text{(I.)}$$

II.

The cork is supported by displacing $\frac{240}{1000}$ of its own bk. of $\text{W}^{\text{r.}}$

∴ Displacing $\frac{1}{1000}$ of bulk supports $\frac{1}{240}$ of weight.

But there are $\frac{760}{1000}$ of bulk of cork not immersed by its own weight.

∴ To immerse these also, $\frac{760}{240}$ of its own wt. must be superimposed.

∴ *Add*$^{l.}$ *unimmersed weight which cork, fully immersed, will support,*

$$= \frac{1000\cdot-240\cdot}{240\cdot}\ \textit{of its own wt.} \doteq \frac{F-B}{B} \quad \text{(II.)}$$

III.

A cu. ft. of iron weighs 7250· oz., but displaces only 1000· oz. of water;

$$\textit{Leaving unsupported}\ \frac{7250\cdot-1000\cdot}{7250\cdot}\ \textit{of its w}^{t.} \doteq \frac{B-F}{B} \quad \text{(III.)}$$

IV.

Fraction of weight of iron requiring extra support is also

$$= \frac{7250\cdot-1000\cdot}{7250\cdot}\ \textit{of its w}^{t.} \doteq \frac{B-F}{B} \quad \text{(IV.)}$$

V.

Hence, evidently, *the fraction of the weight of iron* which *must be excavated* that the hollow vessel may swim in water, also

$$= \frac{7250\cdot-1000\cdot}{7250\cdot} \doteq \frac{B-F}{B} \quad \text{(V.)}$$

EIGHTH EXAMPLE WORKED OUT. I. What weight of flint-glass, Sp. gr. 3·, would 1· cwt. of alder-wood, Sp. gr. ·8, support in distilled water?

II. What weight of alder will support 1· cwt. of glass?

I.

Example completed.

$\because$ 1· cwt. alder offers support for $\dfrac{1000\cdot - 800\cdot}{800\cdot}$ or $\dfrac{1}{4}$ cwt.

And, 1· cwt. glass requires support for $\dfrac{3000\cdot - 1000\cdot}{3000\cdot}$ or $\dfrac{2}{3}$ cwt.

$\therefore$ *No. cwt. glass supported by* 1· *cwt. alder* $= \left(\dfrac{1}{4} \div \dfrac{2}{3}\right)$ *or* $\dfrac{3}{8}$ *cwt.*

II.

No. cwt. alder to support 1· *cwt. glass* $= \left(\dfrac{2}{3} \div \dfrac{1}{4}\right)$ *or* $2\frac{2}{3}$ *cwt.*

PROOF OF FIRST RESULT.

Weight of united bodies $= 1\frac{3}{8}$ *cwt.*

Now, 1· cwt. = 1792· oz.; and 1· cu. ft. alder weighs 800· oz.

$\therefore$ 1· cwt. alder contains $\dfrac{1792}{800}$ or $2\frac{6}{25}$ cu. ft.

Similarly, $\dfrac{3}{8}$ cwt. glass contain $\left(\dfrac{3}{8} \text{ of } \dfrac{1792}{3000}\right)$ or $\dfrac{28}{125}$ cu. ft.

$\therefore$ Bulk of water displaced $= \left(2\frac{6}{25} + \dfrac{28}{125}\right)$ or $2\frac{58}{125}$ cu. ft.

Weight of water displaced $= 1000\cdot$ *oz.* $\times\ 2\frac{58}{125} = 1\frac{3}{8}$ *cwt.*

The united bodies, then, by displacing their own bulk of water, displace also exactly their own weight of it; and, therefore, *just swim* in it.

PROOF OF SECOND RESULT.

$2\frac{2}{3}$ cwt. alder contain $\left(\frac{1792}{800} \times 2\frac{2}{3}\right)$ or $5\frac{73}{75}$ cubic feet.

1 cwt. glass contains $\frac{1792}{3000}$ or $\frac{224}{375}$ cubic feet.

$3\frac{2}{3}$ cwt. water contain $\left(\frac{1792}{1000} \times 3\frac{2}{3}\right)$ or $6\frac{214}{375}$ cubic feet.

Since, then, Bulk of Water = Bulk of united bodies,
And, also, Weight of Water = Weight of united bodies,
$\therefore$ They just swim in it.

EXERCISE CXVI.

Prove every example.

1· How deep would a squared log of fir, 2· ft. thick, Sp. gr. ·55, sink in water of the Dead Sea, Sp. gr. 1·24?

2· The outer length and breadth of a rectangular barge being 40· ft. and 10· ft. 6· in., its inner depth, 6· ft., its sides and ends 9· in., and its bottom 12· in. thick, and Sp. gr. of material 1·5; what weight of cargo will sink it to within three inches of its gunwale in river water, Sp. gr. 1·004?

3· With the same cargo, how much higher will the vessel swim in sea-water, Sp. gr. 1·028?

4· How many cubic feet of coal, Sp. gr. 1·3, would just sink the barge to water's edge,

I. In the river?
II. In the sea?

5· How deep may it be charged with sand, Sp. gr. 2·45, so as to sink within 6· in. of its gunwale,

I. In the river?
II. In the sea?

6· A barge being loaded to the utmost safe point in sea-water, what fraction of the gross weight of vessel and cargo must be discharged so that it may sink no deeper in river water, Sp. grs. as above?

7· A squared block of granite, 8· ft. long, 7· ft. wide, and 5· ft. deep, Sp. gr. 2·688, is to be hollowed into a cistern which shall swim 6· inches clear in water, Sp. gr. 1·. How many cubic feet of the stone must be removed?

From Example 7· it becomes evident how a body, specifically heavier than a liquid, may still be made to swim in that liquid. This, as we see, is done by forming the heavy body into a hollow vessel of such *capacity* and *thinness* that it may displace its own weight of the liquid without being fully immersed.

It is by this means that ships can be made of iron to swim in water.

8· Supposing the side and end walls of the cistern to be left 6· in. thick, how deep must the excavation be, if made

I. In the top of the block?
II. In the side?
III. In the end?

9· How many cubic feet of cork, Sp. gr. ·24, must be attached to a squared log of mahogany, 7· ft. long, 4· ft. wide, and 2· ft. thick, Sp. gr. 1·063, to support it in water, Sp. gr. 1·?

10· If a cubic foot of tin, Sp. gr. 7·3, be supported in distilled water by 18· cu. ft. of fir, what is the Sp. gr. of the wood?

11· What fraction of its own weight, not immersed, will any mass of deal, Sp. gr. ·55, support, being itself fully immersed in water, Sp. gr. 1·?

12· What additional weight, unimmersed, will 1· cwt. of cork Sp. gr. ·24, support when sunk to its own edge,

I. In distilled water?
II. In linseed oil, Sp. gr. ·94?

13· What weight of iron, Sp. gr. 7·25 may be affixed to 1· cwt. of cork, Sp. gr. ·24, so that the united bodies may be fully immersed by their joint weight,

I. In distilled water?
II. In sea-water, Sp. gr. 1·028?
III. In linseed oil. Sp. gr. ·94?

14· A prism of willow-wood, 18· in. square, Sp. gr. ·6, is to be evenly coated, on one side, with as much lead as it will just sustain in water, Sp. gr. 1·; how thick will the lead be?

The length of the prism is not given, as it will not affect the question.

15· A squared log of Spanish cypress, 8· ft. long, 3· ft. wide, and 2· ft. thick, Sp. gr. ·65, is to be uniformly coated with as much copper as it will support in water, Sp. gr. 1·; what may be the thickness of the coating if applied,

I. On the side of the prism?
II. On its end?
III. On the bottom of it?

16· A body of irregular form, when entirely immersed in a vessel quite filled with liquid, causes 3· cubic feet of it to overflow. The body weighs 84·lb. What is its Specific Gravity?

17· What is the Sp. gr. of a body which weighs 2· cwt. 1· qr. 13·lb., and, being wholly immersed, displaces 40· lb. of distilled water?

18· A cube of wood, uniformly dense, measuring 15· inches in the edge, and floating evenly in a rectangular vessel of distilled water, 3· feet long, by 2· feet wide, and of sufficient depth, causes the water to rise $3\frac{3}{4}$ inches in the vessel. What is the Specific Gravity of the wood?

To ascertain this, find:

1· *Area of whole surface of water before immersion of wood.*
2· *Area of surface elevated.* (Exercise cxii. a. Ex. 13·)
3· *Bulk of water displaced, or elevated.*
4· *Weight of ditto, which = weight of the wood.*
5· *Bulk of the wood.*
6· *Weight of* 1· *cu. ft. of wood.*
7· *Specific Gravity of the wood.* (Page 294·)

19· What is the Sp. gr. of cider, of which a gallon weighs 10· lb. 2· oz. $14\frac{2}{25}$ drs.?

20· What is the weight of a tun of wine, Sp. gr. ·991?

THE HYDROMETER.

The Hydrometer is an instrument for ascertaining the Sp. gr. of fluids. These instruments are constructed on two different principles.

I. **Sikes's Hydrometer** is of invariable weight, and, therefore, shews the Sp. gr. of different fluids by the different depths to which it sinks in them.

II. **Fahrenheit's Hydrometer,** or the **Gravimeter,** by varying its weight, is made to sink to the same depth in any fluid, the differences of weight necessary so to sink it shewing the Sp. gr. of the fluids.

III. **Nicholson's Hydrometer** is on the same principle as Fahrenheit's, but can be employed to discover the Sp. gr. of solids as well as of fluids.

NINTH EXAMPLE WORKED OUT, ILLUSTRATING THE PRINCIPLE OF SIKES'S HYDROMETER. A rectangular prism of uniform density sinks for support only 5· inches in distilled water, but sinks $\frac{15}{47}$ of an inch deeper in a certain oil. What is the Sp. gr. of the oil?

Example completed.

In each of the two fluids the prism displaces its own weight of that fluid. (Page 297·)

The length and breadth of the prism, being the same in both cases, do not affect the question. We may, therefore, assume any convenient length and breadth, say 1· inch each.

∴ Bulk of water displaced = 5· cu. inches.
And, Bulk of oil displaced = $5\frac{15}{47}$ cu. inches.
∴ $5\frac{15}{47}$ cu. ft. of oil weigh = 5· cu. ft. of water.
Multiply both sides by 47· (PRINCIPLE XXIV.)
∴ 250· cu. ft. of oil weigh = 235· cu. ft. of water.
Divide both sides by 250· (PRINCIPLE XXIV.)
∴ 1· cu ft. of oil weighs = $\frac{47}{50}$ of 1· cu. ft. of water.
∴ Weight of 1· cu. ft. of oil = $\frac{47}{50}$ of 1000· oz. = 940· oz.
∴ *Specific Gravity of oil* = ·94. (Page 294·)

To obviate the necessity of performing, in every instance, a calculation like the foregoing, the Sp. grs., corresponding to the different depths to which Sikes's hydrometer will sink in different fluids, are recorded in tables. These tables, however, are, of course, only true for hydrometers *exactly similar in form and weight*, and the difficulty of ensuring *exact similarity* in the instruments forms a serious objection to their use. This difficulty, in fact, stimulated the invention of the much more simple and manageable hydrometer of Fahrenheit.

TENTH EXAMPLE WORKED OUT, ILLUSTRATING THE PRINCIPLE OF FAHRENHEIT'S HYDROMETER. A prism weighing 640· grs. sinks in distilled water to a certain depth, when loaded with 360· grs. unimmersed; but, in highly rectified alcohol, it sinks to the same point when sustaining only 169· grs. Required the Sp. gr. of the spirit.

Example completed.

The prism displaces same bulk in both liquids.
∴ Weight of water displaced = (640. + 360·) * = 1000· grs.
And Weight of same bulk of spirit = (640· + 169·) = 809· grs.
∴ *Specific Gravity of Alcohol* = ·809.

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* If the two weights to sink it in water be = 1000·, then the sum of the two weights which sink it to same depth in any other fluid will = Sp. gr. of that fluid.

ELEVENTH EXAMPLE WORKED OUT, ILLUSTRATING THE ACTION OF NICHOLSON'S HYDROMETER IN FINDING SPECIFIC GRAVITY OF A SOLID. A prism which requires 1000· grs. unimmersed, to sink it to a certain depth in distilled water, will sink to same point if loaded with a piece of metal and 460· gr.

Now, if the same piece of metal be transferred and attached to the bottom of the prism, so as to be completely immersed, the prism will then require 560· grs. unimmersed to sink it to same depth as at first. Required Sp. gr. of the metal.

Example completed.

Weight of metal = (1000· − 460·) *grs.* = 540· *grs.*

But, it loses in water (560· — 460·) grs. = 100· grs.

That is to say, the water supports 100· grs. of the metal.

∴ *Weight of same bulk of water* = 100· *gr.*

∴ *Specific Gravity of the metal* = (540· ÷ 100·) = 5·4.

The Sp. gr. of any body is found by dividing its weight by weight of same bulk of distilled water.

For, Sp. gr. of any body is the multiple or part which its weight is of the weight of distilled water, equal bulks of each being compared. (Page 294·)

SIMPLE PROPORTION, OR RULE OF THREE.

Under this rule are usually placed all questions in which, from three given Concrete Quantities, a fourth proportional is to be found.* So that "The Rule of Three"† is the application to Concrete Quantities of the principle of Ratio, explained on pages 145· to 150·, which pages should now be carefully re-studied.

DIRECT AND INVERSE PROPORTION.

Any two Concrete Quantities which are so related that they increase and decrease together, and in the same ratio, are in **Direct Proportion.**

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* It will be presently shewn that every such question may be more naturally and clearly solved by Multiplication and Division, without any reference to Ratio.

† The old Arithmeticians were fond of calling this "*The Golden Rule;*" an appropriate designation, but in a contrary sense to theirs, inasmuch as "The Rule of Three" is, like gold, by very much more ornamental than useful in the ordinary affairs of life. Those who have not made a point of enquiring would be surprised to learn how very rarely "The Rule of Three" is employed by men of business. It is serviceable in enabling us to enunciate the more advanced rules in brief form; and in higher mathematics it becomes aluable.

Thus: I. A certain quantity of goods cost a certain sum. Then, at the same rate,

Twice the quantity will cost *twice the sum.*
Half the quantity will cost *half the sum.*

And so on. Any multiple or fraction of the said quantity of goods will cost the same multiple or fraction of the value of that quantity; so long as the rate remains unchanged, or "constant."

II. If any number of workers complete a certain amount of work, then, in the same time,

Thrice as many such workers will do *thrice as much* work.
One-fifth as many " " *one-fifth as much* work.

From these and similar examples, we see that

I. *Quantity of goods and Gross Value are in direct Proportion;*
When rate is constant.

II. *No. of workers and Work done, are in Direct Proportion;*
When time, and quality of workers, are constant.

III. *Time of working and Work done, are in Direct Proportion;*
When number and quality of workers are constant.

IV. *No. of eaters and Food consumed are in Direct Proportion;*
Time, and daily allowance for each, being constant.

But, when two Concrete Quantities are in such relation that as one *increases* the other *decreases* in an exactly opposite (or reciprocal) ratio, so that, for example, *doubling* either of them compels us to *halve* the other, and so on, those two quantities are in **Inverse Proportion.**

The following cases will illustrate this.

I. If, at a given rate each, a certain No. of articles cost "so much," it is evident that, if the price of each be *doubled*, only *half as many* can be bought for the same amount of money.

II. Supposing a cask of wine to supply a certain number of men with *one pint* each, *only one-eighth of that number* could receive *eight pints each* from the same quantity of wine.

III. The same quantity of food which would furnish a certain number of men with a certain daily allowance for 20· days, would give to each of them *four times as large a daily ration*, if they were only to be fed for *one fourth of the time.*

IV. If a sack of corn last any number of horses one week, it would suffice for *seven times as many* for *one seventh of the time*, each receiving the same daily allowance as before.

V. If a certain weight can be carried "so far" for "so much," only *half the weight* could be carried *twice the distance*, for the same sum, and at the same rate.

VI. But, *twice the weight* could be carried the same distance for the same sum, *at half the rate.*

VII. Also, the whole original weight could be conveyed *twice the distance* for the same sum, *at half the rate.*

VIII. If 50· workers complete a task in any given time, 300· (*six times* 50·) similar workers should do it in *one-sixth of the time.*

IX. The Principal requisite to gain a certain sum of Interest *in ten months* is only *one-tenth* of the Principal necessary to gain the same Interest in *one month*, at the same rate.

X. If two rectangles be of equal Area, but the second *twice as long* as the first, it can only be *half as wide.*

XI. If two rectangular prisms be of equal solidity and thickness, but the first *five times as long* as the second, it can only be *one-fifth as wide.*

XII. If a certain rectangular prism have to sink *twice as deep* for support in one fluid as it does in another, the Specific Gravity of the first fluid is only *half* that of the other.

&c. &c. &c.

From the foregoing examples we see that,

I. *No. of articles and Price of each are in Inverse Proportion;*
When Gross value is constant.

II. *No. fed and allowance to each are in Inverse Proportion;*
Gross amount of food being constant.

III. *Ration and Time of Feeding are in Inverse Proportion;*
Gross amount of food and No. fed being constant.

IV. *No. fed and Time of maintenance are in Inverse Proportion;*
Gross amount of food and ration being constant.

V. *Weight conveyed and Distance are in Inverse Proportion;*
Gross cost and rate of carriage being constant.

VI. *Weight conveyed and Rate are in Inverse Proportion;*
Gross cost and distance being constant.

VII. *Distance and Rate are in Inverse Proportion;*
Weight and gross cost of carriage being constant.

VIII. *No. of workers and time occupied are in Inverse Proportn.;*
Amount of work remaining constant.

IX. *Principal and Time are in Inverse Proportion;*
When Interest and Rate are constant.

X. *Lengths* (of Rectangles) *and Breadths are in Inv. Proportion;*
Areas being constant, but dimensions unlike.

XI. *Lengths and Breadths* (of sq. prisms) *are in Inv. Proportion,*
Only Solidities and Thicknesses being constant.

XII. *Bulks and Specific Gravities are in Inverse Proportion;*
Weights remaining constant.

Direct Proportion. *Quantities are in Direct Proportion when multiplying one of them by any number or fraction causes the other of them to be multiplied by the same number or fraction.*

Inverse Proportion. *Quantities are in Inverse Proportion, when multiplying either of them by any number or fraction causes the other of them to be multiplied by the reciprocal of that number or fraction.* Pages 99· and 145·

This latter definition is frequently, but with less precision, given thus:

Two quantities are in Inverse Proportion when *multiplying* either of them by any number or fraction causes the other of them to be *divided* by the same number or fraction.

In every Simple Proportion (or Rule of Three) question, three Concrete terms are given. Two of these are of the same kind, so that there exists a ratio between them. PRINCIPLE V.

The odd term is of the same kind as the answer sought: so that there exists a ratio between them. PRINCIPLE V.

And these two ratios being equal, the four terms are Proportionals. Page 146·

GENERAL RULE FOR SIMPLE PROPORTION, DIRECT OR INVERSE.

First. *Write the question in an abbreviated form.*
See Examples worked out.

Second. *State the Proportion*, thus:

a. Write the word "As" and the signs of Proportion, leaving spaces for the three given terms, and putting the letter x in the fourth place, to represent the required term (or fourth proportional) until found, thus:

As : :: : x.

b. Put in the third place the term which is of the same kind as the answer sought.

This, as said above, is the odd term, and, in the abbreviated question, has a note of interrogation under it.

c. Consider whether, from the nature of the question, the answer will be greater or less than the third term.

If the answer will be greater than the third term, put the greater of the other two in the second place.

But, if the answer will be less than the third term, put the less of the other two terms in the second place.

d. Put remaining term in the first place.

And now, if the Proportion be correctly stated:

The first and second terms will be of the same kind.

The third term will be of the same kind as the answer.

The second term will be greater or less than the first, according as the answer is to be greater or less than the third. Note, also, that

In Direct Proportion, the term whose value is *sought* stands second.

In Inverse Proportion, the term whose value is *given* stands second.

Third. *Work for the Fourth Proportional*, thus:

- *a.* If either of the first two terms be a Compound Quantity, or if they be of unlike denominations, reduce both to the same name.
- *b.* If the third term be a Compound Quantity, reduce it to its lowest denomination, or leave it in the Compound form, as may be most convenient.*
- *c.* Cancel Common Factors in first and second, and in first and third terms. Page 149·
- *d.* Multiply second and third terms together, and Divide by first.

The result is the fourth proportional, or answer sought, in the same name or names as that or those in which the third term was left.

FIRST EXAMPLE WORKED OUT. A DIRECT PROPORTION. REDUCTION UNNECESSARY. NO CANCELLING. If 21· cwt. cost £53· 3·s. 1½d., what will 11· cwt. cost.?

Example completed.

I. BY PROPORTION.

Abbreviated Question.

	cwt.		£.	s.	d.
If	21·	. . . .	53·	3·	1½
	11·	. . . .		?	

Statement and Work.

cwt. cwt.	£.	s.	d.		cwt.
As 21· : 11· ::	53·	3·	1½	(c. of	21·) : *x*.
			11·		
21· { 7·)	584·	14·	4½	c. of	231·
3·)	83·	10·	7½	c. of	33·
	27·	16·	10½	c. of	11·

Here we have Price in 3rd, because Answer will be price: and 2nd term less than 1st, because Answer will be less than 3rd.

---

* This reduction, although only occasionally serviceable, is, in many Arithmetics, enjoined as an essential part of the process.

II.

The same Example worked out without reference to ratio.

$$\begin{array}{rrrrl}
 & \pounds. & s. & d. & \\
21\cdot \left\{ 7\cdot\right) & 53\cdot & 3\cdot & 1\frac{1}{2} & = \text{Cost of } 21\cdot \text{ cwt.} \\
\cline{2-4}
3\cdot) & 7\cdot & 11\cdot & 10\frac{1}{2} & = \text{Cost of } 3\cdot \text{ cwt.} \\
\cline{2-4}
 & 2\cdot & 10\cdot & 7\frac{1}{2} & = \textit{Cost of one cwt.} \\
 & & & 11\cdot & \\
\cline{2-4}
 & 27\cdot & 16\cdot & 10\frac{1}{2} & = \text{cost of } 11\cdot \text{ cwt.} \\
\cline{2-4}\cline{2-4}
\end{array}$$

This simple and natural method is much more frequently employed than "The Rule of Three," in actual business.

GENERAL RULE FOR WORKING WITHOUT REFERENCE TO RATIO.

First. Find the value of one.

Second. Find, from that, the value of the number required.

When the value of *one* is either given or demanded, only Multiplication or Division will be requisite, according as the quantities move in Direct or Inverse Proportion. Hence,

When either of the three given terms is Unity, reference to Ratio, or Statement of Proportion, is of no utility whatever.

III.

The Example may also be neatly solved as an Equation.

Thus:

$$\because \text{ Ans. sought} = \text{cost of } 11\cdot\text{; and } \overset{\pounds.}{53}\cdot\ \overset{s.}{3}\cdot\text{s. } \overset{d.}{1\tfrac{1}{2}} = \text{cost of } 21\cdot$$

$$\therefore \tfrac{1}{11} \text{ of Answer sought} = \text{cost of } 1\cdot$$

$$\text{Also, } \tfrac{1}{21} \text{ of } \pounds 53\cdot\ 3\cdot\text{s. } 1\tfrac{1}{2}\text{d.} = \text{cost of } 1\cdot$$

And, things = same thing = one another.

$$\therefore \tfrac{1}{11} \textit{ of Answer sought} = \tfrac{1}{21} \textit{ of } \pounds 53\cdot\ 3\cdot\textit{s. } 1\tfrac{1}{2}\textit{d.}$$

Then, multiplying both sides by 11· (PRINCIPLE XXIV.), we have,

$$\textit{Answer sought} = \tfrac{11}{21} \textit{ of } \pounds 53\cdot\ 3\cdot\textit{s. } 1\tfrac{1}{2}\textit{d.} = \pounds 27\cdot\ 16\cdot\textit{s. } 10\tfrac{1}{2}\textit{d.}$$

In this book, a large number of questions which would ordinarily be placed under "The Rule of Three," have been already given under Compound Multiplication and Division. For instance, in Exercise

LXXXIX. B.	Examples	1· to 10·	XCVII.	Examples	31· to 53·
XCII.	„	17· to 51·	XCIX.	„	13· to 60·
XCIII.	„	21· to 64·	C.	„	21· to 52·
XCVI.	„	8· to 19·	CI.	„	5· to 68·

In addition to these two hundred and sixty-eight, there are several Examples in Exercise CIV., and elsewhere, which would generally be included under "The Rule of Three."

SECOND EXAMPLE WORKED OUT. AN INVERSE PROPORTION. NO REDUCTION OR CANCELLING. In what time will 24· men do as much work as 13· men do in 11· days?

Example completed.

I. BY PROPORTION.

Abbreviated Question.

If 13· men .... 11· days.
24· men .... ?

Statement and Work.

Here are two terms *workers* (13· men and 24· men).
The odd term is *time*.
The answer will be time.

Put time (11· days) *in the third term.*

Will the answer be more or less than the third term? An increased No. of workers will complete the task in a shorter time. For Time and No. of workers are in Inverse Proportion. See page 309· VIII.

Therefore, 24· men will do the work in a shorter time than 13· men.

The answer will be less than the third term; therefore
The second term must be less than the first.
Put 13· (*men*) *in second place, and* 24· (*men*) *in first.*

$$\begin{array}{llllll} & \text{men.} & & \text{men.} & & \text{days.} & & \text{men.} \\ \text{As} & 24\cdot & : & 13\cdot & :: & 11\cdot & (\text{time for } 13\cdot) & : \quad x. \\ & & & & & \underline{13\cdot} & & \\ & & & & 24\cdot \left\{ \begin{array}{l} 8\cdot)\overline{143\cdot} \\ 3\cdot)\overline{\ 17\tfrac{7}{8}} \end{array} \right. & \begin{array}{l} = \text{time for } \ 1\cdot \text{ man.} \\ = \text{time for } \ 8\cdot \text{ men.} \end{array} & & \end{array}$$

$$x = \overset{\text{d.}}{\frac{11 \times 13}{24}} = \overset{\text{d.}}{\frac{143}{24}} = \underline{\underline{5\tfrac{23}{24}}} = \text{time for } 24\cdot \text{ men.}$$

Ans. 5·d. 11½ ho., taking a working day as = 12· hours.

II.

The same Example worked without reference to ratio.

If we proceed in obedience to the general rule given on page 312·, the resulting work will be precisely similar to the foregoing. For, in every Inverse Proportion Example, the *value of one* is brought out by multiplying second and third terms together, which thus completes the first step of the general rule for working without reference to ratio.

Without recourse, then, to Proportion, the question would be reasoned out thus:

Time for 13· *men* = 11· *days.*

But, since each of the 13· men works 11· days, one man, to d the whole, must work 13· times 11· days.

$$\therefore\ \textit{Time for } 1\cdot \textit{ man} = 11\cdot \textit{ days} \times 13\cdot = 143\cdot \textit{ days.}$$

Now, 24· men, working together, will do as much in one day as one man does in 24· days.

$$\begin{aligned}\therefore\ \textit{Time for } 24\cdot \textit{ men} &= \tfrac{1}{24} \textit{ of time for one man.}\\ &= \tfrac{1}{24} \text{ of } 143\cdot \text{ days.}\\ &= 5\tfrac{23}{24} \text{ days} = 5\cdot\text{d. } 11\tfrac{1}{2} \text{ h.}\end{aligned}$$

III.

Or, reversing the order of the Multiplication and Division, (PRINCIPLE XVIII.) the question may be solved and explained thus:

$$\because\ \textit{Time for } 13\cdot \textit{ men} = 11\cdot \textit{ days.}$$

And 24· times as many men would do the work in $\frac{1}{24}$ of the time.

$$\therefore\ \textit{Time for } (24\cdot \textit{ times } 13\cdot \textit{ or}) \ 312\cdot \textit{ men} = \tfrac{1}{24} \textit{ of } 13\cdot\textit{d.} = 5\tfrac{1}{2}\textit{h.}$$

But, 24· men, being $\frac{1}{13}$ of 312· men, will require 13· times as long.

$$\therefore\ \textit{Time for } 24\cdot \textit{ men} = 5\tfrac{1}{2} \textit{ ho.} \times 13\cdot = 5\cdot\textit{d. } 11\tfrac{1}{2} \textit{ ho.}$$

IV.

The same Example solved as an Equation.

$$\begin{aligned}&\because\ 11\cdot \text{ days} = \text{time for } 13\cdot \text{ men,}\\ &\text{And, answer sought} = \text{time for } 24\cdot \text{ men;}\\ &\therefore\ 11\cdot \text{ days} \times 13\cdot = \text{time for one man.}\\ &\text{Also, answer} \times 24\cdot = \text{time for one man.}\\ &\text{And things} = \text{same thing} = \text{one another.}\\ &\therefore\ \text{Answer } \times 24\cdot = 11\cdot \text{ days} \times 13\cdot\end{aligned}$$

∴ Dividing both sides by 24· we have (PRINCIPLE XXIV.)

$$\begin{aligned}\text{Answer, or time for } 24\cdot \text{ men,} &= (11\cdot\text{d.} \times 13\cdot) \div 24\\ &= 5\cdot\text{d. } 11\tfrac{1}{2} \text{ ho.}\end{aligned}$$

THIRD EXAMPLE WORKED OUT. REDUCTION AND CANCELLING IN FIRST TWO TERMS. What will 12· cwt. 15¾ lb. cost, at the rate of £5· 4·s. 0¾d. for 13· cwt. 3· qr. 14· lb.?

I.

Example completed by Proportion.

Abbreviated Question.

	cwt.	qr.	lb.		£.	s.	d.
If	13·	3·	14·	. . . .	5·	4·	0¾
	12·	0·	15¾	. . . .		?	

Statement and Work.

```
      cwt. qr. lb.     cwt. qr. lb.       £.  s.  d.          cwt. qr. lb.
   As 13· 3· 14·  :  12· 0· 15¾  ::   5·  4·  0¾ (c. of 13· 3· 14·) : x.
       4·              4·                      7·
      55· qrs.        48· qrs.      8·)36·  8·  5¼  c. of 97· 0· 14·
      28·             28·              4· 11·  0½ ⅝ c. of 12· 0· 15¾
    1554· lb.       1359· lb.
       4·              4·
Or, As 6216· qr. lb. : 5439· qr. lb.
Or, As  888·         :  777·
Or, As    8·         :    7·
```

II.

Or thus: reducing also third term, and abstaining from Cancelling.

Statement and Work.

```
 cwt. qr. lb.        cwt. qr. lb.       £.  s. d.           cwt. qr. lb.
As 13· 3· 14·   :   12· 0· 15¾   ::   5· 4· 0¾  (c. of 13· 3· 14·)  :  x.
    4·               4·                20·
   55· qr.          48· qr.           104· s.
   28·              28·                12·
  444·             399·              1248· d.
 1110·             960·                 4·                    qr.-lb.
 1554· lb.        1359·              4995· far. = cost of 6216.
    4·               4·              5439.                          qr.-lb.
 6216· qr.-lb.    5439· qr.-lb.     44955·  c. of      9· times 6216·
                                   149850·  c. of     30· times 6216·
                                  1998000·  c. of    400· times 6216·
                            far. 24975000·  c. of   5000· times 6216·
                          6216·)27167805·  c. of   5439· times 6216·
                                 4·)4370⅝ c. of 5439· qr.-lb. (PRIN. XX.)
                              12·)1092½d. ⅝ q. = do.
                                 20·)91·s. 0½d. ⅝ q. = do.   cwt. qr. lb.
                                  £4· 11·s. 0½d. ⅝ q. = c. of 12· 0· 15¾.
```

This is the unwieldy method until recently recommended for *every example* in "The Rule of Three" when *any* of the terms are compound quantities.

If the long Division, which we have suppressed, were written out at full, the work would occupy *twenty-two lines* under third term, instead of *three*, which, as is shewn above, are sufficient.

The third term should never be "reduced to its lowest name" unless the multiplier (2nd term) be too large for convenient Compound Multiplication. Page 311· *b.*

IMPLICIT CHANGES OF UNIT.

The 6216·, being employed to divide the farthings, in last Example, are implicitly changed into farthings, because farthings can only be divided by (that is, distributed into lots of) farthings. (PRINCIPLE XIV.)

Wishing to ascertain the $\frac{1}{6216}$th part of 27167805· far., we proceed to find how many lots of 6216· far. are contained in 27167805· far.

$$\begin{aligned} \text{Now, } 27167805\cdot \text{ fars.} &= 4370\tfrac{5}{8} \text{ lots of } 6216\cdot \text{ fars. each.} \\ &= 6216\cdot \text{ lots of } 4370\tfrac{5}{8} \text{ fars. each.} \\ \therefore \tfrac{1}{6216} \text{ of } 27167805\cdot \text{ fars.} &= 4370\tfrac{5}{8} \text{ fars.} \quad \text{See p. 218}\cdot \end{aligned}$$

In the work under third term of this example there are, also, other six implicit changes of unit, which the pupil will do well to discover and note.

III.

The same example worked out without reference to ratio.

$$\begin{array}{lrcl} \text{Cost of } \overset{\text{cwt.}}{13}\cdot \overset{\text{qr.}}{3}\cdot \overset{\text{lb.}}{14}\cdot \text{ or } \overset{\text{stone.}}{111}\cdot & = & & \overset{\text{£.}}{5}\cdot\ \overset{\text{s.}}{4}\cdot\ \overset{\text{d.}}{0\tfrac{3}{4}} \\ \text{Cost of } \qquad 14\cdot \text{ or } \quad 1\cdot & = & \tfrac{1}{111} \text{ of } & 5\cdot\ 4\cdot\ 0\tfrac{3}{4} \\ \text{Cost of } 12\cdot 0\cdot 15\tfrac{3}{4} \text{ or } 97\tfrac{1}{8} & = & (\tfrac{1}{111} \text{ of } & 5\cdot\ 4\cdot\ 0\tfrac{3}{4}) \times 97\tfrac{1}{8} \\ & = & \tfrac{7}{8} \text{ of } & 5\cdot\ 4\cdot\ 0\tfrac{3}{4} \\ & = & & 4\cdot\ 11\cdot\ 0\tfrac{1}{2}\ \tfrac{5}{8}\text{q}. \end{array}$$

IV.

Or thus: by Rule of Three in Vulgar Fractions.

$$\begin{array}{llcccl} \text{As} & \overset{\text{cwt.}}{13}\cdot \overset{\text{qr.}}{3}\cdot \overset{\text{lb.}}{14}\cdot & : & \overset{\text{cwt.}}{12}\cdot \overset{\text{qr.}}{0}\cdot \overset{\text{lb.}}{15\tfrac{3}{4}} & :: & \overset{\text{£.}}{5}\cdot \overset{\text{s.}}{4}\cdot \overset{\text{d.}}{0\tfrac{3}{4}} \ : \ x. \\ \text{Or, As} & \overset{\text{cwt.}}{13\tfrac{7}{8}} & : & \overset{\text{cwt.}}{12\tfrac{9}{64}} & :: & \overset{\text{s.}}{104\tfrac{1}{16}} \ : \ x. \\ \text{Or, As} & \tfrac{111}{8} & : & \tfrac{777}{64} & :: & \overset{\text{s.}}{\tfrac{1665}{16}} \ : \ x. \\ \text{Or, As} & \tfrac{1}{8} & : & \tfrac{7}{64} & :: & \overset{\text{£.}}{\tfrac{333}{64}} \ : \ x. \end{array}$$

$$\begin{aligned} \therefore x &= \overset{\text{£.}}{\tfrac{333}{64}} \times \tfrac{7}{64} \div \tfrac{1}{8} \\ &\doteq \overset{\text{£.}}{\tfrac{333}{64}} \times \tfrac{7}{64} \times \tfrac{8}{1} \ (\text{PRIN. XVII.}) \\ &= \tfrac{7}{8} \text{ of } \overset{\text{£.}}{\tfrac{333}{64}} \\ &= \overset{\text{£.}}{\tfrac{2331}{512}} \ (\text{PRIN. IX.}) \\ &= \overset{\text{£.}}{4}\cdot\ \overset{\text{s.}}{11}\text{s. } \overset{\text{d.}}{0\tfrac{1}{2}}\ \tfrac{5}{8}\text{q}. \end{aligned}$$

Or thus: By Rule of Three in Decimals.

$$\text{As } \overset{\text{cwt.}}{13\cdot}\ \overset{\text{qr.}}{3\cdot}\ \overset{\text{lb.}}{14\cdot}\ :\ \overset{\text{cwt.}}{12\cdot}\ \overset{\text{qr.}}{0\cdot}\ \overset{\text{lb.}}{15\tfrac{3}{4}}\ ::\ \overset{£.}{5\cdot}\ \overset{\text{s.}}{4\cdot}\ \overset{\text{d.}}{0\tfrac{3}{4}}\ :\ x.$$

$$\text{Or, As } \overset{\text{cwt.}}{13{\cdot}875} : \overset{\text{cwt.}}{12{\cdot}140625} :: \overset{£.}{5{\cdot}203125}(\text{c. of } \overset{\text{cwt.}}{13{\cdot}875}) : x.$$

$$\text{Or, As } 8\cdot\ :\ 7\cdot$$

$$\begin{array}{rl}
 & 7\cdot \\ \hline
8\cdot) & 36{\cdot}421875 \text{ c. of } \overset{\text{cwt.}}{97{\cdot}125} \\ \hline
£ & 4{\cdot}552734375 \text{ c. of } 12{\cdot}140625 \\ \hline
 & 20\cdot \\ \hline
\text{s.} & 11{\cdot}0546875 \\ \hline
 & 12\cdot \\ \hline
\text{d.} & 0{\cdot}65625 \\ \hline
 & 4\cdot \\ \hline
\text{q.} & 2{\cdot}625 = \tfrac{1}{2}\text{d.} + \tfrac{5}{8}\text{q.}
\end{array}$$

$$\therefore\ x = £4\cdot\ 11{\cdot}\text{s. } 0\tfrac{1}{2}\text{d. } \tfrac{5}{8}\text{q.}$$

Although not in this one, Decimals, in many examples, shorten the work. The quantities being first expressed as Vulgar Fractions, attention to Exercise LXXI. will render it easy to decide when the employment of Decimals will be possible or advantageous.

EXERCISE CXVII.

EXAMPLES IN THE RULE OF THREE, OR SIMPLE PROPORTION, DIRECT AND INVERSE; IN INTEGERS, IN VULGAR FRACTIONS, AND IN DECIMALS.

Work every example in two ways :

I. *By Simple Proportion, or Rule of Three.*
II. *By Multiplication and Division, without reference to Ratio.*

Explain every line. Reduce as little as possible. Cancel as much as possible.

A.

THE RULE OF THREE IN INTEGERS. DIRECT PROPORTIONS.

1· If 3· articles cost 3·s. 6¾d., what will 11· cost, at the same rate?
2· When 9· cost £1· 11·s. 10½d. what will 7· cost?
3· What is the value of 10·, of which 6· cost £2· 10·s. 5½d.?
4· Find weight of 5· when 6· weigh 89·lb. 11·oz. 5·dwt. 6·gr.?
5· The weight of 2· being 18·lb. 7·℥ 5·ʒ 2·℈ 10·gr. find weight of 9·?
6· If 4· men build 91·yd. 2·ft. 2·in. of hedging, what length will 7· build?
7· Supposing 82·yd. 1·qr. 3·na. 1·in. of cloth, to make 5· dresses, how much will make 7·?
8· If 3· men can mow 42·ac. 2·r. 36·po. 28·yd., what area will 11· men mow?

9· Five bins containing 28·lo. 4·qr. 1·bu. 3·pe., how much will 3· such bins hold?

10· When 51· cost £2· 16·s. 3¾d., what is the value of 68· ?

11· What is the value of 144·, when 57· cost £3· 17·s. 2¼d. ?

12· At what rate per quarter do 91·lb. cost £150· 18·s. 2·d. ?

13· How much will 117· cost, at £14· 6·s. for 52· ?

14· When £7· 3·s. 2¾d. will pay for 55· what is the value of 209· ?

15· If 11· packages weigh 161·T. 4·cwt. 2·qr. 4·lb. 13·oz. 13·dr., what do 8· such packages weigh?

16· The weight of 6· being 87·T. 18·cwt. 3·qr. 10·lb. 4·oz. 10·dr., find weight of 7·

17· If 3· weigh 43·T. 19·cwt. 1·qr. 19·lb. 2·oz. 5·dr., what will 8· weigh?

18· If 4· pieces measure collectively 35·EE. 2·na. 1·in., what would the united length of 9· such pieces be?

19· If the joint length of 5· pieces be 82·yd. 1·qr. 3·na. 1·in., what will 11· pieces measure?

20· If in 4· hours a steam-ship travel 70·mi. 1·fur. 22·po. 4·yd., how far should she go in 10· hours?

21· When seven equal lengths amount to 160·yd. 1·ft. 6·in. 6·li., what do 5· such lengths amount to?

22· Three ingots weighing in all 44·lb. 11·oz. 12·dwt. 15·grs., find the weight of eight?

23· If 11· equal bars weigh 164·lb. 10·oz. 12·dwt. 23·grs., what will seven weigh?

24· Supposing 2· packages to contain, altogether, 18·lb. 7·℥ 5·ʒ 2·℈ 10·gr., ascertain the weight of half a score such packets.

25· Four equal areas comprise 139·su.yd. 5·ft. 10·′ 10·″, what extent would 7· such spaces fill?

26· To make eight equal allotments requires 113·ac. 3·r. 31·po. 24¼yd. How much land will 5· of them cover?

27· A cellar consists of two compartments, which together will contain 11·lo. 2·qr. 5·bu. 2·pe. It is proposed to enlarge the cellar by the addition of other three such compartments. What would it then contain?

28· Seven equal masses of stone contain 244·cu.yd. 3·ft. 6·′ 11½″. Required the solidity of nine such blocks.

29· A body, moving with uniform speed, traverses, in 3· hours, a distance of 52·mi. 5·fur. 7·po. ¼yd., what distance would it travel in eight hours at the same rate?

30· If 4· hours be requisite for travelling 70·mi. 1·fur. 22·po. 4·yd., what distance in 3· hours' journey?

31· Nine pieces of tape measuring, together, 148·yd. 1·qr. 3·na., what should seven such pieces measure?

32· If threescore sheep can be grazed in a field whose area is 42·ac. 2·r. 36·po. 28·yd., what extent of pasture of like quality will suffice for 140· sheep?

33· Fifty tons of hay being produced by a field measuring 71·ac. 34·po. 26½yd., what extent of ground of similar fertility will yield ninety tons?

34· If 200· tiles cover a space of 69·su.yd. 7·ft. 5·′ 5·″, what area will 500· tiles cover?

hen 385· articles cost £637· 5·s. 1¼d., what do 1463· cost?

d value of 1287· at £15· 9·s. 10·d. for 572·

37· If 465· weigh 7·cwt. 2·qr. 2·lb. 13·oz. what will 1881· weigh?

38· When 9·cwt. 3·qr. 19·lb. 8·oz. is the weight of 468· equal packages, what is the weight of 1053· of them?

39· Ascertain the weight of 8281· articles, of which 3248· weigh 259·T. 11·cwt.

40· What will the joint length of 90· pieces be, when 618· measure 695·yd. 1·qr.?

41· If 5· weigh 73·T. 5·cwt. 2·qr. 22·lb. 9·oz. 3·dr., what will 4· weigh?

42· The united length of 591· pieces being 1625·yd· 1·qr., what will 101· of them measure?

43· Required the gross length of 98· equal pieces, 837· such measuring 5335·yd. 3·qr. 2·na.

44· A rectangle, 5· chains wide, has an area of 71·ac. 34·po. 26½yd. what is the extent of another rectangle, of the same length, but 9· chains wide?

45· A rectangular-sided cistern, 8· feet deep, will contain 46·lo. 6·bu., how much will fill it to the depth of 3· feet?

46· If the value of 8· articles be exactly 7·s. 10⅔d., what will 13· be worth?

A superfluous term is frequently given, as in each of the following six questions.

47· If 13·cwt. 2·qr. 13·lb. be carried 40·mi. 4·fur. for 18·s., how far would the same weight be carried for £2· 14·s.?

Here, the weight conveyed being the same for both distances, the exact amount of the weight needs not have been given, for the increase of distance is due only to the increased payment. Hence, the amount of weight may be disregarded in the computation, and the question considered as running thus:

"If a *certain weight* be carried 40·mi. 4·fur. for 18·s., how far would it be conveyed for £2· 14·s.?"

48· Supposing the carriage of 2·T. 15·cwt. 3·qr., for 348· mi., cost £3· 12·s. 6·d., what would the conveyance of the same weight 145· miles cost?

49· If 2·T. 17·cwt. 2·qr. 20·lb. can be conveyed 54· miles for 38·s. what weight will be carried the same distance for £5·?

50· If £350· gain £13· 2·s. 6·d. interest in 9· months, how much will £181· gain in the same time, at the same rate?

51· The interest on £175· for 18· months being £6· 11·s. 3·d., what amount of principal will gain £7· 10·s. in the same time?

52· If £1040· gain £52· in 12· months, in what time will the same principal gain £169· at the same rate?

Frequently, only a portion of the question is solved by Proportion.

53· A person who spends at the rate of £12· 8·s 6·d. in seven weeks, is able to lay by £24· 14·s. per annum. What is his yearly income?

Find, by Proportion, his annual *expenditure*, which + annual *saving* = *Annual Income.*

54· If £3· 18·s. will provide keep for a horse for 65· days, how much more will it cost to keep him a year?

55· Bought 400· qrs. of wheat for £996· 18·s. 1½d., but 35· qrs. being rendered useless by damp, for what sum must I sell 17· qrs. so as neither to lose nor gain on them?

EXERCISE CXVII. B.

THE RULE OF THREE IN INTEGERS. INVERSE PROPORTIONS.

1· If a certain quantity of food will suffice for 6· men, for 15· days, how long will it last 9· men at the same rate?

2· How many men can subsist for 7· days on the quantity of food which will serve 10· men for 168· days?

3· In what time will 11· horses consume as much corn as would last 29· weeks 6· days for 8· horses?

4· When the crew of a ship comprises only 8· persons, each may receive 4·lb. 8·oz. Avoir. of food daily. What would each receive if there were 9· of them?

5· Supposing, instead of 8·, there were 12· of them, by how much would the daily ration of each be diminished?

6· Nine loaves, at 1·s. 1¼d. each, cost as much as 6· others; what are these worth apiece?

7· How many eightpenny loaves are equivalent to 120· ninepenny ones?

8· Seven yards of Irish linen are exchanged for 9· yd. of Holland at 12¼d. What is the former worth per yard?

9· The contents of 5· packages, each 1·lb. 12·oz., being made up into 7· equal packets, what will each of these weigh?

10· What is the value of one article, when 7· such are equivalent to 11· at 5¼d. each?

11· Eighteen workmen having completed a certain job, are entitled to £9· 7·s. each. Now, suppose that 17· of them had done the task, what would each have earned?

12· A dealer employing, instead of true weights, a set which only weigh at the rate of 15·oz. to the pound, of how much does he defraud a customer to whom he sells the pretended weight of 1·qr. 4·lb. 12·oz.?

13· Supposing his weights to be at the rate of 13·oz. to the pound, what is the true weight of that which he pretends to be 5·cwt. 1·qr. 20·lb.?

14· If a piece of work could be perfected, in the required time, by 108· persons, working 17· hours per day, how many more must be employed so that they need work only 12· hours per day?

15· If 240· gain £28· in 18· months, what amount of principal will gain the same interest in 8· months?

16· If 3·qr. 20·lb. be carried 6· miles for a certain sum, what weight would be taken 13· miles for the same amount?

17· How far will 11· tons be carried for the sum which would pay for the conveyance of 14· tons, 187· miles?

18· If 2·cwt. 2·qr. 5·lb. be transported a certain distance for 13·d.

per cwt., what weight, carried the same distance, would cost the same sum at 15·d. per cwt.?

19· Fourteen tons being conveyed by water, the same distance, and for the same sum, as 5· tons by land, what is the rate per ton by water, the land carriage costing 4·s. 8·d. per ton?

20· If the shilling loaf weigh 8·lb. when wheat is selling at £3· 12·s. per quarter, what is the price of wheat when the loaf is enlarged to 9·lb. without increase of cost?

21· A sixpenny loaf weighing 3·lb. 6·oz. when wheat is at 11·s. per bushel, what should it weigh when wheat fetches only 9·s. per bushel?

22· In what time will 8· men build a wall which would occupy 9· men 72· days?

23· The consumption of 117· tons of coal per day being requisite to effect certain work by steam engines in 4· days, what quantity per day would suffice if 9· days were allowed for the work?

24· What is the fare per mile by coach, which will carry one 20· miles for the same sum as one might post 13· miles, at 15·d. per mile?

25· Sixty men being sufficient to excavate a trench in 8· days, how many could do it in 5· days?

26· To complete the walls of a certain edifice in 90· days, it is requisite that they should be built at the rate of 5·ft. in height every day; at what rate must they rise to be finished in 75· days?

27· In what time will £900· gain as much interest as £1000· gains in 117· days, rate, of course, constant?

28· What principal, at £5· per cent., per annum, will gain the same interest, in the same time as £450· 15·s. 10·d. will gain at £3· per cent., per annum?

29· Of two rectangles, equal in area, the first is 15·ft. 2·in. long, and 11·in. wide. The second is 14·in. wide. How long is it?

30· The rate of carriage by river being 4·d., and by rail 9·d. per mile, for a certain weight, how far can it be conveyed by water for the same sum as 52· miles by land?

31· From a garrison of 2850· soldiers, how many must be detached, so that the daily ration of those left behind may be increased from 13·oz. to 15·oz.?

32· How many persons may be furnished with 17·oz. of beef daily for 3· months, out of the same quantity which would supply 12·oz. per diem to 153· consumers for the same length of time?

33· Two dozen tenpenny loaves can be made out of a certain quantity of flour; how many, worth sixteenpence each, could be made from the same?

34· An army having provisions enough to supply each man with 19· oz. daily for 112· days, how long will the stock last if the ration be reduced to 16·oz.?

35· A ship starts with a stock of biscuit calculated to furnish her crew with 14·oz. each, per diem, for 200· days. But, on the eighteenth evening of the voyage, it is discovered that 39· days' allowance has been rendered useless by a leak.

What must the ration of each now be, that the stock may still last the full time?

36· A rectanglar-sided block of stone, 7·ft. wide, and 2·ft. 9·in. thick, is of equal solidity with another square prism of the same length as itself, but 11·ft. wide. How thick is the latter?

37· Two cisterns (hollow square prisms) will hold equal quantities of water. The first is 13·ft. wide, the second 15·ft wide, and their depths are equal. How long is the first, the length of the second being 20·ft. 7·in.?

38· A rectangular-sided tank, 8· ft. long, and 6· ft. deep, being quite full of water, is discharged into a square prismatic cistern of the same width and depth as the tank, but 9· ft. long. How deep will the water stand in the cistern?

39· How much must the tank be deepened to allow of its being shortened 1· ft. without diminishing its capacity?

40· How long must a rectangle 2· ft. wide be to have an Area of 1· square yard?

41· If £155· gain £5· 16·s. 9·d. in 9· months, what principal will gain the same interest in 15· months?

42· A certain estate being divided into 7· equal fields, each of them contains 42· ac. 2· r. 36· po. 28· yd. Suppose it re-arranged into 3· equal farms, what would each contain?

43· The specific gravity of mahogany being 1·065, and the sp. gr. of beech ·852, what thickness of the former wood will weigh as much as 15· in. thick of the latter, the other dimensions remaining constant?

44· Two prisms, the one of apple-tree wood, and the other of ash, are of similar form, and like dimensions, the thickness of each being 16¼ inches. Now, to make them of equal weight, one inch must be planed away from the thickness of the ash, of which the sp. gr. is ·845; what is the weight of a cubic foot of the apple-tree?

45· A certain rectangular-sided vessel weighs the same, whether containing asphaltum to the depth of 5· ft. 4¼ in. or sea-water to the depth of 8· ft. 7⅛ in. What is the sp. gr. of the asphaltum, the weight of a cubic foot of sea-water being 2· qr. 8· lb. 4· oz.?

The greater part of the questions in the foregoing section are given in EXERCISE CXVIII. A. *as Direct Proportions in Vulgar Fractions. See "Conversion of Inverse into Direct Proportion," page* 330.

FOURTH EXAMPLE WORKED OUT. If 19· men earn £6· 7·s. 5½d. in 3· days, how many men would earn £9· 1·s. 1½d. in the same time, at the same rate?

The time (3· days) is a superfluous term.

Abbreviated Question.

	men.	£.	s.	d.
If	19·	6·	7·	5½
	?	9·	1·	1½

I.

Statement and Work.

$$\text{As } \overset{\pounds.\ \text{s.}\ \text{d.}}{6\cdot\ 7\cdot\ 5\tfrac{1}{2}} : \overset{\pounds.\ \text{s.}\ \text{d.}}{9\cdot\ 1\cdot\ 1\tfrac{1}{2}} :: \overset{\text{men}}{19\cdot} \text{ (earning } \overset{\pounds.\ \text{s.}\ \text{d.}}{6\cdot\ 7\cdot\ 5\tfrac{1}{2}}) : x$$

$$\text{Or, As } \overset{\text{halfp.}}{3059\cdot} : \overset{\text{halfp.}}{4347\cdot} :,\ \overset{\text{men.}}{19\cdot} \text{ (earning } \overset{\text{halfp.}}{3059\cdot}) : x$$

$$\begin{array}{rrl}
 & 4347\cdot & \\ \cline{2-2}
 & 133\cdot & \text{earning} \quad 7\cdot \text{ times } \overset{\text{halfp.}}{3059\cdot} \\
 & 760\cdot & \text{earning} \quad 40\cdot \text{ times } 3059\cdot \\
 & 5700\cdot & \text{earning } 300\cdot \text{ times } 3059\cdot \\
 & 76000\cdot & \text{earning } \underline{4000\cdot} \text{ times } 3059\cdot \\ \cline{2-2}
3059\cdot) & 82593\cdot & \text{earning } 4347\cdot \text{ times } 3059\cdot\ {}^{*} \\ \cline{2-2}
 & 27\cdot & \text{earning } 4347\cdot \text{ hfp. or } \pounds 9\cdot\ 1\text{s}\cdot\ 1\tfrac{1}{2}\text{d.} \\ \cline{2-2} \cline{2-2}
\end{array}$$

* Or 3059 times 4347· halfpence.

II.

When the third term is much less than the second, it is more convenient to make the third the Multiplier. This demands a new form of explanation, which the student will find interesting to trace, as in the present example.

Statement and Work.

$$\text{As } \overset{\text{halfp.}}{3059\cdot} : \overset{\text{halfp.}}{4347\cdot} :: \overset{\text{men.}}{19\cdot} \text{ (ea. earning } \tfrac{1}{19} \text{ of } \overset{\text{halfp.}}{3059\cdot}) \text{ A. } : x.$$

$$\begin{array}{rrl}
 & 19\cdot & \\ \cline{2-2}
 & 39123\cdot & \doteq \text{ men, earning } 4347\cdot \text{ times } \tfrac{9}{19} \text{ of } 3059\cdot \text{ B.} \\
 & 43470\cdot & \doteq \text{ men, earning } 4347\cdot \text{ times } \tfrac{10}{19} \text{ of } 3059\cdot \text{ C.} \\ \cline{2-2}
3059\cdot) & 82593\cdot & \doteq \text{ men, earning } 4347\cdot \text{ times } 3059\cdot \text{ hp. D.} \\ \cline{2-2}
 & 27\cdot & \doteq \text{ men, earning } 4347\cdot \text{ hp. or } \pounds 9\cdot\ 1\cdot\text{s. } 1\tfrac{1}{2}\text{d.} \\ \cline{2-2} \cline{2-2}
\end{array}$$

A. because 19· men earn 3059· hfp. 1· man earns $\frac{1}{19}$ of that sum.
B. 9· men earn $\frac{9}{19}$ of 3059 hfp. C. 10· men earn $\frac{10}{19}$ of that sum.
And 9· times 4347· = 4347· times 19· &c. PRINCIPLE VI.
D. 4357· times 3059· halfp. = 3059· times 4347· halfp.

III.

Cancelling may be employed in this Example.

Statement and Work.

$$\text{As } \overset{\text{halfp.}}{3059\cdot} : \overset{\text{halfp.}}{4347\cdot} :: \overset{\text{men.}}{19\cdot} \text{ (earning } \overset{\text{halfp.}}{3059\cdot}) : x.$$

$$\text{Or, As } 161\cdot : 4347\cdot :: \overset{\text{man.}}{1\cdot} \text{ (earning } \tfrac{1}{19} \text{ of } \overset{\text{halfp.}}{3059\cdot}) : x.$$

$$\text{Or, As } 1\cdot : 27\cdot :: \overset{\text{man.}}{1\cdot} : \overset{\text{men.}}{27\cdot} \text{ (earning } \tfrac{27}{19} \text{ of } \overset{\text{halfp.}}{3059\cdot})$$

or 27· times 161· h.p. = 4347· h.p.

Here, we first divide first and third by 19·; and then divide the second and the new first by 161·

IV.

When the explanation is not required, the cancelling and result are shewn thus:

$$x = \frac{\cancel{19}\cdot m \times \overset{27\cdot}{\cancel{4347}}\cdot}{\underset{\cancel{161}\cdot}{\cancel{3059}\cdot}} = 27\cdot \text{ men.}$$

V.

The same example worked by two Divisions, without reference to ratio.

$$\begin{array}{r|rrl} & \text{£.} & \text{s.} & \text{d.} \\ 19\cdot) & 6\cdot & 7\cdot & 5\tfrac{1}{2} = \text{earnings of } 19\cdot \text{ men.} \\ \hline & & 6\cdot & 8\tfrac{1}{2} = \text{earnings of } 1\cdot \text{ man.} \end{array}$$

The No. of times, then, that 6·s. 8½d. can be paid out of £9· 1·s. 1½d. will concide with the No. of men who earn the latter sum.

$$\text{No. of men} \doteq \overset{\text{£.}}{9\cdot}\ \overset{\text{s.}}{1\cdot}\ \overset{\text{d.}}{1\tfrac{1}{2}} \div \overset{\text{s.}}{6\cdot}\ \overset{\text{d.}}{8\tfrac{1}{2}} = \overset{\text{men.}}{27\cdot} \text{ (Ex. LXXXIX.)}$$

EXERCISE CXVII. C.

RULE OF THREE IN INTEGERS. DIRECT PROPORTIONS.

Work each example in, at least, three ways:

1· *Making second term multiplier.*
2· *Making third term multiplier.*
3· *Without reference to ratio.*
} *Explain every line.*

Cancel as much as possible.

1· If 16· articles cost £218· 16·s., how many, at the same rate, may be bought for £95· 14·s. 6·d.?

2· What weight of beef may be purchased for 3·s. 6¾d., supposing 11·lb. to cost 13·s. 0¾d.?

3· When 7·cwt. cost £1·4·s. 9½d. what weight will cost £1·11·s. 10½d?

4· The labour of four men being requisite to build 91· yd. 2· ft. 2· in. of hedging, how many will complete 160· yd. 1· ft. 6½ in. in the same length of time, and working at tbe same rate?

5· The capacity of 5· bins sufficing exactly for the reception of 28·lo. 4·qr. 1·bu. 3·pe. how many such bins will hold 17· lo. 1·qr. 4·bu. 1·pe.?

6· Four slabs of equal surface will cover an area of 139· su. yd. 5· ft. 10·′ 10·″, how many such would be required to cover 244· su. yd. 3· ft. 6·′ 11½″?

7· The gross weight of 11·hhds. being 161·T. 4·cwt. 2·qr. 4·lb. 13·oz. 13·dr., how many hhds. will weigh 117·T· 5· cwt. 13·lb· 11·oz. 8·dr.?

8· Of two rectangles, the first is 5· chains wide, and has an area of 71·ac. 34·po. 26½yd., while the second, being of the same length, has an area of 156·ac. 2·r. 28·po. 22·yd.; how wide is it?

9· At 10·s. 2½d. per stone, how many lb. will cost 8·s. 0¼d.?

10· How many yds. for £1· 14·s. 0¾d., when 4· yds. cost 13·s. 7½d.?

11· What number will 5·s· 1¼d. pay for, at 8·s. 9·d. per dozen?

12· If 10· lengths are equal to 229·yd. 11·in., how many such lengths can be cut from 160·yd. 1·ft. 6½in.?

13· If £7· 15·s. be the value of 31·lb., how much is worth £176· 10·s.?

14· If 18·lb. cost 15·s. 4½d., what weight may be bought for £272· 1·s. 0½d.?

15· If 3·hhds. weigh 43 T. 19·cwt. 1·qr. 19·lb. 2·oz. 5·dr., how many will weigh 117·T. 5·cwt. 13·lb. 11·oz. 8·dr.?

16· Twenty-four equal allotments comprising, in all, 837·su.yd. 8·ft. 5·′, how many such can be made from 1047·su.yd. 3·ft 9·′ 3·″?

17· The united weight of 10· equal packages being 93·lb. 2·℥ 5·ʒ 10·gr., how many can be made from 83·lb. 10·℥. 6·ʒ. 15·gr.?

18· If 25· equal parcels weigh 59·lb. 11·oz. 10·dwt. 4·gr., how many such parcels will weigh 164·lb. 10·oz. 12·dwt. 23·gr.?

19· Of two rectangular-sided bins, the first is 4·ft. deep, and will contain 23·lo. 3·bu.; the second is of the same length and breadth as the first, but contains 20·lo. 7·bu. 2·pe. 1·gal. How deep is it?

20· The *ends* of two rectangular fields of equal width, are separated by a hedge 10·links thick, which it is proposed to remove. Now, the area of the larger field is 128·ac. 30·po. 23½yd., and that of the smaller one, 85·ac. 1·r. 33·po. 25¾yd. What will the length of the united fields be, the larger being 9· chains long?

21· At 13·s. 7·d for 6·oz. 15·dwts. 20·gr. what would 1·lb. 3·℥. 2·ʒ. 15·gr. cost?

EXERCISE CXVII. D.

RULE OF THREE IN INTEGERS. DIRECT PROPORTIONS.

Work variously, as before, and explain every line.

1· If 34·lb. 6·oz. Avoir. cost £1· 2·s. 11·d., what will 3·qr. 21·lb. 9·oz. cost?

2· When 5·sc. 8·lb. are worth £1· 7·s., what weight is worth £2· 9·s. 3·d.?

3· What weight may be bought for £13· 2·s. 11·d., when 7·sc. 6·lb. cost £3· 0·s. 10·d.?

4· If a pig, weighing 6·sc. 8·lb. be worth £4· 16·s., what is the weight of another, worth £8· 10·s. 3·d. at the same rate?

5· At £1· 16·s· for 5·sc. 8·lb., what is the value of 13·sc. 5·lb.?

6· Find the cost of 8·cwt. 3·qr. 16·lb., when 3·cwt. 1·qr. 11·lb. cost £3· 18·s. 1½d.?

7. If 5· English Ells cost £4· 11·s. 6·d., what will 207½yd. cost, at the same rate?

8. The rent of 41·ac. 1·r. 30·po. being £120· 18·s. 7¼d., what is the rent of 61·ac. 2·r., at the same rate?
9. Find rent of 65·ac. 3·r. 20·po., at £826· 10·s. 11¼d. for 314·ac. 3·r. 20·po.
10· 2·qr. 6·lb. costing £15· 10·s., what will 3·cwt. 17·lb. cost?
11· 7·cwt. 1·qr. 7·lb. of pork being worth £10· 12·s. 0¾d., how much will 5·s. 5¼d. buy?
12· Compute the cost of 17·sc. 13·lb. of pork, when 22·sc. 5·lb. cost £6· 19·s. 0¾d.
13· If a certain pipe discharge 92· gal. 1· qt. of water in 15· minutes 22½ seconds, how much will it discharge in 35· min. 15· sec.?
14· Supposing 15·lb. 6·oz. of wax to be worth 1·qr. 18·lb. 2·oz. of sugar, how much sugar would be of equal value with 1·qr. 7·lb. 14·oz. of wax?
15· There are two ricks of hay of the same mean breadths and heights. The first, being 6·ft. 8·in. long, is computed to contain 2· Tons. 15· cwt. The other is 11·ft. 9·in. long; what should its weight be, supposing its density to be equal to that of the first?
16· An oblong room, which is 14·ft. 9·in. wide, has an area of 390· su. ft. 10·′ 6·″; what will be its area, if it be widened by 21· inches?
17· Assuming that the shadows cast by any two vertical objects, at the same moment, are in direct proportion to their heights, what is the altitude of a tower which projects a shadow 100· ft. 4·in. long, at the same instant that a vertical rod 6·ft. 3· in. high casts a shadow 7·ft. 2·in. long?
18· A's income being £550· 10·s., and the income-tax upon it £20· 12·s. 10½d., what is B's income, on which at the same rate, the tax is £14· 5·s. 6¾d.?
19· If £3· 10·s. 4½d. be the value of 3·qr. 21·lb., what weight is worth £1· 3·s. 5½d.?
20· If 14·sc. 9½lb. cost £7· 16·s. 9¾d., what weight will cost £5· 5·s. 1·d.?
21· A square meadow contains 4·ac. 3·r. 15·po., and its length is 30·po. 5·yd. By what addition to its length will its area be increased to 7·ac. 3·r.?
22· How many boxes of soap, each 1·cwt. 2·qr., can I buy for £73· 10·s., at the rate of £4· 9·s. 10·d. for 2·cwt. 3·qr.?
23· If 7·s. 6·d. be the price of 10·cwt. 2·qr. of coal, what quantity can be bought for £25·?
24· Find how much coffee may be bought for £34· 6·s 8·d., when 5·lb. 14·oz. cost 9·s. 9½d.
25· How many stone of 14·lb. will cost £4· 13·s. 9·d., if 9·sc. 10·lb. cost £4· 18·s. 11½d.?
26· A rectangular parterre is 5·yd. 1·ft. wide, and contains 47·su. yd. 1·ft., what will its area be if its width be reduced to 4·yd. 2·ft. 8·in.?
27· The cost of 1·cwt. 2·qr. 26·lb. being £5· 5·s. 1·d., what will 2· cwt. 2·qr. 9·lb. 8·oz. cost?

RULE OF THREE IN VULGAR FRACTIONS.

The methods exemplified in the preceding examples is equally applicable to the solution of questions in which one or more of the terms is a Vulgar Fraction, or Mixed Number.

FIFTH EXAMPLE WORKED OUT. RULE OF THREE IN VULGAR FRACTIONS. If 11·cwt. 21·lb. cost £1· 6·s. $1\frac{1}{4}$d., how much would 17·cwt. 1·qr. 4·lb. cost?

Abbreviated Question.

$$\text{If } \overset{\text{cwt.}}{11\tfrac{3}{16}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \overset{\text{s.}}{26\tfrac{5}{48}}$$
$$17\tfrac{2}{7} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots ?$$

I.

Statement and Work.

$$\text{As} \quad \overset{\text{cwt.}}{11\tfrac{3}{16}} \quad : \quad \overset{\text{cwt.}}{17\tfrac{2}{7}} \quad :: \quad \overset{\text{s.}}{26\tfrac{5}{48}} \quad : \quad x$$

$$\text{Or as} \quad \overset{\text{cwt.}}{\frac{179}{16}} \quad : \quad \overset{\text{cwt.}}{\frac{121}{7}} \quad :: \quad \overset{\text{s.}}{\frac{1253}{48}} \quad : \quad x$$

$$\therefore x = \frac{1253}{48}\text{ s.} \times \frac{121}{7} \div \frac{179}{16} \doteq \frac{\cancel{1253}}{\underset{3}{\cancel{48}}}\text{ s.} \times \frac{121}{\cancel{7}} \times \frac{\cancel{17}}{\cancel{179}}$$

$$= \frac{121}{3}\text{ s.} = \overset{\text{s.}}{40\tfrac{1}{3}} = £2\cdot\ 0\cdot\text{s. } 4\cdot\text{d.}$$

II.

The explanation of the work of Rule of Three in Vulgar Fractions cannot conveniently be given with each step of the process. The result in the foregoing example is traced precisely as in Rule of Three in Integers, thus:

$$\overset{\text{s.}}{\frac{1253}{48}} = \text{cost of } \overset{\text{cwt.}}{\frac{179}{16}}$$

$$\overset{\text{s.}}{\frac{1253}{48}} \times \frac{121}{7} = \text{cost of } \frac{121}{7} \text{ of } \frac{179}{16} \text{ cwt.} \quad \text{PRIN. IX.}$$

$$\overset{\text{s.}}{\frac{1253}{48}} \times \frac{121}{7} \div \frac{179}{16} \doteq \overset{\text{s.}}{\frac{1253}{48}} \times \frac{121}{7} \times \frac{16}{179}$$

$$= \text{cost of } \frac{16}{179} \text{ of } \frac{121}{7} \text{ of } \frac{179}{16} \text{ cwt.}$$

$$= \text{cost of } \frac{121}{7} \text{ cwt. or } 17\tfrac{2}{7} \text{ cwt.}$$

III.

The same example worked out without reference to ratio.

$$\text{Cost of } \frac{179}{16} \text{ cwt.} = \frac{1253}{48} \text{ s.}$$

$$\text{Cost of } 1\cdot \text{ cwt.} = \frac{1253}{48} \text{ s.} \div \frac{179}{16} \doteq \frac{1253}{48} \text{ s.} \times \frac{16}{179}$$

$$\text{Cost of } \frac{121}{7} \text{ cwt.} = \frac{1253}{48} \text{ s.} \times \frac{16}{179} \times \frac{121}{7}$$

$$= \frac{121}{3} \text{ s.} = 40\tfrac{1}{3}\text{s.} = £2\cdot\ 0\cdot\text{s. } 4\cdot\text{d.}$$

This example could not be worked out by Decimals. Why?

EXERCISE CXVII. E.

RULE OF THREE IN VULGAR FRACTIONS. DIRECT PROPORTIONS.

1· At £21· for $\frac{1}{2}$ a Ton, what will $1\frac{1}{14}$cwt. cost?

2· If $3\frac{1}{4}$cwt. cost £$5\frac{11}{16}$, what weight is worth £$1\frac{7}{8}$?

3· What will $7\frac{6}{16}$cwt. cost, at $5\frac{7}{16}$s. for $\frac{3}{16}$cwt?

4· If $2\frac{1}{3}$ miles be travelled in $\frac{13}{18}$ of an hour, what time will suffice to pass over a distance of $4\frac{4}{13}$ miles?

5· By saving 4·s. in the pound of my income I lay by £20· 2·s. 6·d. a year; how much should I save if I spent only 10·d. in every shilling?

6· Three shillings and sixpence in the pound of a person's income amounting to £3· 7·s. 1·d., what would 5·d. in the shilling amount to?

7· What is his income on the above suppositions?

8· By gaining $2\frac{1}{2}$d. *in the shilling* of a certain selling-price, I bag £4· 8·s. $2\frac{3}{4}$d., what would my gain be at $2\frac{1}{2}$d. *on the shilling* of the cost price?

9· What is the amount of outlay?

10· A's and B's steps in walking are of equal lengths, but A takes 5· to B's 3· and thus gains 140·yd. 2·ft. 8·in. in a certain time. How much would he gain in the same time, if he took 8· steps to B's 5·?

11· How far does each walk in the first instance?

12· How far would each walk in the second instance?

13· If adding 3· gallons of water to 21· of brandy will make a mixture worth $28\frac{7}{8}$s. per gal., what will a compound be worth per gal. of which 5· parts are water, and 28· parts brandy?

14· In making up crayons, of whiting and plaster of Paris, the directions were misunderstood, and 2· parts of plaster were put *in every* 5· of the mixture instead of *to every* 5· of whiting. How much whiting must be added to 3·lb. of the mixture to restore the due proportions of the ingredients?

To every pound of plaster there is $1\frac{1}{2}$lb. of whiting, instead of $2\frac{1}{2}$lb.

Quantity of whiting in 3·lb. of crayons =

$$\text{As } 1\tfrac{1}{2} : 1\cdot :: \overset{\text{lb.}}{\tfrac{9}{5}} : x$$

15· If $1\frac{3}{14}$st. cost $7\frac{1}{12}$s. how much may be bought for £$7\frac{1}{3}$?
16· If $1\frac{2}{5}$ bushels weigh $89\frac{5}{8}$lb., what will $\frac{3}{4}$ of a peck weigh?
17· If $93\frac{5}{8}$cwt. be worth £$367\frac{153}{320}$, what weight will be worth £$86\frac{7}{20}$?
18· If $\frac{21}{40}$ of a quarter of corn weigh $3\frac{199}{448}$ qr. of a cwt., what will $7\frac{7}{8}$ quarters weigh?
19· At 9·s. $4\frac{1}{2}$d. $\frac{15}{16}$q. for $7\frac{159}{256}$lb. avoir., how much will £$15\frac{1}{32}$ pay for?
20· What weight is worth £$20\frac{11}{16}$, at $13\frac{19}{24}$s. for $22\frac{27}{32}$lb.?
21· If $\frac{16}{25}$ of $\frac{5}{8}$ of $1\frac{4}{5}$ cost $\frac{3}{4}$ of 9· times $\frac{7}{25}$ of £1·, how much will $67\frac{7}{8}$ cost?
22· If $4\frac{1}{5}$ acres will pasture a flock for 70· days, for how long will 21·9 acres serve the same flock?
23· What weight will cost £$37\frac{1}{3}$, at £$26\frac{19}{80}$ for $14\frac{11}{112}$ cwt.?
24· If a woman earn 13·s. 6·d. per week by closing ladies' boots, at the rate of 9· pairs in 4· days, what would she earn per week, if she could close 7· pairs in every 2· days?
25· If pipe A discharge 16· gal. of water in 5· seconds, and pipe B 18· gal. in 7·sec., how much will B discharge in the same time that A discharges $62\frac{2}{9}$ gallons?
26· A certain weight, lightened by long use, is discovered, at last, to make 33·lb. 15·dr. appear to be 34·lb. 2·oz. What fraction of the said weight is worn away?

The Examples in Exercise CXVII. D. may now also be worked by Vulgar Fractions.

EXERCISE CXVII. F.

EXAMPLES IN THE RULE OF THREE IN DECIMALS.

DIRECT PROPORTIONS.

1· If 5·25 cwt. cost £39·375, what will 23·75 cwt. cost?
2· When 72· articles cost £34· 8·fl. 7·c. 5·m., how much will 62·8 cost?
3· How many will £30· 4·f. 1·c. $8\frac{3}{4}$m. pay for, at £6·975 for 14·4?
4· Supposing the value of 314· articles to be £192· 4·fl. 8·c. 2·m., how many will cost £5· 5·fl. 1·c. 7·m.?
5· If 122·35 art. cost £15·29375, how many will cost £3·4875?
6· If 1· Ton, 4· cwt. cost £21· 3·fl., what will 4· Tons, 14· cwt. 3· qrs. cost in £. s. d.
7· If 12· cwt. 3· qrs. 18·lb. 6·oz. cost £3· 3·fl. 6·m., what weight will cost half a guinea?
8· If 19·75 cwt. cost £30· 12·s. 3·d., how much will ·875 of a ton cost?
9· If 2·8125 ac. produce 20·7421875 qrs., what quantity would 5·625 ac. produce at the same rate?
10· If 2· ac. 3·r. 6·4po. produce hay worth £3· 9·s. 9·d., what value should 12· ac. 37·6 po. produce?
11· At the rate of £30· 8·s. $4\frac{1}{2}$d. for 31·4 articles, what will 36· cost in Decimal Coinage?
12· If 36·8 gal. of spirit be contained in 42· gal. $1\frac{1}{2}$ pt. of a mixture, how much spirit is there in $616\frac{127}{128}$ gal. of a similar compound?
13· If 19· ac. 2·r. be worth £23·75 per annum, what will 678·6 ac. be worth?

14· Supposing 33·75 qrs. of wheat to shrink, under certain conditions, to 31·25 qrs., what value is lost on that which shrinks to 681·0546875 qrs. under similar circumstances, reckoning the grain at 42/6 per quarter?

INVERSE (OR CONTRARY) RATIO AND PROPORTION.

The Inverse or Contrary Ratio of any two quantities is the Ratio of their Reciprocals. Page 145·

To prove this, take any two Nos., say, 7· and 8·
Their Reciprocals are $\frac{1}{7}$ and $\frac{1}{8}$.
Now, Inverse Ratio of 8· to 7· is Ratio of 7· to 8·, the Nos. being written in *Inverse* or *Contrary* order.
$\therefore$ 7· is to 8· *inversely* as 8· is to 7.
But 8· is to 7· as $\frac{8}{56}$ is to $\frac{7}{56}$, or, as $\frac{1}{7}$ is to $\frac{1}{8}$.
$\therefore$ 7· is to 8· *inversely* as $\frac{1}{7}$ is to $\frac{1}{8}$.

That is, Inverse Ratio of 7· to 8· = Direct Ratio of $\frac{1}{7}$ to $\frac{1}{8}$. And to make of these two Ratios a Direct Proportion, we must invert the order, either of the two Numbers, or of their two Reciprocals.

$$\therefore\ 8\cdot : 7\cdot :: \frac{1}{7} : \frac{1}{8}$$
$$\text{or}\ 7\cdot : 8\cdot :: \frac{1}{8} : \frac{1}{7}$$

In both which statements the Numbers appear in *Inverse* or *Contrary* order to their Reciprocals.

2nd. No. : *1st. No.* :: *Recip. of 1st.* : *Recip. of 2nd.*
1st. No. : *2nd. No.* :: *Recip. of 2nd.* : *Recip. of 1st.*

CONVERSION OF INVERSE INTO DIRECT PROPORTION.

Every question in Inverse Proportion may be transformed into a Direct Proportion, by employing, instead of the first two terms, their Reciprocals.

Then, Reciprocal of first term of original proportion becomes the second term of the new proportion, and Reciprocal of second term becomes the first term.

This conversion of Ratios demands, of course, a considerable change in the form of the question, which change the following examples will illustrate.

I. A QUESTION AS AN INVERSE PROPORTION IN INTEGERS. If 9· men complete a work in 144· days, in what time will 16· men do it?

Abbreviated Question.

	men.		days.
If	9·	....	144·
	16·	....	?

Statement and Result.

$$\text{As } \overset{\text{men.}}{16\cdot} \;:\; \overset{\text{men.}}{9\cdot} \;::\; \overset{\text{days.}}{144\cdot} \text{ (for } \overset{\text{m.}}{9\cdot}\text{)} \;:\; \overset{\text{days.}}{81\cdot} \text{ (for } \overset{\text{m.}}{16\cdot}\text{)}$$

The two Nos. of days are Inversely Proportional to the two Nos. of men.

II. The same Question as a Direct Proportion in Fractions. If a work be completed in 144· days, when $\frac{1}{9}$ of it falls to the lot of each man employed, in what time will it be done, if the No. of men be so increased, that each will have to do only $\frac{1}{16}$ of the task?

Abbreviated Question.

$$\begin{array}{lcl} & \text{for ea.} & & \text{days.} \\ \text{If} & \frac{1}{9} & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & 144\cdot \\ & \frac{1}{16} & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & ? \end{array}$$

Now, as the men are supposed to work at the same speed, whether each does $\frac{1}{9}$ or $\frac{1}{16}$, it is plain, that *the smaller the share* each has to do, *the shorter the time* he and the others will be required to work. The time to complete the work, in the second instance, will be greater or less than 144· days, exactly as the second share of labour falling to each man is greater or less than the portion due to him in the first case.

Statement and Result.

As $\frac{1}{9}$: $\frac{1}{16}$ :: 144· (when ea. does $\frac{1}{9}$) : 81· (when ea. does $\frac{1}{16}$)

Once more. Comparing the two forms of the question, in order to discover how the Direct Proportion arises from the Inverse Proportion, we note that

I. The Times are *Inversely* as the Numbers of men.
The *more* men, the *shorter* time.
The *fewer* men, the *longer* time.

II. But, the Times are *Directly* as the shares of work.
The *larger* a man's share, the *longer* time.
The *smaller* his share, the *shorter* time.

And these two Proportions are linked by the fact, that

III. The Shares are *Inversely* as the Nos. of men.
The *more* men, the *smaller* share, and the *shorter* time.
The *fewer* men, the *larger* share, and the *longer* time.

Where we see both Proportions at once, and their connection. There are, in fact, three Proportions included under III., for it is there shewn that

I. If we take *men* and *shares* together, we have an *Inverse Proportion.*

II. If we take *men* and *times* together, we have an *Inverse Proportion.*

III. If we take *times* and *shares* together, we have a *Direct Proportion.*

RE-CONVERSION OF DIRECT INTO INVERSE PROPORTION.

If, in the Direct Proportion of Fractions, the first two terms be reduced to a Common Denominator, and this denominator cancelled, the statement will return to that of the Inverse Proportion of Integers. Thus:

As $\frac{1}{9}$: $\frac{1}{16}$ :: 144· (each man doing $\frac{1}{9}$) : x.

Or, reducing first two terms to a Com. Denr.

As $\frac{16}{144}$: $\frac{9}{144}$ :: 144· (each man doing $\frac{16}{144}$) : x.

Then, dividing first and second terms by $\frac{1}{144}$.

As 16· : 9· :: 144· (each man doing $\frac{16}{144}$) : x.

$$\begin{array}{r l} & 144\cdot \\ & 9\cdot \\ \hline 16\cdot) & 1296\cdot \text{ a man doing } \tfrac{144}{144} \text{ or all the work.} \\ \hline & 81\cdot \text{ each man doing } \tfrac{9}{144} \text{ or } \tfrac{1}{16} \text{ of it.} \end{array}$$

This last statement coincides with that of the Inverse Proportion in Integers, though the 16· and the 9· are not here *days*, as they are *there*, but 144ths. of the work.

This is their meaning in the statement. As soon, however, as the 9· is employed as a Multiplier, either here or in the former process, it ceases to represent any concrete quantity. See Pages 170· and 149·

EXERCISE CXVIII. A.

CONVERSION OF INVERSE PROPORTION OF INTEGERS INTO DIRECT PROPORTION OF FRACTIONS, AND THE CONVERSE.

Direct Proportions in Fractions.	*The same Questions as Inverse Proportions, from* CXVII. B. *p.* 320·
1· A certain quantity of food will last 15· days, when each person fed receives $\frac{1}{6}$ of it. How long would it last, if the number of eaters were so increased, that each would receive only $\frac{1}{9}$ of it, at the same daily ration as before?	1· If a certain quantity of food will suffice for 6· men for 15· days, how long will it last 9· men at the same rate?
2· If ten men consume per day $\frac{1}{168}$ of a certain quantity of food, how many would consume $\frac{1}{7}$ of it per day?	2· How many men can subsist for 7· days, on the same quantity of food which will serve 10· men for 168· days?
3· A stock of corn will last 29· weeks 6· days, when each horse fed is allowed $\frac{1}{8}$ of it. How long would it last if each horse got $\frac{1}{11}$, at the same daily rate?	3· In what time will 11· horses consume as much corn as would serve 29· weeks 6· days, for 8· horses?

4· One eighth of a ship's crew's daily allowance being 4·lb. 8·oz. avoir., what is $\frac{1}{9}$ of it?	4· When the crew of a ship comprises only 8· persons, each may receive 4·lb. 8·oz. avoir. of food daily. What might each receive if there were 9· of them?

Previously to copying any one of the remaining questions in this Exercise, write out the question having the same number in Exercise cxvii. b. *p.* 320· *Work that question by Inverse Proportion. Under that work, write the question as here given, and work it by Direct Proportion of Fractions. This method will exhibit the contrast between the two forms of the same question, and between the two modes of solving it.*

5· Supposing, (see last example,) instead of $\frac{1}{9}$, each person were entitled to only $\frac{1}{12}$, by how much would his daily ration then be diminished?
6· One ninth of a certain quantity of bread is worth $13\frac{1}{2}$d., what is $\frac{1}{8}$ of it worth?
7· If $\frac{1}{8}$ of a certain loaf be equivalent to $\frac{1}{9}$ of another, how many of the former are worth 120· of the latter?
8· One seventh of a yard of Irish linen being of the same value as $\frac{1}{9}$ of a yard of Holland, what is the Irish worth per yard, the Holland costing $12\frac{1}{4}$d. per yard?
9· What is the weight of $\frac{1}{7}$ of a quantity of goods, $\frac{1}{8}$ of which weighs 1·lb. 12·oz.?
10· What is the value of $\frac{1}{7}$ of a lot of goods, if $\frac{1}{11}$ of it be worth $5\frac{1}{4}$d.?
11· If, by doing $\frac{1}{18}$ of a piece of work, a man earns £9· 7·s., what would he earn by doing $\frac{1}{17}$ of it?
12· A dealer, each of whose weights is only $\frac{1}{16}$ of what it should be, sells what he calls 1·qr. 4·lb. 12·oz.; of how much weight does he thus defraud his customer?
13· Supposing his weights to be only $\frac{15}{16}$ of what they should be, what is the true weight of that which he pretends to be $5\frac{3}{7}$ cwt.?
14· How many men must come to the assistance of 108·, so that $\frac{1}{12}$ of a certain work may be done in the same time as $\frac{1}{17}$ of it would have occupied?
15· What principal will gain the same interest, in the same time, at the rate of £$3\frac{1}{2}$ per month, as £240· will gain at the rate of £$1\frac{5}{8}$ per month?
16· One sixth of the distance that $3\frac{5}{7}$qr. are carried, is equal to $\frac{1}{13}$ of the distance that a smaller weight can be carried for the same sum. What is that weight?
17· One fourteenth of the weight carried 187· miles, is equal to $\frac{1}{11}$ of that conveyed a greater distance for the same money. Required this distance?
18· Supposing that $\frac{1}{15}$ of a cwt. were to cost as much in carriage as $\frac{1}{13}$ now does, what weight would be conveyed for the sum paid, at present rate, for 2·cwt. 2·qr. 5·lb., distance constant?
19· One fourteenth of the weight conveyed by water, is equal to

$\frac{1}{8}$ of that carried the same distance, for the same money, by land. What is the rate by water, the land carriage being £$\frac{7}{30}$ per ton?

20· What is the price of wheat, if $\frac{1}{9}$ of a certain-priced loaf is as heavy as $\frac{1}{8}$ of a loaf of that price was, when wheat stood at £3·6 per quarter?

21· If the price of wheat fall, so that $\frac{1}{8}$ of a bushel costs only what $\frac{1}{11}$ did, when the sixpenny loaf weighed 3·lb. 6·oz., what should the weight of the loaf be made?

22· A wall can be completed in 72· days, when each man employed builds $\frac{1}{9}$ of it; how long would it take if each such man were required to do only one-eighth of it?

23· To effect a certain work, by steam-engines, at the rate of $\frac{1}{4}$ per day, requires the consumption of 117· tons of coal daily. How much would suffice for one day's fires, supposing only $\frac{1}{9}$ of the work to be completed in each day?

24· The fare by post-chaise being 15· pence per mile, what is the rate by mail, which would carry one as far for $\frac{1}{20}$ of any sum, as one could post for $\frac{1}{13}$ of the same amount?

25· If sixty men can excavate $\frac{1}{8}$ of a trench in a day, how many would dig $\frac{1}{3}$ of it in that time?

26· To complete $\frac{1}{90}$ of a building per day, it is requisite that the walls should rise at the rate of 5·ft. in a given time. At what rate must they be elevated so that $\frac{1}{75}$ of the work may be done per day?

27· When every £1· of principal gains $\frac{1}{1000}$ of a certain amount of interest, the whole is gained in 117· days. In what time would it be gained at the rate of $\frac{1}{900}$ for every £1· of principal?

28· What principal will produce the same interest in any given time as £450· 15·s. 10·d., supposing £$\frac{1}{5}$ to gain as much as £$\frac{1}{3}$ gained in the former case?

29· Of two rectangles, equal in area, $\frac{1}{11}$ of the first is of the same width as $\frac{1}{14}$ of the second, both fractions being rectangular, longitudinal slices, (page 289·), what is the length of the second, the first being 15·ft. 2·in. long?

30· The conveyance of any given weight by river, costing only as much per $\frac{1}{4}$ of a mile as by rail for $\frac{1}{9}$ of a mile, how far could it be carried by water for the same expense as 52· miles by land?

31· From a garrison of 2850· soldiers, how many must be detached, so that those left may receive as much food for $\frac{1}{15}$ of a day, as they at first received for $\frac{1}{13}$ of that time?

32· How many men will consume in $\frac{1}{17}$ of a given time, as much beef as 153· men will consume in $\frac{1}{12}$ of that time?

33· If $\frac{1}{10}$ of a certain loaf be equal to $\frac{1}{16}$ of another, how many of the latter are equivalent to 24· of the former?

34· An army having provisions for 112· days, how long will the stock last, if the quantity originally calculated for $\frac{1}{19}$ of any given time, be made to suffice for $\frac{1}{16}$ of that time?

35· If a ship's stock of biscuit had been secured from injury, each man might have received 14·oz. per day for the entire voyage. But, after some days at sea, it was discovered that, a certain portion having been spoiled, $\frac{1}{182}$ of the remainder must last as long as $\frac{1}{148}$ of it was intended for. What must the ration now be?

36· Two square prisms are of equal lengths, and of equal solidities, but of unlike widths and thicknesses. Now, if the first were $\frac{1}{7}$ of its present width, and the second $\frac{1}{11}$ of its present width, they would differ only in thickness. How thick is the second, the first being 2·ft. 9·in. thick?

37· Two cisterns (hollow square prisms,) will hold equal quantities of water, and are of equal depths. Now, the same width which, in the second will contain $\frac{1}{15}$ of a gallon, will in the first contain $\frac{1}{13}$ of a gallon. How long is the first, the second being 20·ft. 7·in. long?

38· A rectangular-sided tank, 6·ft. deep, being quite full of water, is discharged into a (square prismatic) cistern, of the same width and depth. Now, $\frac{1}{9}$ of length of cistern $= \frac{1}{8}$ of length of tank. How deep will the water stand in the cistern?

Or thus:

How deep is a cistern, which will contain in $\frac{1}{9}$ of its length, as much water as a tank 6·ft. deep will contain in $\frac{1}{8}$ of its length: breadths being equal?

39· How much must the tank be deepened, so that the length which originally contained $\frac{1}{8}$ of a gal. shall contain $\frac{1}{7}$ of a gal.?

40· There are two rectangles, each having an area of 1· square yard. The first is 3·ft. long, and $\frac{1}{3}$ of its width is equal to $\frac{1}{2}$ the width of the other. How long is the latter?

41· What principal will gain £$\frac{1}{15}$ in the same time that £155· will gain £$\frac{1}{5}$, rate being constant?

42· What is the extent of $\frac{1}{3}$ of an estate of which $\frac{1}{7}$ contains 42·ac. 2·r. 36·po. 28·yd.?

TRACING THE PROCESS IN INVERSE PROPORTION EXAMPLES.

The conversion of Inverse Proportion of Integers into Direct Proportion of Fractions, explained on pages 330· and 331·, and illustrated by the Examples in EXERCISE CXVII. B., and EXERCISE CXVIII. A., furnishes us with a means of elucidating parts of the process of Inverse Proportion which would otherwise be obscure.

This obscurity occurs when the third term of an Inverse Proportion is multiplied by a second term containing more than one place of figures, each being employed as a separate multiplier.

SIXTH EXAMPLE WORKED OUT. If a certain work be completed in 149· days, by 352· men, how long would it occupy 47· men, working at the same rate?

Abbreviated Question.

	men.		days.
If	352·	....	149·
	47·	....	?

Statement and Work.

As $\overset{\text{men.}}{47\cdot}$: $\overset{\text{men.}}{352\cdot}$ :: $\overset{\text{days.}}{149\cdot}$ (for $\overset{\text{men.}}{352\cdot}$) : x.

$352\cdot$

$\overline{298\cdot}$ for $\frac{1}{2}$ of $\overset{\text{men.}}{352\cdot}$ or $\overset{\text{men.}}{176\cdot}$ A.

$7450\cdot$ for $\frac{1}{50}$ of $352\cdot$ or $7\frac{1}{25}$ B.

$44700\cdot$ for $\frac{1}{300}$ of $352\cdot$ or $1\frac{13}{75}$ C.

$47\cdot)\overline{52448\cdot}$ for $\frac{1}{352}$ of $352\cdot$ or $1\cdot$ D.

$$\frac{\overset{\text{d.}}{149\cdot} \times 352\cdot}{47} = \frac{\overset{\text{d.}}{52448}}{47} = \underline{\underline{1115\tfrac{43}{47}}} \text{ for } 47\cdot \text{ men.} \qquad \text{E.}$$

A. *Twice* as long for *half* as many men.
B. *Fifty times* as long for *one fiftieth* of them.
C. *Three hundred times* as long for *one three-hundredth* of them.
D. *Three hundred and fifty two* times as long for $\frac{1}{352}$ of them, or 1· man.
E. *One forty-seventh* of that time for 47· men.

The explanations given above of the five lines of the work, are evidently correct. Yet there is still a gap. The process is not clearly traced. For the meanings appended to the three intermediate lines A, B, and C, do not account for, or lead down to, that of the fourth line D, produced by adding them. For $\frac{1}{2} + \frac{1}{50} + \frac{1}{300}$ is not equal to $\frac{1}{352}$.

A step in the reasoning is concealed. And that step is made evident by considering what the question would be, if converted into a Direct Proportion, by employing, instead of the first and second terms, their Reciprocals. The question would then be:

"A certain work is completed in 149· days, when $\frac{1}{352}$ of it is done by each man employed. How many days would it occupy, were $\frac{1}{47}$ of it done by each, working at the same rate of speed as before?"

And now, reverting to the former work, we can easily explain it, tracing the process in such a manner, as to shew clearly how each line derives its value from its predecessors.

Statement and Work.

As $\overset{\text{men.}}{47\cdot}$: $\overset{\text{men.}}{352\cdot}$ :: $\overset{\text{days.}}{149\cdot}$ (for $\overset{\text{men.}}{352\cdot}$ each doing $\frac{1}{352}$) : x.

$352\cdot$

$\overline{298\cdot}$ for $\frac{1}{2}$ of $\overset{\text{men.}}{352\cdot}$ each doing $\frac{2}{352}$ A.

$7450\cdot$ for $\frac{1}{50}$ of $322\cdot$ each doing $\frac{50}{352}$ B.

$44700\cdot$ for $\frac{1}{300}$ of $352\cdot$ each doing $\frac{300}{352}$ C.

$47\cdot)\overline{52448\cdot}$ for $\frac{1}{352}$ of $352\cdot$ or $1\cdot$ m. doing all. D.

$\underline{\underline{1115\frac{43}{47}}}$ for $47\cdot$ men.

A. Half as many men, each must do twice as much.
B. $\frac{1}{50}$ as many „ each must do 50· times as much.
C. $\frac{1}{300}$ as many „ each must do 300· times as much.
D. $\frac{1}{352}$ as many „ each must do 352· times as much.

But, in all five cases, the speed at which a man works remains unchanged. For, when half the original no. of men complete the task they have twice as long to do it in. When one fiftieth of the no. are sufficient fifty times as long are allowed, and so on.

Hence, it is easy to perceive how the significations of line D is connected with those of lines A, B, and C.

For:
days
In the first 298· he would do $\frac{2}{352}$ of the task.
In the next 7450· he would do $\frac{50}{352}$ of the task.
In the last $\underline{44700}$· he would do $\frac{300}{352}$ of the task.
∴ In the whole $\underline{\underline{52448}}$· he would do $\frac{352}{352}$, or all the work.

EXERCISE CXVIII. B.

INVERSE PROPORTIONS.

Trace the process, and explain every line.

1· If 73· horses can be kept for 78· days, on a certain quantity of corn, how long can 13· be fed, at the same rate, on the like quantity?

2· At what rate per day can 96· men be supplied out of the quantity of food which would give 8· lb. 10· oz. per day, to 95· men, for the same number of days?

3· Supposing each of 143· men can have daily $14\frac{7}{8}$ oz. of food for 47· days, how much must the ration be reduced so that the same stock may serve 168· men for the same length of time?

4· If 91· persons can complete a piece of work in 185· days, in what time would 37· do it?

5· Supposing 153· men can finish a house in the required time by working 12· hours a day, at what rate must the remainder work to effect the same result, if 45· of them be withdrawn?

6· If a loaf, of a fixed price, weigh 7· lb. 13· oz. 4· dr., when wheat is at 11·s. 5·d. per bushel, what should it weigh when wheat is at 13·s. 11·d. per bushel?

7· When wheat is at 11·s. 1·d. per bushel, a certain-priced loaf weighs 3· lb. 6· oz., what is the price of wheat when a loaf weighing 4· lb. 2· dr. can be bought for the same sum?

8· How many yards of linen, $4\frac{1}{4}$ yd. wide, will line 46· yd. 3· qr. of cloth $4\frac{1}{2}$ yd. wide?

9· How wide is the cloth of which 48· yd. 3· qr. are lined by 55· yd. 1· qr. of canvas, 3· qr. 3· na. wide?

10· How many yards of carpet, $1\frac{3}{4}$ yd. wide, will cover the floor of a rectangular passage 7· ft. 6· in wide, and 168· ft. long?

11· Of two equal rectangular fields the first is 19·69 ch. long, and 1·31 ch. wide; the second is 1·79 ch. wide; how long is it?

12· If a certain commodity cost £7· 16·s., at a given rate per lb. avoir., what would it cost at the same rate per lb. Troy?

13· The contents of 59· bales, each weighing 17· cwt. 2· qr. 15· lb. 5· oz., are re-packed into 735· equal parcels. What will each of these weigh?

14· How many boxes of soap, each containing 2· cwt. 3· qr., can be filled from 14· frames, each containing 1· ton, 15· cwt. 3· qr.?

15· What is the true length of a road which, being measured by a Gunter's chain, stretched 9· inches, appears to be 79· ch. 29· ft. 4· in. long?

16· How much is a Gunter's chain stretched, if it make a real length of 70· ch. 1· ft. 9· in., appear to be only 69·5 chains?

17· What is the true length of that which appears to be 140· ch. 35· ft. when measured by a G. C. which has shrunk 4· links?

18· How long is a chain which gains 5· ch. 41· ft. on 134· ch. 60· ft. of true measure?

19· How large a bulk of Portland stone, Sp. gr. 2·63 will counterpoise 8·21875 cu. ft. of American ebony, Sp. gr. 1·49?

20· A certain weight of oil, Sp. gr. ·936, fills a certain rectangular-sided vessel to the depth of 3· ft. 3¼ in.; to what height would the same weight of molten lead rise in the same vessel, Sp. gr. of the metal being 11·25?

21· At a certain rate per lb. Troy, a parcel of goods costs £4· 14·s. 9½d., what would it cost at the same rate per lb. avoir.?

22· A Gunter's chain being 1· link short, how long will 109·98 true chains appear when measured by it?

BARTER, OR EXCHANGE OF GOODS.

When two quantities of goods are exchanged, one for the other, it is implied that the gross values of the two lots are equal. If, then, the one lot consist of a greater number of things than the other, it must comprise articles of a less individual value than those in the other.

No. of articles and price of each are in Inverse Proportion.

When Gross Value is constant. Pages 308· and 309·

Barter questions, therefore, fall under our first case of Inverse Proportion, and there is no necessity for treating them under a separate rule, as is frequently done. There are, however, certain peculiarities about the tracing of the process in Barter questions, when worked by Proportion.

First example worked out. How many yards of cloth at 12·s. should be equitably exchanged for 108· yds. at 17·s.?

Abbreviated Question.

	s.		yds.
If at	17·	....	108·
at	12·	....	?

Statement and Work.

$$\text{As } \overset{\text{s.}}{12\cdot} : \overset{\text{s.}}{17\cdot} :: \overset{\text{yds.}}{108\cdot}\ (@\ \overset{\text{s.}}{17\cdot} \doteq \tfrac{1}{17}\text{ no. of yds, }@\ \overset{\text{s.}}{1}) : x$$

$$\begin{array}{rrl}
 & 17\cdot & \\ \cline{2-2}
 & 756\cdot & = \tfrac{7}{17}\text{ no. of yds. }@\ 1\cdot\text{s.} \\
 & 1080\cdot & = \tfrac{10}{17}\text{ no. of yds. }@\ 1\cdot\text{s.} \\ \cline{2-2}
 & 12\cdot)\,1836\cdot & = \text{no. yds. }@\ 1\cdot\text{s.} \\ \cline{2-2}
x = \dfrac{\overset{9\cdot}{\cancel{108}\cdot} \times 17\cdot}{\cancel{12}} = & 153\cdot & = \text{no. yds. }@\ 12\cdot\text{s.} \\ \cline{2-2} \cline{2-2}
\end{array}$$

The multiplication is here, purposely, effected in two lines, rather than in one, that the values of the two intermediate lines may be shewn.

Both quantities of goods in this example being measured by lineal units, there is a ratio between those two quantities. (PRINCIPLE V.)

And we state a proportion which is logical and true, in saying:

$$\text{As } \overset{\text{s.}}{12\cdot} : \overset{\text{s.}}{17\cdot} :: \overset{\text{yds.}}{108\cdot} : \overset{\text{yds.}}{153\cdot}$$

$$\text{For, } \overset{\text{s.}}{12\cdot} = \tfrac{12}{17}\text{ of } \overset{\text{s.}}{17\cdot}\text{; and } \overset{\text{yds.}}{108\cdot} = \tfrac{12}{17}\text{ of } \overset{\text{yds.}}{153\cdot}$$

But, suppose the question had been, as it might have been:

"How many gallons of gin, at 12·s. per gallon, are exchangeable for 108· yds. of cloth, at 17·s. per yard?"

We could not say:

$$\text{As } \overset{\text{s.}}{12\cdot} : \overset{\text{s.}}{17\cdot} :: \overset{\text{yds.}}{108\cdot}\ \textit{of cloth} : \overset{\text{gal.}}{153\cdot}\ \textit{of gin,}$$

which is an absurd statement; for there is no ratio between yards of cloth and gallons of gin: (PRINCIPLE V.) 108· yds. of cloth being neither part nor multiple of 153· gals. of gin. In such examples, namely, where quantities of unlike kind are bartered, we cannot form the question into a proportion, unless *we change the unit* of one of those unlike quantities into the same name as that of the other quantity; or, consider *both* quantities as made up of units of any *similar* kind.

$$\text{As } \overset{\text{s.}}{12\cdot} : \overset{\text{s.}}{17\cdot} :: \overset{\text{yds.}}{108\cdot} : \overset{\text{yds.}}{153\cdot}$$

∵ Then, 108· yds. at 17/ are equivalent to 153· yards at 12/.
And 153· yds. at 12/ are equivalent to 153· gal. at 12/.
∴ 108· yds. at 17/ are equivalent to 153· gal. at 12/.

So that, in fact, we work out a true proportion, which gives us the fourth term of same kind as the third, and we then *change the unit* of the result into the name required by the question.

Also, instead of working for *yards* at 12·s., and then supposing those *yards* exchanged for the same no. of *gallons* at the same price, we might have supposed the 108· yds. at 17·s. to be changed for 108· gals. at 17·s., and then our statement would have been

$$\text{As } \overset{\text{s.}}{12\cdot} : \overset{\text{s.}}{17\cdot} :: \overset{\text{gal.}}{108\cdot}\ (\text{at } 17\cdot\text{s.}) : \overset{\text{gal.}}{153\cdot}\ (\text{at } 12\cdot\text{s.})$$

The answer coming out at once in the required name.

This change of unit is particularly to be noted in examples where the result requires reduction, as in next example.

SECOND EXAMPLE WORKED OUT. How many gallons of beer at 5·d. per qt. are worth 6· lb. 4· oz. of tea at 4/6 per lb.?

Abbreviated Question.

lb. oz.
If at 5·d. . . . . 6· 4·
at 4/6 . . . . ?

Statement and Work.

d. d. lb. oz.
As 5· : 54· : : 6· 4· : x
16
100·oz. @ 54·d. per lb. = $\frac{1}{54}$ wt. @ 1·d. per lb.
54·
400·oz. @ $\frac{54}{4}$d. per lb. = $\frac{4}{54}$ wt. @ 1·d. per lb.
5000·oz. @ $\frac{54}{50}$d. per lb. = $\frac{50}{54}$ wt. @ 1·d. per lb.
5·)5400·oz. at 1·d. per lb.
16·)1080·oz. at 5·d. per lb.
67½ lb. at 5·d. $\doteq$ no. qts. at 5·d.

gal. qt. pt.
$x = 67\frac{1}{2}$ lb. $\doteq 67\frac{1}{2}$ qt. = 16· 3· 1·

We might, at starting, have changed the unit, and treated the question as running thus, (substituting qts. for lb.)
"How many qts. at 5·d. are worth 6¼ qts. at 4·s. 6·d.?"
And then, worked by fractions, the statement and result would be

d. d. qts. qts.
As 5· : 54· : : 6¼ (at 54·d.) : 67½ (at 5·d.)

Proportion is, however, a needlessly roundabout and laborious method of working out questions in Barter. They are much more simply solved

Without reference to Ratio.

s. d.
4· 6· = v. of 1·lb. of tea.
6¼
1· 7· 0· = v. of 6·lb. of tea.
1· 1½ = v. of ¼ lb. of tea.
v. of 1· qt. = 5·d. 1· 8· 1½ = v. of 6¼lb. of tea, or of the beer.
same in far. = 20· q.)1350· q. = v. of beer, in farthings.
4·)67½ $\doteq$ no. of qts. at 5·d.
16· gal. 3· qts. 1· pt.

THIRD EXAMPLE WORKED OUT. If 40·yd. 2·qr. 2·na. of cloth at 6·s. per yard, be equitably exchanged for 4·lb. 15·dwts. of old silver, what is the latter rated at per lb.?

With the foregoing concrete quantities, all three being unlike, the statement of a proportion is impossible. For, as the third term of the proportion must be that one which is of the same kind as the answer sought, the only semblance of a proportion which we can derive from the given question is

$$\text{As}\quad \overset{\text{lb.}}{4\cdot}\ \overset{\text{dwt.}}{15\cdot} \quad : \quad \overset{\text{yd.}}{40\cdot}\ \overset{\text{qr.}}{2\cdot}\ \overset{\text{na.}}{2\cdot} \quad :: \quad \overset{\text{s.}}{6\cdot} \quad : \quad x.$$

which is absurd; for it means "Whatever part or multiple the quantity of silver is of the quantity of cloth, the same part or multiple 6·s. are of the answer sought." In such a case, we reduce the two terms of commodity into mixed numbers, and consider the units whose price is given as changed into units of the same kind as those whose price is sought. In the present example, then,

$$\because\ \overset{\text{yd.}}{40\cdot}\ \overset{\text{qr.}}{2\cdot}\ \overset{\text{na.}}{2\cdot} = \overset{\text{yd.}}{40\tfrac{5}{8}};\quad \text{and}\ \overset{\text{lb.}}{4\cdot}\ \overset{\text{dwt.}}{15\cdot} = \overset{\text{lb.}}{4\tfrac{1}{16}},$$

the price of a yard being given, and the price of a pound being sought, we change the yards into pounds, and say in the

Abbreviated Question.

$$\begin{array}{lcr} \text{If } \overset{\text{lb.}}{40\tfrac{5}{8}} & \ldots\ldots\ldots\ldots & \text{at } \overset{\text{s.}}{6\cdot} \\ \quad 4\tfrac{1}{16} & \ldots\ldots\ldots\ldots & \text{at } ? \end{array}$$

Statement and Work.

$$\text{As } 4\tfrac{1}{16} \quad : \quad 40\tfrac{5}{8} \quad :: \quad \overset{\text{s.}}{6\cdot} \quad : \quad x.$$

$$\text{Or, as } \tfrac{65}{16} \quad : \quad \tfrac{325}{8} \quad :: \quad \tfrac{6}{1} \quad : \quad x.$$

$$\therefore x = \frac{\overset{\text{s.}}{6}}{1} \times \frac{325}{8} \div \frac{65}{16} = \frac{\overset{\text{s.}}{6} \times \overset{5\cdot}{\cancel{325}} \times \overset{2\cdot}{\cancel{16}}\cdot}{\cancel{8} \times \cancel{65}\cdot} = \overset{\text{s.}}{60}\cdot = \overset{\pounds}{3}\cdot \text{ per lb.}$$

PROOF.

40· yd. 2· qr. 2· na. at 6·s. per yd. cost £12· 3·s. 9·d.

4· lb. 15· dwt. at £3· per lb. cost £12· 3·s. 9·d.

EXERCISE CXIX.

MISCELLANEOUS EXAMPLES IN SIMPLE PROPORTION, DIRECT AND INVERSE, INCLUDING BARTER.

1· *Trace the process, explaining every line.*

2· *Prove every example, without reference to ratio.*

1· If 68· men, by completing a certain work, earn £2· 16·s. 3¾d. each, what sum would have been due to each of 51· men for performing the same task?

2· Two exactly similar houses are built, the first by 144· men, each earning £3· 17·s. 2¼d., while, on the second, each man gets £9· 15·s. The rate of wage and the quality of workers being constant, how many men are engaged on the second house?

3· The grain produced by 9·4375 ac. being accurately measured, was found to be 82· qr. 4· bu. 2· pe. 1· gal. At the same rate, what should be the produce of 10·59375 ac.?

4· An estate containing 59· ac. 3· r. 20· po., estimated at £56· 17·s. 6·d. per acre, is exchanged for 12· ac. 2· r. 30· po. of building land. At what rate per acre is the latter valued?

5· At £125·6 for 6·4 ac. how much land is worth £160· 9· fl. 2· c. 5· m.?

6· If 9· men do as much as 14· women, how many women should do as much as 54· men?

7· Supposing 7· sheep eat as much as 2· cows, how long will that amount of food serve 100· sheep, which will keep 10· cows for 3· weeks?

10· cows eat as much as $\frac{70}{2}$ *sheep, &c.*

8· The area of a rectangle being 424· su. feet, and its width 16· ft., how much must be taken from its length to reduce its area by 11· su. yd. 7· ft.?

9· A man bought 40· sacks of oats, each 1· cwt. 1· qr. 20· *lb.*, *at* 9·s. 9·d. per sack, and sold the whole at 10/3 per *sack of* 7· sc. 8· lb. How much did he gain?

10· How many sacks did he sell?

11· How much did he gain by 40· sacks *sold?*

12· How many sacks must he buy to have 12· more than that number to sell, respective weights as above?

13· What did each sack which he sold cost him?

14· At what rate, per sack *bought*, did he sell?

15· If 2· lb. 6· ℥. 4· ʒ. 1· ℈. 10· gr. cost £2· 9·s. 1½d., what will 3· oz. 7· dwt. 22· gr. cost?

16· How many yards at 2·s. 9¼d. are equivalent to 1· cwt. 1· qr. 21· lb. at £6· 13·s. per cwt.?

17· At what rate per cwt. will 17· cwt. 1· qr. be worth as much as 2· oz. 13· dwt. 16· grs. at £3· 11·s. 3·d. per oz.?

18· Supposing 4· cwt. 1· qr. 7· lb. of sugar to be equitably exchangeable for 10· yds. 1· na. of broadcloth, at £1· 8·s. 6·d. per yard, what is the computed price of 1· cwt. of sugar?

19· If 2· qr. 13· lb. at £15· 10·s. 4·d. per cwt. be of equal value with 1· hhd. 17· gal. 2· qt., what is the price of a gallon?

20· At what rate per lb. will 10· oz. 1· dwt. 6· gr. be worth as much as 4· oz. 5· dr. at £2· 4·s. 4·d. per lb.?

21· Bought a quantity of drug at £4· 17·s. 6·d. per lb. Avoir., and sold it at the same price per lb. Apoth., thus gaining £10· 11·s. 8·d. What was the weight bought?

22· What is the real length of that which appears to be 40· ch. 55· ft. 11¼ in., when measured by a Gunter's chain, contracted one foot?

COMPOUND PROPORTION, OR DOUBLE RULE OF THREE.

When two or more Simple Proportions are so connected that the fourth term of each of them becomes the third term of the next, these several Simple Proportions, taken together, form one Compound Proportion.

EXAMPLE OF A COMPOUND PROPORTION, MADE UP OF FOUR SIMPLE PROPORTIONS.

First Simple Proportion. If 37· men build 15· sup. roods of a wall, how much will 74· men build in the same time, at the same rate of speed?

Statement and Result.

men. men. roods. men. roods. men.
As 37· : 74· : : 15· (for 37·) : 30· (for 74·)

Second Simple Proportion, introducing two new terms with fourth term of foregoing Simple Proportion. If those 30· roods be built in 27· days, how much should be built in 63· days, of the same length, by same no. of men, working at same rate of speed?

Statement and Result.

days. days. roods. days. roods. days.
As 27· : 63· : : 30· (in 27·) : 70· (in 63·)

Fourth of preceding proportion becomes Third of this.

Third Simple Proportion, introducing two new terms with fourth term of Second Simple Proportion. Supposing those 70· roods to be built by working 10· hours a day, how many would be built by same no. of men, in same no. of days, at same rate of speed, but working 12· hours a day?

Statement and Result.

hours. hours. roods. roods.
As 10· : 12· : : 70· (in 10· hour days) : 84· (in 12· hour days)

Fourth of Second Proportion becomes Third of this.

Fourth Simple Proportion, introducing two new terms with fourth term of Third Simple Proportion. Supposing those 84· roods to be the surface built when the wall is 3· ft. thick, what area would be built, by same no. of men, in same no. of days, of same length, and at same rate of speed, if the wall were 2· ft. thick?

Statement and Result.

$$\text{As } \overset{\text{ft.}}{2\cdot} : \overset{\text{ft.}}{3\cdot} :: \overset{\text{roods.}}{84\cdot} \text{ (of } \overset{\text{ft.}}{3\cdot} \text{ thick)} : \overset{\text{roods.}}{126\cdot} \text{ (of } \overset{\text{ft.}}{2\cdot} \text{ thick)}$$

Fourth of Third Proportion becomes Third of this.

Now, this ultimate result, namely, 126· roods, which is fourth term of the last proportion, is produced by multiplying third term of first proportion continually by all four second terms, and dividing the product continually by all four first terms, or by their continued product. For,

$$\text{4th term of First Prop.} = \overset{\text{roods.}}{15\cdot} \times 74\cdot \div 37\cdot = \overset{\text{r.}}{\frac{15\cdot \times 74\cdot}{37\cdot}} \text{ (PRIN. XV.)} = \overset{\text{roods.}}{30\cdot}$$

$$\text{4th term of Second Prop.} = \overset{\text{roods.}}{30\cdot} \times 63\cdot \div 27\cdot = \overset{\text{r.}}{\frac{15\cdot \times 74\cdot \times 63\cdot}{37\cdot \times 27\cdot}} = \overset{\text{roods.}}{70\cdot}$$

$$\text{4th term of Third Prop.} = \overset{\text{roods.}}{70\cdot} \times 12\cdot \div 10\cdot = \overset{\text{r.}}{\frac{15\cdot \times 74\cdot \times 63\cdot \times 12\cdot}{37\cdot \times 27\cdot \times 10\cdot}} = \overset{\text{roods.}}{84\cdot}$$

$$\text{4th term of Fourth Prop.} = \overset{\text{roods.}}{84\cdot} \times 3\cdot \div 2\cdot = \overset{\text{r.}}{\frac{15\cdot \times 74\cdot \times 63\cdot \times 12\cdot \times 3\cdot}{37\cdot \times 27\cdot \times 10\cdot \times 2\cdot}} = \overset{\text{roods.}}{126\cdot}$$

COMPOUND PROPORTION (OR DOUBLE RULE OF THREE) QUESTION, MADE UP OF THE PRECEDING FOUR SIMPLE PROPORTION QUESTIONS.

If 37· men, in 27· days, of 10· hours each, *build* 15· sup. roods of a wall, 3· ft. thick, what area of a wall 2· ft. thick, will 74· such men build in 63· days, of 12· hours each?

In the same way, five or six, or any number of duly-connected Simple Proportions may be combined into one Compound Proportion. And it will be seen that

I. Every Compound Proportion question must contain an odd number of terms.

II. The odd term (as in Simple Proportion) is of the same kind as the answer sought.

We shall call this the *Common Third Term.*

III. The other terms go in couples, two of a kind.

TO SOLVE ANY COMPOUND PROPORTION QUESTION.

The method of working out Compound Proportion is merely an extension of that for Simple Proportion.

First. *Write the question in an abbreviated form.*

See Examples worked out.

Second. *State the Proportion*, thus:

- *a.* Enclose, in four vertical braces, two columns, the left-hand one for the set of first terms, and the other for the set of second terms.
- *b.* Write signs of Proportion and of Multiplication as in Examples worked out.
- *c.* Put the odd term (which is like the answer sought) as *Common Third Term.*
- *d.* Take first couple of like terms, and, *disregarding the other couples*, consider these two terms in connection with the *Common Third*, and state as in Simple Proportion.
- *e.* Proceed similarly with each of the remaining couples.

Third. *Work for the result.*

- *a.* Cancel Common Factors in firsts and common third, and in firsts and seconds.
- *b.* Multiply Common Third by continued product of all the seconds remaining after cancelling, and Divide by continued product of all the remaining firsts.

NOTE. If any of the terms be compound, perform the necessary reduction before stating. See second example worked out.

FIRST EXAMPLE WORKED OUT. NO REDUCTION NECESSARY. If 37· men, in 27· days, of 10· hours each, build 15· sup. roods of a wall, 3· ft. thick, what area of a similar wall, but 2· ft. thick, will 74· such men build in 63· days, of 12· hours each?

Abbreviated Question.

	men	days	ho.	roods	ft. th.
If	37·......	27·......	10·......	15·......	3·
	74·......	63·......	12·......	?	2·

The Statements.

FIRST COUPLE. *The two nos. of men.* "Will 74· men build more or less than 37· men, other things remaining constant?" "More."

∴ *Put the greater*, (74· men,) *second.* A Direct Proportion.

Second Couple. *The two nos. of days.* "In 63· days will more or less be built than in 27· days, other things remaining constant?" "More."

∴ *Put the greater*, (63· days,) *second.* A Direct Proportion.

Third Couple. *The two lengths of day.* "In a day of 12· hours, will more or less be built than in a day of 10· hours, other things remaining constant?" "More."

∴ *Put the greater*, (12· hours,) *second.* A Direct Proportion.

Fourth Couple. *The two thicknesses.* "Will the same amount of labour complete a greater or less Area of a wall 2· ft. thick than of one which is 3· ft. thick?" "Greater."

∴ *Put the greater*, (3· ft.) *second.* An Inverse Proportion.

The Statements and Work.

I.

Without Cancelling.

$$\text{As}\left\{\begin{array}{c}37\cdot\\ \times\\ 27\cdot\\ \times\\ 10\cdot\\ \times\\ 2\cdot\end{array}\right\}:\left\{\begin{array}{c}74\cdot\\ \times\\ 63\cdot\\ \times\\ 12\cdot\\ \times\\ 3\cdot\end{array}\right\}::\begin{array}{r}\text{roods}\\ 15\cdot\\ 167832\cdot\\ \hline 19980\cdot)2517480\cdot(126\cdot\text{ roods}\\ 19980*\\ \hline 51948\\ 39960\\ \hline 119880\\ 119880\\ \hline\hline\end{array}:x$$

$$\text{Or, As }\overline{\underline{\underline{19980\cdot}}}:\overline{\underline{\underline{167832\cdot}}}$$

* For the explanation of each step in the process, and, particularly of this number, 2517480·, see III. below.

II.

With Cancelling.

$$x=\frac{\overset{3\cdot}{\cancel{15}}\cdot\text{ roods}\times\overset{\cancel{2}\cdot}{\cancel{74}}\cdot\times\overset{7\cdot}{\cancel{63}}\cdot\times\overset{6\cdot}{\cancel{12}}\cdot\times\cancel{3}\cdot}{\cancel{37}\cdot\times\underset{\cancel{3}\cdot}{\cancel{27}}\cdot\times\underset{\cancel{2}\cdot}{\cancel{10}}\cdot\times\cancel{2}\cdot}=\overset{\text{r.}}{3}\cdot\times 7\cdot\times 6\cdot=126\cdot\text{rds.}$$

III.

The "tracing of the process" in such an example as this is extremely interesting. To do this clearly, it will be requisite to multiply the "Common Third Term" continually by the several seconds, and then to divide the product, continually, by the several firsts.

Thus:

15· roods, 3·ft. th., by 37·m., in 27·d., of 10· ho. each.
74·
60·*
1050·*
1110·r. 3·ft th., by (74· times 37·)m., in 27·d. of 10· ho. ea.
63·
3330·*
66600·*
69930·r. 3·ft. th., by (74· t. 37·)m., in (63· t. 27·)d. of 10· ho. ea.
12·
839160·r. 3·ft. th., by (74· t. 37·)m., in (63· t. 27·)d. of (12· t. 10·)ho.
3·
37)2517480·r. 1·ft. th., by (74· t. 37·)m., in (63· t. 27·)d. of (12· t. 10·)ho.
27·)68040·r. 1·ft. th., by 74·m., in (63· t. 27·)d. of (12· t. 10·)ho.
10·)2520·r. 1·ft. th., by 74·m., in 63· d. of (12· t. 10·)ho.
2·)252·r. 1·ft. th., by 74·m., in 63· d., of 12· ho.
126·r. 2ft. th., by 74·m., in 63· d., of 12· ho.

Which was to be found.

IV.

The same example worked without reference to ratio.

Area of wall built, when

$$\overset{\text{ft.}}{3\cdot}\text{ th., by }\overset{\text{m.}}{37\cdot}\text{, in }\overset{\text{d.}}{27\cdot}\text{ of }\overset{\text{ho.}}{10\cdot} = \overset{\text{roods}}{15\cdot} \qquad =\text{A.}$$

$$\overset{\text{ft.}}{1\cdot}\text{ th., by }\overset{\text{m.}}{37\cdot}\text{, in }\overset{\text{d.}}{27\cdot}\text{ of }\overset{\text{ho.}}{10\cdot} = 3\text{ t. A.} = \overset{\text{r.}}{15\cdot\times3\cdot} \qquad =\text{B.}$$

$$\overset{\text{ft.}}{1\cdot}\text{ th., by }\overset{\text{m.}}{1\cdot}\text{, in }\overset{\text{d.}}{27\cdot}\text{ of }\overset{\text{ho.}}{10.} = \tfrac{1}{37}\text{of B.} = \frac{\overset{\text{r.}}{15\cdot\times3\cdot}}{37\cdot} \qquad =\text{C.}$$

$$\overset{\text{ft.}}{1\cdot}\text{ th., by }\overset{\text{m.}}{1\cdot}\text{, in }\overset{\text{d.}}{1\cdot}\text{ of }\overset{\text{ho.}}{10\cdot} = \tfrac{1}{27}\text{of C.} = \frac{\overset{\text{r.}}{15\cdot\times\ 3\cdot}}{37\cdot\times27\cdot} \qquad =\text{D.}$$

$$\overset{\text{ft.}}{1\cdot}\text{th., by }\overset{\text{m.}}{1\cdot}\text{, in. }\overset{\text{d.}}{1\cdot}\text{ of }\overset{\text{ho.}}{1.} = \tfrac{1}{10}\text{of D.} = \frac{\overset{\text{r.}}{15\cdot\times\ 3\cdot}}{37\cdot\times27\cdot\times10\cdot} \qquad =\text{E.}$$

$$\overset{\text{ft.}}{1\cdot}\text{ th., by }\overset{\text{m.}}{74\cdot}\text{, in }\overset{\text{d.}}{1\cdot}\text{ of }\overset{\text{ho.}}{1\cdot} = 74\cdot\text{ t. E.} = \frac{\overset{\text{r.}}{15\cdot\times\ 3\cdot\times74\cdot}}{37\cdot\times27\cdot\times10\cdot} \qquad =\text{F.}$$

$$\overset{\text{ft.}}{1\cdot}\text{ th., by }\overset{\text{m.}}{74\cdot}\text{, in }\overset{\text{d.}}{63\cdot}\text{ of }\overset{\text{ho.}}{1\cdot} = 63\cdot\text{ t. F.} = \frac{\overset{\text{r.}}{15\cdot\times3\cdot\times74\cdot\times63\cdot}}{37\cdot\times27\cdot\times10\cdot} \qquad =\text{G.}$$

---

* The explanations of these intermediate lines are omitted for the avoidance of confusion.

$$\overset{\text{ft.}}{1\cdot}\text{ th., by }\overset{\text{m.}}{74\cdot}\text{, in }\overset{\text{d.}}{63\cdot}\text{ of }\overset{\text{ho.}}{12\cdot} = 12\cdot\text{ t. G.} = \overset{\text{r.}}{\frac{15\cdot\times3\cdot\times74\cdot\times63\cdot\times12\cdot}{37\cdot\times27\cdot\times10\cdot}} = \text{H.}$$

$$\overset{\text{ft.}}{2\cdot}\text{ th., by }\overset{\text{m.}}{74\cdot}\text{, in }\overset{\text{d.}}{63\cdot}\text{ of }\overset{\text{ho.}}{12\cdot} = \tfrac{1}{2}\text{ of H.} = \overset{\text{r.}}{\frac{15\cdot\times3\cdot\times74\cdot\times63\cdot\times12\cdot}{37\cdot\times27\cdot\times10\cdot\times2\cdot}}$$

$= 126\cdot$ roods, as before, by Compound Proportion.

SECOND EXAMPLE WORKED OUT. REDUCTION REQUIRED BEFORE STATING. If the cost of excavating a rectangular ditch, 95· ft. 3· in. long, 6· ft. 9· in. wide, and 6· ft. deep, be £5· 19·s. 0¾d., what value of labour will be expended in digging another such trench, in similar ground, but 81· ft. 9· in. long, by 5· ft. 9· in. broad, and 4· ft. 3· in. deep?

Here, all the seven terms will require to be reduced before the *work* can be commenced.

It happens, in the present example, that all the six given terms of lineal measure can be reduced to quarter-feet.

Abbreviated Question; before reducing the terms.

	ft. in.		ft. in.		ft. in.		£ s. d.
If	95· 3·	(long) ...	6· 9·	(wide) ...	6· 0·	(deep) ...	5· 19· 0¾
	81· 9·	„ ...	5· 9·	„ ...	4· 3·	„ ...	?

Abbreviated Question; after reducing the terms.

	qr-ft.		qr-ft.		qr-ft.		farthings.
If	381·	(long)	27·	(wide)	24·	(deep)	5715·
	327·	„	23·	„	17·	„	?

Statements and Work.

$$\text{As}\left\{\begin{matrix}381\cdot\\ \times\\ 27\cdot\\ \times\\ 24\cdot\end{matrix}\right\} : \left\{\begin{matrix}327\cdot\\ \times\\ 23\cdot\\ \times\\ 17\cdot\end{matrix}\right\} :: \overset{\text{farthings}}{5715\cdot} : x.$$

$$\therefore\ x = \frac{\overset{\overset{5\cdot}{\cancel{635}\cdot}}{\cancel{5715}}\cdot\text{far.}\times\overset{109\cdot}{\cancel{327}}\cdot\times23\cdot\times17\cdot}{\underset{\cancel{127}\cdot}{\cancel{381}}\cdot\times\underset{3\cdot}{\cancel{27}}\cdot\times24\cdot} = \frac{\overset{\text{farthings}}{213095\cdot}}{72\cdot} = £3\cdot\ 1\cdot\text{s. }7\tfrac{3}{4}\text{d. }\cancel{\tfrac{47}{72}}\text{q.}$$

EXERCISE CXX.

EXAMPLES IN COMPOUND PROPORTION, OR DOUBLE RULE OF THREE.

Work each example in two ways :

I. *By Compound Proportion,*
II. *By Multiplication and Division, without reference to Ratio: explaining every line.*
III. *Resolve each question into the several Simple Proportion questions of which it is composed.* See Pages 343· and 344·.

A.

1· If £28· will pay for the board of 16· persons for 7· weeks, for how many will £39· pay for 4· weeks ?
2· Calculating that 16· horses plough 128· acres in 8· days, in what time will 14· horses plough 56· acres ?
3· How many loaves will suffice 19· people for 23· days, supposing that 34· persons, in 6· days, will consume 51· such loaves ?
4· Granting that a man can walk 315· miles in 14· days of 9· hours each, in how many such days will he go 160· miles, allowing himself one hour per day for repose ?
5· If 63· cwt. be carried 19· miles for 4·s. 6·d., how far should 57· cwt. be conveyed for 3·s.?
6· What number of loaves at 10·d. each, will serve 32· persons for 9· days, if 14· persons in 11· days require 55· loaves at 7·d.?
7· If 48· bushels of corn last 15· horses for 13· days, for what length of time will 32· bushels suffice for 5· horses ?
8· If £14· be the cost of maintaining 8· boys for 7· weeks, what would 25· cost for 11· weeks ?
9· How long could 300· soldiers be provisioned for £577· 10·s., if the keep of 114· for 19· days cost £379· 1·s.?
10· If £48· pay for the board and lodging of 33· people for 16· days, what sum will defray the like expenses of 15· persons for 77· days ?
11· How many loaves at 7·d. will serve 14· persons for 11· days, if 48· loaves at 15·d. suffice sixteen persons for 18· days ?
12· If 39· horses can be maintained 28· days for £19· 10·s., how many can be supported 49· days for £14· ?
13· In how many days will 32· horses plough 256· acres, working at the same rate as 7· horses which plough 28· acres in 4· days ?
14· A man, walking per day 8· hours incessantly, travels 160· miles in 8· days. How much longer must he walk per day, at the same rate of speed, to complete 630· miles in 28· days ?
15· What is the price, each, of 90· loaves, which will provide 16· boys with the same daily allowance for 9· days, as 77· loaves at 5·d. would furnish to 7· boys for 22· days ?
16· How many workmen will mow 128· acres in 8· days, when 21· can mow 84· acres in 4· days ?
17· Supposing a man could walk 210· miles in 7· days of 9· hours each, how many hours per day might he allow himself to

rest if he were required to walk, at the same rate, only 80· miles in 4· such days?

18· For how many people will 218½ lb. of beef serve for 11½ days, supposing that 34· consume 102· lb. in 3· days?

19· How many people will consume twenty eighteenpenny loaves in 4· days, at the same rate as 77· consume 35· elevenpenny loaves in two days?

20· If 15· cwt. 3· qr. be carried 4¾ miles for 1·s. 1½d., how far should 14· cwt. 1· qr. be carried for eighteenpence?

21· If 19· men by excavating 50· yards of trenching in 12· days, earn 4·s. per day, in what time ought 24· excavators to clear out 45· yards, each being paid 4·s. 6·d. per day?

22· If £4· 5·s. 6·d. will maintain 6· people for 19· days, for what length of time will £52· 10·s. pay the current expenses of 300· people?

23· Supposing that 110· loaves at 3½d. be sufficient for 28· persons for 5½ days, how long would 144· loaves at 5·d. last 64· people?

24· How much interest would £100· gain in 1· year, at the same rate that £103· 10·s. gain £3· 0·s. 4½d. in 7· months?

25· What would be the cost of digging a trench 29· ft. 3· in. long, 6· ft. 9· in. broad, and 2· ft. 3· in. deep, if another trench, in similar ground, 31· ft. 6· in. long, 7· ft. 3· in. broad, and 4· ft. 6· in. deep, cost £2· 10·s. 9·d.?

26· If 16· men, by raising 5130· Tons of ore, in 57· days, earn 2·s. 6·d. per day, what would be a fair daily wage for each of 20· men, who should raise 2250· Tons in 24· days?

27· What amount of Principal would gain £1· 18·s. 9¾d. Interest, in 4½ months, if £27· 17·s. 6·d. in 8· months, gain 18·s. 7·d.?

28· If the keep of 19· horses, for 22· days, cost £5· 4·s. 6·d. when oats are at 19/ per quarter, what would it cost to maintain 35· horses for 15· days, when oats are 25/4 per quarter?

29· For what sum should 28· cwt. 2· qr. be carried 14· miles, if 31· cwt. 2· qr. be conveyed 19· miles for 6·s. 9·d.?

30· What weight should be carried 42· miles for 4·s. 6·d., if 10· cwt. 2· qr. be carried 85½ miles for 10·s. 1½d.?

EXERCISE CXX. B.

1· If 15· men mow 48· acres in 4· days, how many acres will 30· men mow in 3· days?

2· Supposing 250· sheep can be pastured on 8· acres of land, for 3· weeks, how much land will be requisite for 420· sheep for 15· weeks?

3· If a trench 29· ft. 3· in. long, by 6· ft. 9· in. wide, and 2· ft. 3· in. deep, cost in labour £1· 1·s. 11¼d., what will another cost which is 31· ft. 6· in. long, 7· ft. 3· in. wide, and 4· ft. 6· in. deep?

4· The excavation of a trench 16· chains long, 4·5 ft. wide, and 4·5 ft. deep, cost £39· 6·fl., how many chains in length of a similar trench 12·5 links wide, and 7·5 feet deep, should be dug for £105· 17·s. 6·d.?

5· If $\frac{9}{16}$ cwt. be carried 54· miles for 8⅜s., how far will 1⅞ cwt. be transported for £1⅜?

6· Supposing that £103· 10·s. gain £1· 18·s. 9¾d. in 4½ months, in what time will £27· 8·fl. 7·c. 5·m. gain 18·s. 7·d.?

7· If 72· men complete a wall 450· ft. long, 10· ft. high, and 15· in. thick, in 9· days, and thus earn 5·s. 6·d. per day, how much per day would each of 48· men deserve, who should build, in 7· days, a wall of similar material, but 320· ft. long, 9· ft. high, and 25· inches thick?

8· If 8· men earn 4·s. 6·d. per day each, by completing in 9· days, a wall 72· ft. long, 12· ft. 6· in high, and 3· ft. 6· in. thick, what length of wall, 14· ft. high, and 4½ ft. thick, should 15· men build, in 16· days, so as to earn 6·s. 9·d. per day each?

9· If 18· men, by digging a trench 199· ft. 6· in. long, by 12· ft. wide, and 8· ft. 6· in. deep, in 14· days, each earn 5·s. 11¼d. per day, in what time should 21· men complete an excavation 91· ft. 7· in. long, 9· ft. wide, and 5· ft. 8· in. deep, to earn 3·s. 3¼d. per day, per man?

10· If 24· men excavate a trench 99· ft. 9· in. long, 12· feet wide, and 8· ft. 6· in. deep, in 42· days, thus earning each 5·s. 11¼d. per day, how many men should complete a trench 274· ft. 9· in. long, 9· ft. wide, and 2· ft. 10· in. deep, in 5· days, to earn, per man, 3·s. 3¼d. per day?

11· If £5· 1·s. 6·d. be the value of a log 57· ft. 3· in. long, by 4· ft. 10· in. wide, and 4· ft. 10· in. thick, what is the length of a log of similar wood, which, being 6· ft. 9· in. wide, and 2· ft. 3· in. thick, costs £1· 1·s. 11¼d.?

12· What length of embankment 9· ft. wide, and 2¼ ft. high, will cost £19· 16·s. in building, if 7· chains long, 1· pole wide, and 7· ft. 6· in. high, cost £105· 17·s. 6·d.?

13· If 27· men, by building a wall 42· ft. long, 9⅓ ft. high, and 1· ft. 3· in. thick, in 4· days, of 10· hours each, earn 4·s. 6·d. each, per day, how much per day would each of 15· men earn, by building in 6· days of 12· hours each, a similar wall, 19· ft. long, 7· ft. high, and 2· ft. 1· in. thick?

14· If a rectangular-sided block of stone 9· ft. 11· in. long, 2· ft. 2· in. wide, and 1· ft. 9· in. thick, can be quarried, dressed, and raised to a height of 99· ft., by 6· men, in 2· days of 10· hours, each man earning 5·s. 10·d. per day, what would be a proportionate payment per day for each of 8· men, for like preparation and raising to the height of 42· ft. in 4· days of 9· hours, a block 23· ft. 10· in. long, 2· ft. 10· in. wide, and 2· ft. 3· in. thick?

PERCENTAGES.*

The chief commercial use of percentages, or allowances upon the hundred, is to compute

I. **Commission,** which is a payment for disbursing, receiving, or remitting money for another person.

II. **Brokerage,** charge for buying or selling goods, shares, &c., for another person.

III. **Interest,** payment for the loan of money.

IV. **Insurance Premium**, previous payment for undertaking to replace loss by fire, shipwreck, &c., or to pay a certain sum at a person's death.

The sum on which any one of the above named percentages is charged or computed is called **Principal.**

Any percentage is, evidently, equal to as many one-hundreth parts of the Principal as there are units in the **Rate**. Thus:

1· per cent. $= \frac{1}{100}$ of P. $=$ P $\times \frac{1}{100}$ (Prin. ix.) $\doteq$ P. $\div$ 100· (Pr. xvii.)
2· per cent. $= \frac{2}{100}$ of P. $=$ P $\times \frac{2}{100}$ $\doteq$ P. $\times$ 2· $\div$ 100·
&c. &c. &c.

To find any percentage of a given Principal.

Multiply Principal by Rate and Divide by 100·

Or, (Principle xvii.) Multiply $\frac{1}{100}$th part of Principal by the no. of units and parts of a unit in the Rate.

Now, the one-hundredth part of any number is found by removing its digits two places to the right. Whence the ordinary form of the rule is

Multiply by the Rate, and cut off the two right-hand figures of the Product.

Which would be more correctly expressed thus:

Multiply by the no. of units and parts of a unit in the Rate, and remove digits of Product two places to the right, or Decimal Point two places to the left.

First Example worked out. Required the Commission or Brokerage on £648· 11·s. 8·d. at £2¼ per cent.

Example completed.

	£	s.	d.	
	648·	11·	8·	
			2¼	units in Rate.
	1297·	3·	4·	
	162·	2·	11·	
	14·59	6·	3·*	
	20·			
s.	11·86			
	12·			
d.	10·35			
	4·			
q.	1·4			

* In adding the two lines which produce this one we remove D.P. of pounds two places to the right, which is equivalent to dividing the pounds by 100· We then reduce the decimal part to shillings.

Ans. £14· 11·s. 10$\frac{1}{4}$d. $\frac{2}{5}$q. $= \dfrac{2\frac{1}{4}}{100}$ of £648· 11·s. 8·d.

When the Rate is a convenient fraction of £100· we take the same fraction of the Principal, by one Division.

SECOND EXAMPLE WORKED OUT. Find the Interest on £6432· 13·s. 9·d. for one year, at £5· per cent. per annum.

Example completed.

$\because$ £5· $= \frac{1}{20}$ of £100· $\therefore$ Interest $= \frac{1}{20}$ of Principal.

	£	s.	d.	
20·)	6432·	13·	9·	Principal.
	321·	12·	8$\frac{1}{4}$	Interest at £5· per cent.

THIRD EXAMPLE WORKED OUT. What is the Commission on £5843· 15·s. 10·d. at £$\frac{1}{8}$ per cent.

Example completed.

$\because$ $\frac{1}{8}$ of £1· $= \frac{1}{800}$ of £100· $\therefore$ Com. $= \frac{1}{800}$ of P. $\doteq$ P. $\div$ 800· (PR. XVII.)

	£	s.	d.	
800·)	5843·	15·	10·	Principal.
	7·	6·	1$\frac{1}{80}$	Commission at £$\frac{1}{8}$ per cent.

EXERCISE CXXI.

A.

What is the Commission, or Brokerage, or Interest for one year, or Insurance Premium, on

	£	s.	d.	£		£	s.	d.	
1·	1600·	14·	8·	at 1· per cent.	7·	464·	3·	6·	at $\frac{1}{2}$ p. c.
2·	3453·	11·	6·	at 2· per cent.	8·	954·	15·	0·	at $\frac{1}{4}$ p. c.
3·	7148·	13·	7·	at 3· per cent.	9·	846·	12·	6·	at $\frac{1}{8}$ p. c.
4·	9463·	16·	6·	at 4· per cent.	10·	593·	7·	6·	at $\frac{3}{4}$ p. c.
5·	3285·	11·	8·	at 5· per cent.	11·	942·	17·	6·	at $\frac{3}{8}$ p. c.
6·	1943·	6·	8·	at 6· per cent.	12·	398·	2·	6·	at $\frac{5}{8}$ p. c.

B.

Find the Interest, for one year, on each of the following sums of Principal, at each of the given rates per cent. per annum.

I. At £5·	III. At £3$\frac{1}{4}$	V. At £4$\frac{1}{4}$	VII. At £6$\frac{1}{8}$
II. At £3·	IV. At £3$\frac{1}{2}$	VI. At £5$\frac{3}{8}$	VIII. At £10·

	£	s.	d.		£	s.	d.		£	s.	d.
1·	3164·	15·	0·	5·	8562·	5·	0·	9·	9345·	13·	6·
2·	7395·	12·	6·	6·	6758·	2·	6·	10·	8594·	12·	6·
3·	9164·	7·	6·	7·	6185·	11·	0·	11·	5468·	3·	9·
4·	2943·	17·	6·	8·	3416·	18·	9·	12·	3547·	8·	9·

To find Interest for any number of years.

Multiply Principal by Rate, and by number of years, and Divide by 100·

The **Amount** is the sum of Principal and Interest.

To find Interest for any number of days.

For this purpose there is a neat little special rule, the origin and contrivance of which will be shewn by the

FOURTH EXAMPLE WORKED OUT. What is the Interest of £648· 12·s. 6·d. for 49· days, at £5· per cent. per annum?

$\because$ Int. of £100· for 365· days = £5·

$\therefore$ Int. of £1· for 365· days = £$\frac{5}{100}$

$\therefore$ Int. of £1· for 1· day = $\frac{\text{£}5\cdot}{36500}$

$\therefore$ Int. of £648· 12·s. 6·d. for 1· day $= \frac{\text{£}648\cdot\ 12\cdot\text{s.}\ 6\cdot\text{d.} \times 5\cdot}{36500\cdot}$

$\therefore$ Int. of £648· 12·s. 6·d. for 49· days $= \frac{\text{£}648\cdot\ 12\cdot\text{s.}\ 6\cdot\text{d.} \times 5\cdot \times 49\cdot}{36500\cdot}$ A

Now, this Denominator, or Divisor, (PRINCIPLE XV.) 36500·, is "*constant*," that is to say, it must occur in every such example.

And, 36500· is a convenient Divisor, inasmuch as it requires us to divide only by the number expressed by the three significant figures (365), the two ciphers merely causing the two right-hand figures of Dividend to be "cut off," or, as we more correctly say, removed two places to the right. (PRINCIPLE XXII.)

Twice 36500·, or 73000·, would, however, be a still more convenient Divisor, giving us only a number expressed by *two* significant figures (73) to divide by.

In order, then, to secure this more convenient Divisor, we double the constant Denominator 36500· in the above result A; and, therefore, also double the numerator; (PRINCIPLE XIII.) which last reduplication we effect by doubling the Rate.

$$\therefore \text{Int. of } \overset{\text{£.}}{648\cdot}\ \overset{\text{s.}}{12\cdot}\ \overset{\text{d.}}{6\cdot} \text{ for } \overset{\text{days}}{49\cdot} \text{ at } \overset{\text{£}}{5\cdot} \text{ p. c., pr. an.}$$

$$= \frac{\overset{\text{£.}}{648\cdot}\ \overset{\text{s.}}{12\cdot}\ \overset{\text{d.}}{6\cdot} \times 10\cdot \times 49\cdot}{73000\cdot} = \overset{\text{£.}}{4\cdot}\ \overset{\text{s.}}{0\cdot}\ \overset{\text{d.}}{7\tfrac{3}{4}}\ \overset{\text{q.}}{\tfrac{231}{365}}$$

To find Simple Interest for any number of days.

Multiply Principal by twice the Rate, and by the number of days, and divide by 73000·

EXERCISE CXXI. C.

Find Simple Interest and Amount of

1· £3416· 18·s. 6·d., 4½ years, £5· per cent., per annum.
2· £6005· 17·s, 6·d., 7¼ years, £3¼ p. c., pr. an.

3· £3484· 12·s. 6·d., 5$\frac{3}{4}$ years, £4$\frac{1}{8}$ p. c., pr. an.
4· £914· 5·s., 5· years and 2· months, £4$\frac{1}{3}$ p. c., pr. an.
5· £463· 10·s., 11· months, £7$\frac{1}{2}$ p. c., pr. an.
6· £500· 12·s. 6·d., 18· mo., £3$\frac{3}{8}$ p. c., pr. an.
7· £7864· 11·s. 9·d., 4· yrs. 5· mo., £4· 15·s. p. c., pr. an.
8· £3000·, 7· years, 8· months, £8· 12·s. 6·d. p. c., pr. an.
9· £1614· 10·s., from June 24th, 1863·, to April 11th, 1864·, at £4· per cent., per annum.*
10· £7368· 15·s., from Aug. 12th, 1856·, to Midsummer, 1860·, at £6· per cent., per annum.*
11· £634· 3·fl. 4·c. 9·m., for 100· days, at £3$\frac{7}{8}$ per cent., per annum.
12· £748· 3·c. for 219· days, at £5$\frac{1}{4}$ p. c., per an.
13· £5643· 8·fl. 5·m., from Sep. 30th to Dec. 12th, at £3$\frac{3}{4}$ per cent., per annum.*

COMPOUND INTEREST.

In calculating Compound Interest, the Amount, made up of Principal and Simple Interest, at the end of the first year,† becomes the Principal for the second year.† Simple Interest is calculated on this sum for the second year,† and the Amount at the end of that year,† is taken as Principal for the third year;† and so on.

Fifth Example worked out. Find Compound Interest and Amount of £648· 11·s. 8·d. for 3· years, at £5· per cent., per annum.

I. When Interest is paid yearly.
II. When Interest is paid half-yearly.

Example completed.

I.

INTEREST PAID YEARLY.

At £5· per cent., per annum,
1· year's Simple Interest = $\frac{1}{20}$ of Principal.

	£.	s.	d.	
20·)	648·	11·	8·	= Principal, 1st year.
	32·	8·	7·	= Interest, 1st year.
20·)	681·	0·	3·	= Principal, 2nd year.
	34·	1·	0$\frac{2}{4}$ $\frac{3}{8}$q.	= Interest, 2nd year.
20·)	715·	1·	3$\frac{2}{4}$ $\frac{3}{8}$	= Principal, 3rd year.
	35·	15·	0$\frac{3}{4}$ $\frac{3}{100}$	= Interest, 3rd year.
	750·	16·	3$\frac{3}{4}$ $\frac{63}{100}$	= *Amount* in 3· yrs. Comp. Int.
	648·	11·	8·	= Original Principal, to subtract.
	102·	4·	7$\frac{3}{4}$ $\frac{63}{100}$	= *Compound Interest*, in 3· yrs. an. payments.

Note.—At £5· per cent., per annum, Compound Interest, paid annually, any Principal will double itself in 14$\frac{1}{5}$ years, nearly.

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* Include the last day, but not the first. † Or other stipulated term.

II.

INTEREST PAID HALF-YEARLY.

At £5· per cent., per annum,
Half-a-year's Simple Interest = $\frac{1}{40}$ of Principal.

£.	s.	d.	
40·)648·	11·	8·	= Principal, 1st half-year.
16·	4·	$3\frac{1}{2}$	= Interest, 1st half-year.
40·)664·	15·	$11\frac{1}{2}$	= Principal, 2nd half-year.
16·	12·	$4\frac{3}{4}\ \frac{3}{20}$q.	= Interest, 2nd half-year.
40·)681·	8·	$4\frac{1}{4}\ \frac{3}{20}$	= Principal, 3rd half-year.
17·	0·	$8\frac{1}{2}\ \frac{23}{80}$	= Interest, 3rd half-year.
40·)698·	9·	$0\frac{3}{4}\ \frac{143}{800}$	= Principal, 4th half-year.
17·	9·	$2\frac{1}{2}\ \frac{28143}{32000}$	= Interest, 4th half-year.
40·)715·	18·	$3\frac{1}{2}\ \frac{1863}{32000}$	= Principal, 5th half-year.
17·	17·	$11\frac{1}{4}\ \frac{1217863}{1280000}$	= Interest, 5th half-year.
40·)733·	16·	$3\frac{0}{4}\ \frac{12383}{1280000}$	= Principal, 6th half-year.
18·	6·	$10\frac{3}{4}\ \frac{25612383}{51200000}$	= Interest, 6th half-year.
752·	3·	$1\frac{3}{4}\ \frac{26107703}{51200000}$	= Amount in 3· yrs. C. I. pd. half-yearly.
648·	11·	8·	= Original Principal, to deduct.
103·	11·	$5\frac{3}{4}\ \frac{26107703}{51200000}$	= Comp. Int. 3· yrs. hf.-yrly. payments.

COMPARISON OF COMPOUND INTEREST PAID HALF-YEARLY AND YEARLY.

£.	s.	d.	q.	
103·	11·	$5\frac{3}{4}$	$\frac{26107703}{51200000}$	= Comp. Int. paid yearly.
102·	4·	$7\frac{3}{4}$	$\frac{63}{100}$	= Comp. Int. paid half-yearly.
1·	6·	$9\frac{3}{4}$	$\frac{45051703}{51200000}$	= Increase, by half-yearly payments.

COMPARISON OF SIMPLE AND COMPOUND INTEREST.

£.	s.	d.	q.	
102·	4·	$7\frac{3}{4}$	$\frac{63}{100}$	= Compound Interest, Yearly payments.
97·	5·	9·		= Simple Interest, for the 3· years.
4·	18·	$10\frac{3}{4}$	$\frac{63}{100}$	= Excess of Comp. Interest, paid yearly.

£.	s.	d.	q.	
103·	11·	$5\frac{3}{4}$	$\frac{26107703}{51{,}200{,}000}$	= Comp. Interest, Half-yearly payments.
97·	5·	9·		= Simple Interest.
6·	5·	$8\frac{3}{4}$	$\frac{26107703}{51;200{,}000}$	= Excess of Comp. Int., paid half-yearly.

For other methods, see "Compound Interest by Decimals."

EXERCISE CXXII.

EXAMPLES IN COMPOUND INTEREST.

A

Find the Amount, at Compound Interest, of each of the following sums; the Interest being paid and made Principal yearly.

1· £350· for 2· years, at £5· per cent., per annum.
2· £400· for 3· years, at £4· per cent., per annum.
3· £564· for 4· years, at £2· per cent., per annum.
4· £945· for 4· years, at £1· per cent., per annum.
5· £763· for 5· years, at £3½ per cent., per annum.
6· £695· 10·s. for 5· years, at £3· per cent., per annum.
7· £49· 15·s. for 4· years, at £2½ per cent., per annum.
8· £685· 2·s. 6·d. for 4½ years, at £6· per cent., per annum.
9· £836· 17·s. 6·d. for 3¾ years, at £7½ per cent., per annum.
10· £1000· for 7· years, at £5· per cent., per annum.

B

1· What would £1300· amount to, in 3· years, at £4· per cent., per an., Compound Interest, paid half-yearly?
2· Find amount of £15000·, in one year, at £10· per cent, per an., Comp. Int., paid quarterly.
3· Required the amount of £1000· at the end of 3½ years, at £4· per cent., per an., Comp. Int., paid half-yearly.
4· Find the Compound Interest of £7000· for three years, half-yearly payments, at £10· per cent., per an.
5· What would £3000· amount to in 3· years, at £5· per cent., per annum, Compound Interest, computing
I. Annual payments?
II. Half-yearly payments?
III. Quarterly payments?

SIMPLE INTEREST BY DECIMALS.

Hundredth parts being Decimal Fractions, per centages can be conveniently worked out by decimal methods.

For, Interest of £1· for 1· year,

£	£	£	£	£	£
At 1· per cent. =	·01	At ½ p. c. =	·005	At 1½ p. c. =	·015
At 2· per cent. =	·02	At ¼ p. c. =	·0025	At 1¼ p. c. =	·0125
At 3· per cent. =	·003	At ¾ p. c. =	·0075	At 3¾ p. c. =	·0375
At 4· per cent. =	·004	At ⅛ p. c. =	·00125	At 5⅛ p. c. =	·05125
&c.	&c.	&c.	&c.	&c.	&c.

To express the Simple Interest of £1· for one year, in the Decimal form.

Divide Rate by 100·

This is most conveniently done by first reducing Rate to the decimal form, and then removing every digit two places to the right. PRIN. XXI.

SIXTH EXAMPLE WORKED OUT. Express, in the decimal form, the Interest of £1· for 1· year at £⅝, at £2⅜, and at £6¼ per cent. per annum.

I.

At £⅝ per cent. per annum.

Rate, or Int. of £100· for 1· year = £⅝ = £·625
Int. of £1· for 1· year = $\frac{1}{100}$ of Rate = £·00625

II.

At £2⅜ per cent., per annum.

Rate, or Int. of £100· for 1· year = £2⅜ = £2·375
Int. of £1· for 1· year = 1/100 of Rate = £·02375

III.

At £6¼ per cent., per annum.

Rate, or Int. of £100· for 1· year = £6¼ = £6·25
Int. of £1· for 1· year = 1/100 of Rate = £·0625

To find, by Decimals, the Simple Interest of any Principal, for any number of years, at any Rate per cent., per annum.

First. Express, decimally, Interest of £1· for 1· year at given Rate.

Second. Multiply by No. of years, and by No. of pounds in Principal.

The student will notice that the main difference between this method of finding Interest, and the former one is, that here dividing by 100· is the first step in the process, and there it is the last.

Seventh Example worked out. Calculate the Simple Interest and Amount of £3428· 15·s. for 4½ years at £3¼ per cent. per annum.

Example completed.

I.

£.	
·0375	= Int. of £1· for 1· year, at £3¾ p. c. pr. an.
4·5	= No. of years.
·01875	= Int. of £1· for ½ year.
·15	= Int. of £1· for 4· years.
·16875	= Int. of £1· for 4½ years.
3428·5	= No. of £. in Principal.
·084375	= Int. of 10·s. or £·5 for 4½ years.
1·35	= Int. of 8· for 4½ years.
3·375	= Int. of 20· for 4½ years.
67·5	= Int. of 400· for 4½ years.
506·25	= Int. of 3000· for 4½ years.
£578·559375	= Int. of £3428·5 for 4½ years.

20·
s. 11·1875
12·
d. 2·25
4·
q. 1·

	£.	s.	d.
Interest of 3428· (£.) 10· (s.) for 4½ (years) at 3¾ (£.) p. c., pr. an. =	578·	11·	2¼
Principal =	3428·	10·	0·
Amount =	4007·	1·	2¼

II.

Or thus: by way of proof.

£. £. s. yr. £.
34·285 = Int. of 3428· 10· for 1· at 1· p. c., pr. an.
9·
yrs. £.
2·)308·565 = Int. of 3428· 10· for 9· at 1· p. c., pr. an.
4·)154·2825 = Int. of 3428· 10· for 4½ at 1· p. c., pr. an.
4·
617·13 = Int. of 3428· 10· for 4½ at 4· p. c., pr. an.
38·570625 = Int. of 3428· 10· for 4½ at ¼ p. c., pr. an.
£578·559375 = Int. of 3428· 10· for 4½ at 3¾ p. c., pr. an.

EXERCISE CXXIII.

Find, by the Decimal method, the Simple Interest of

A

	£.	s.	d.							
1.	943·	10·	0·	for	7· years,	at £5·	per	cent.,	per	annum.
2·	6148·	5·	0·	„	4½ yrs,	at £2½	„	„	„	„
3·	9014·	7·	6·	„	3¼ yrs,	at £1¼	„	„	„	„
4·	6359·	12·	6·	„	2¼ yrs,	at £3¾	„	„	„	„
5·	763·	15·	0·	„	5· yrs,	at £7½	„	„	„	„
6·	548·	2·	6·	„	6· mo.	at £2·	„	„	„	„

B.

	£.	fl.	c.	m.							
1·	345·	6·	9·	8·	for	14· years,	at £4·	per	cent.,	per	annum.
2·	1432·	7·	8·	3·	„	10· years,	at £3·	„	„	„	„
3·	769·	3·	0·	2·	„	9· years,	at £2¼	„	„	„	„
4·	64·	0·	1·	5·	„	7¼ years,	at £1¾	„	„	„	„
5·	305·	1·	3·	5·	„	9· mo.	at £5⅛	„	„	„	„

COMPOUND INTEREST BY DECIMALS.

If the Rate be £5· per cent., per annum, then £·05 will be the *Simple Interest* of £1· for the first year, and £1·05 will be the *Amount* of £1· at the end of that year: = £1· × 1· 05.

And this Amount, (£1·05,) being taken as Principal for the 2nd year, the *Simple Interest* on it will be ·05 of £1·05 = £1·05 × ·05 (PRINCIPLE IX.)

And the *Amount* of £1·05 at the end of the 2nd year will be 1·05 of £1·05 = £1·05 × 1·05 = $1{\cdot}05^2$ see INVOLUTION, Page 53·

Hence, Amount, at Compound Interest, £5· per cent., per annum, of

£1· at end of 1st year = £1· × 1·05 = £1·05 = P. for 2nd yr.
£1· at end of 2nd year = £1·05 × 1·05 = £$1{\cdot}05^2$ = P. for 3rd yr.
£1· at end of 3rd year = £$1{\cdot}05^2$ × 1·05 = £$1{\cdot}05^3$ = P. for 4th yr.
£1· at end of 4th year = £$1{\cdot}05^3$ × 1·05 = £$1{\cdot}05^4$ = P· for 5th yr.
&c. &c. &c.

Similarly, Amount of £1·, Compound Interest, at end of

	At £1· p. c.	At £2· p. c.	At £3½ p. c.	At £4¾ p. c.
1st year,	£1·01	£1·02	£1·035	£1·0475
2nd year,	£$1{\cdot}01^2$	£$1{\cdot}02^2$	£$1{\cdot}035^2$	£$1{\cdot}0475^2$
3rd year,	£$1{\cdot}01^3$	£$1{\cdot}02^3$	£$1{\cdot}035^3$	£$1{\cdot}0475^3$
4th year,	£$1{\cdot}01^4$	£$1{\cdot}02^4$	£$1{\cdot}035^4$	£$1{\cdot}0475^4$
5th year,	£$1{\cdot}01^5$	£$1{\cdot}02^5$	£$1{\cdot}035^5$	£$1{\cdot}0475^5$
&c.	&c.	&c.	&c.	&c.

To find Amount of £1· for any no. of years, at Compound Interest.

First. Express, in the Decimal form, the Amount of £1· for 1· year at given Rate.

Second. Raise this Amount to that Power of which the Index is equal to the number of years. See INVOLUTION; Page 53·

Then, ∵ Amount of £1· for 5· years at £4· p. c. = $1{\cdot}04^5$
Amount of £2· „ „ „ „ „ = $1{\cdot}04^5 \times 2{\cdot}$
Amount of £3· „ „ „ „ „ = $1{\cdot}04^5 \times 3{\cdot}$
&c. &c &c.

To find Amount of any Principal for any no. of years, at Compound Interest.

First. Find the Amount of £1· (as above) for given no. of years, at given Rate.

Second. Multiply this Amount by no. of £. in Principal.

If P represent Principal; A the Amount; R the Interest of £1· for one year; and n, the no. of years; then this rule will be concisely expressed thus:

$$A = P \times (1 + R)^n$$

EXERCISE CXXIV.

Find, by the Decimal method, the Amount, at Compound Interest, of each of the following Sums.

A.

Annual payments.

£. s. d. £.
1· 450· 12· 6· for 4· years, at 5· per cent. per annum.
2· 3645· 7· 6· for 3· years, at 4· per cent. per annum.
3· 7365· 10· 0· for 5· years, at 2½ per cent. per annum.
4· 415· 2· 6· for 4· years, at 3· per cent. per annum.
5· 965· 15· 0· for 2· years, at 3½ per cent. per annum.
6· 6148· 17· 6· for 5· years, at 4½ per cent. per annum.
7· 8416· 5· 0· for 6· years, at 7· per cent. per annum.
8· 5418· 0· 0· for 7· years, at 6· per cent. per annum.

B.

1· £1500· for 3· years, at £5· p. c., pr. an. half yearly payments.
Here will be six payments, at £2½ per cent.
Amount $= 1{\cdot}025^6 \times 1500{\cdot}$

2· £7648· for 2½ years, at £4½ p. c., pr. an., quarterly payments.
3· £4184· at £3· pr. cent., pr. an., half-yearly, for 4· years.
4· £8413· at £2· pr. cent., pr. an., paid quarterly, for 3· years.
5· £3216· for 4½ years, half-yearly, at £2½ pr. cent., pr. annum.

DISCOUNT.

Discount is an allowance made for paying a debt before it is due.

Present Worth is that which remains to be paid after deducting Discount from the debt,

Discount = Interest on Present Worth.

Thus; If £105· be due one year hence, and Interest be at the rate of £5· per cent., per annum, present worth of £105· would be £100·, and Discount £5· For, £100· in one year, would gain £5·, and would thus amount, at the year's end, to £105·, which is the sum then due, and for which, therefore, a present payment of £100· is equivalent.

A Bill of Exchange or Promissory Note is a written engagement to pay the sum of money named thereon, either "on demand," or at a stated time "after date," or (if sent from abroad) "after sight" of the document.

A great part of Banking business consists in "discounting" such drafts; that is to say, advancing cash on them before they fall due.

Three "*days of grace*" are added to the time of a Bill of Exchange or Promissory Note. So that, for example, one dated June 2nd, and drawn for one month, would not be presented for redemption until July 5th, unless that day should be Sunday, in which case the bill would fall due on the Saturday, one of the "days of grace" being thus lost.

When Bankers advance cash on Bills of Exchange or Promissory Notes, it is their custom to deduct the *Interest* of the debt, instead of the true *Discount*. This practice is to the advantage of the Bankers.

For, supposing the Bill or Note to promise payment of £405· at the end of 3· months, Interest being at £5· per cent., per annum. Then, Discount on £405· = £5· and Present Worth = £400· For £400· in 3· months, would gain the £5· necessary to make up £405· at the end of that time. But the Banker would withhold

		£.	s.	d.
Interest on £405· for 3· months	=	5·	1·	3·
Instead of Disct. on £405· for 3· months	=	5·	0·	0·
Gain to Banker	=		1·	3·

METHOD OF FINDING TRUE DISCOUNT.

We shall deduce the rule for finding true Discount from the following two

Examples worked out. Find the true Discount, and the Present Worth of

I. £476· due one year hence; Int. at £5· pr. c., pr. an.
II. £1000· due 4· months hence, at £3½ p. c., pr. an.

Examples completed.

I.

At £5· p. c., pr. an., £100· in 1· year, will amount to £105·

∴ Discount on £105· due 1· year hence = £5·
Discount on £1· ,, ,, ,, ,, $= \frac{\pounds 5}{105}$
Discount on £476· ,, ,, ,, ,, $= \frac{\pounds 5\cdot \times 476\cdot}{105\cdot}$
= £22· 13·s. 4·d.

Present Worth = £476· — £22· 13·s. 4·d. = £453· 6·s. 8·d.

II.

At £3½ p. c., pr. an., £100· in 4· months, will amount to £101⅙

∴ Discount on £101⅙, due 4· mo. hence, = £1⅙
Discount on £1· ,, ,, ,, ,, $\frac{\pounds 1\frac{1}{6}}{101\frac{1}{6}} = \frac{\pounds 7}{607}$
Discount on £1000· ,, ,, ,, ,, $= \frac{\pounds 7\cdot \times 1000\cdot}{607\cdot}$
= £11· 10·s. 7½d. $\frac{510}{607}$q.

Present Worth = £1000· — Discount = £988· 10·s. 4¼d. $\frac{97}{607}$q.

Here, we note that

I. Interest on £100· = Discount on Amount of £100·
II. Discount on £1· ≐ Int. on £100· ÷ Amt. of £100·

To find true Discount on any debt.

First. Find Interest and Amount of £100· for given time, at given rate.

Second. Divide the first of these by the second.
This will give Discount on £1·

Third. Multiply by number of £. in the debt.

To find Present Worth.

Subtract Discount from debt.

The above rules are usually given as two Proportions: thus:

For the Discount, say

As (*Amt. of* £100·) : (*Int.* of £100·) :: (*Debt*) : (*Its Disct.*)

For the Present Worth, say

As (*Amt. of* £100·) : (£100·) :: (*Debt*) : (*Its Present Worth.*)

EXERCISE CXXV.

A.

Find Discount and present Worth of

£ s. d. £
1· 247· 12· 6· due 2· years hence, at 2½ p. c., pr. an.
2· 633· 9· 3· „ 6· months „ at 4· „ „ „ „
3· 1835· 6· 7· „ 3· months „ at 3· „ „ „ „
4· 1879· 5· 7½ „ 2· months „ at 5· „ „ „ „
5· 1108· 18· 2¼ „ 5· years 11· mths. „ at 5· „ „ „ „
6· 514· 12· 10¾ „ 2· years 5· mths. „ at 4½ „ „ „ „
7· 77· 12· 10· „ 8· months „ at 4· „ „ „ „
8· 692· 16· 3· „ 75· days „ at 6· „ „ „ „
9· 373· 17· 11¼ „ 9· months „ at 3¼ „ „ „ „
10· 350· 12· 6· due Feb. 15th, 1865·, but paid April 22nd, 1864·, at £3· per cent., per annum.

B.

Ascertain by how much the Simple Interest exceeds the Discount on

1· £495· 5·s., due at the end of 2½ years, at £2· p. c., pr. an.
2· £1254· 10·s. 6·d., due 3· mo. hence, at £4· p. c., pr. an.
3· £924· 9·s. 11·d., paid at Midsummer, instead of at the following Christmas, at £3· p. c., pr. an.
4· £3743· 0·s. 7½d., paid one month before due, at £5· p. c., pr. an.
5· £1964· 13·s. 2¼d. prepaid 5· yrs. 11· mo., at £2½ p. c., pr. an.
6· £232· 18·s. 6·d., due 8· mo. hence, at £4· p. c., pr. an.
7· £978· 16·s. 2¾d., due Lady Day, 1866·, but paid Christmas, 1863·, at £2· 8·s. 4·d. p. c., pr. an.
8· Find by how much the Banker's Discount exceeds the true Discount on a bill of exchange, for £806· 8·s. payable 4· months after date, drawn March 1st, and cashed April 22nd, reckoning Interest at £4· per cent., per annum.
9· Find the same, on a bill for £4044· payable 3· months after date, drawn Feb. 5th, 1864·, and discounted Feb. 24th. Interest at £5½ per cent., per annum.

STOCKS, OR THE FUNDS.

The British Government has, from time to time, during the last 200· years, borrowed of the people large sums of money,* which still remain unpaid, and constitute "THE

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* The English National Debt dates no further back than 1688· At the commencement of the Revolution the public debt was less than one million; during the reign of William III. it was augmented to £15;000,000· When George the First came to the throne, it had reached £54;000,000· He and his successor reduced the debt, but at the outset of the American war in 1775·

NATIONAL DEBT," usually called "THE FUNDS," or "THE STOCKS:" of which the Interest is regularly paid by the Government, and absorbs a large portion of the taxes.*

The original lenders of much of this debt being long since dead, their rights, as creditors of the Government, have, either by inheritance or by purchase, passed into the hands of other persons. For, a creditor of the nation who desires to recover his capital can only do so by selling his right to it. The purchaser then assumes the seller's position as a public creditor to that amount.

The "Price of the Funds" at any given time, is the sum then obtainable for the right to £100· of the National Debt. This price, from many causes, is continually fluctuating.†

The value of the Funds, at any moment, depends principally on the Rate of Interest which there is then a prospect of obtaining by other investments or employments of money.

Suppose, for example, £3· per cent., per annum, were the highest Interest that a capitalist could obtain by lending his money for employment in any commercial enterprise. Then, since Interest measures the value of money, and, since £100· of the "Three per cent., Consols" would, at such a time, be as productive of Interest as £100· elsewhere employed, £100· of that kind of Stock would be worth £100· in cash.‡ At such a time, "*The Funds are at Par.*"

If, however, not so much Interest could be obtained elsewhere, as on the N. D. then, £100· of that debt, being productive of more Interest than £100· otherwise employed, would be worth more than £100· in money. "*The Funds would be at a Premium.*"

These two cases very rarely occur.

When, as is most frequently the case, more Interest than is paid on the N. D. can be obtained elsewhere, then, £100· of the N. D., not producing so much Interest as £100· otherwise invested, are worth less than £100· in cash. "*The Funds are at a Discount.*"

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the public debt was upwards of £128;000,000· At the end of that war, £249;000,000· The great war with France added £601;000,000 to the amount. But the highest point the National Debt has ever reached was on the 1st of February, 1817· when it arrived at the sum of £865;000,000· It now stands at about £800;000,000·

* The subjoined table of public expenditure is interesting and suggestive.

	1845·	1850·	1855·	1860·
	£	£	£	£
Interest on debt	28;253,872·	28;091,590·	27;647,899·	26;200,000·
Army..........	8;854,296·	8;955,061·	29;377,349·	15;800,000·
Navy..........	6;809,873·	6;437,883·	19;014,708·	13;900,000·
Civil charges ..	5;324,672·	6;747,340·	8;465,832·	9;500,000·

† The lowest point to which Consols have fallen was £47⅜, in 1797· The highest quotation was £107·, in the year 1737·

‡ *Or a little more, on account of the better security.*

Now, it is the prospect or presence of war, scarcity, or other calamity, that makes capitalists unwilling to risk their money in commercial enterprise, unless induced to do so by a high Rate of Interest. Hence, (as that paid on the N. D. does not vary,) when times or prospects are bad, *The price of the Funds declines.*

On the other hand, when public affairs and private business are in a prosperous or a hopeful condition, money is more freely employed, the general Rate of Interest therefore declines, and *The price of the Funds advances.*

It is now seen how good news elevates The Funds, and bad news depresses them.

And, hence, the price of The Funds has become a sort of commercial pulse, which is watched and studied as an index of the feeling and opinion of men of business as to public events and prospects.

Calculations in Stocks require no special rule, being effected by the application of processes already elucidated.*

It is to be noted that the Broker's Commission for buying or selling is £⅛ or 2·s. 6·d. per cent. on the *nominal* amount of stock bought or sold. In all the following questions this Commission will be included in the quoted price.

FIRST EXAMPLE WORKED OUT. What amount will purchase £4500· in the £3· per cents., at £92·?

Example completed.

I.

$$\text{Cost of a nominal } \pounds 100\cdot = \pounds 92\cdot$$

$$\text{Cost of ,, ,, } \pounds 1\cdot = \frac{\pounds 92\cdot}{100\cdot}$$

$$\text{Cost of ,, ,, } \pounds 4500\cdot = \frac{\pounds 92\cdot \times 4500\cdot}{100\cdot} = \overset{\pounds}{4140}\cdot$$

II.

Or thus, by a Simple Proportion.

Abbreviated Question.

If £100· (nominal)	. . . .	£92· (money.)
4500· ,,	. . . .	?

Statement and Result.

As £100· (stock) : £4500· (stock) :: £92· (money) : £4140· (money)

SECOND EXAMPLE WORKED OUT. What nominal amount in the £4· per cents., at £93·, can be purchased for £3534· in money?

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* See, however, page 370·

Example completed.

I.

$$\text{Nominal Amount bought for } \overset{£}{93\cdot}\text{ (money)} = \overset{£}{100\cdot}$$

$$\text{,,} \quad \text{,,} \quad \text{,,} \quad \text{for £1}\cdot \quad \text{,,} \quad = \frac{100}{93}$$

$$\text{,,} \quad \text{,,} \quad \text{,,} \quad \text{for £3534}\cdot \text{ ,,} \quad = \frac{100\cdot \times 3534\cdot}{93\cdot} = \overset{£}{3800\cdot}$$

II.

Or thus, by a Simple Proportion.

Abbreviated Question.

If $\overset{£}{93\cdot}$ (money) . . . . . . $\overset{£}{100\cdot}$ (stock)
3534· ,, ... ?

Statement and Result.

As $\overset{£}{93\cdot}$ (money) : $\overset{£}{3534\cdot}$ (money) :: $\overset{£}{100\cdot}$ (stock) : $\overset{£}{3800\cdot}$ (stock)

THIRD EXAMPLE WORKED OUT. What is the price, per cent., of that Stock of which £1050· are worth £1008· in cash?

Example completed.

I.

$$\text{Price of } \overset{£}{1050\cdot}\text{ (stock)} = \overset{£}{1008\cdot}\text{ (money)}$$

$$\text{Price of} \quad £1\cdot \quad \text{,,} \quad = \frac{£1008}{1050} \quad \text{,,}$$

$$\text{Price of } £100\cdot \quad \text{,,} \quad = \frac{£1008\cdot \times 100\cdot}{1050\cdot} = \overset{£}{96\cdot}$$

II.

Or thus, by a Simple Proportion.

Abbreviated Question.

If $\overset{£}{1050\cdot}$ (stock) .... $\overset{£}{1008\cdot}$ (cash)
100· (stock) .... ?

Statement and Result.

As $\overset{£}{1050\cdot}$ (stock) : $\overset{£}{100\cdot}$ (stock) : : $\overset{£}{1008\cdot}$ (cash) : $\overset{£}{96\cdot}$ (cash)

FOURTH EXAMPLE WORKED OUT. What annual income accrues from £3000· invested in the £3· per cents., at *£75·?*

Example completed.

I.

∵ Every £75· of the £3000· will buy a nominal £100· of stock.
∴ From every £75· invested, the annual income is £3·
From every £1· invested, ,, ,, ,, $\frac{\pounds 3}{75}$
From £3000· invested, ,, ,, ,, $\frac{\pounds 3\cdot \times 3000\cdot}{75\cdot} = \overset{\pounds}{120}\cdot$

II.

Or thus, by a Simple Proportion.

Abbreviated Question.

If $\overset{\pounds}{75}\cdot$ (invested) .... $\overset{\pounds}{3}\cdot$ (annual income)
3000· (invested) .... ?

Statement and Result.

As $\overset{\pounds}{75}\cdot$ (inv.) : $\overset{\pounds}{3000}\cdot$ (inv.) :: $\overset{\pounds}{3}\cdot$ (ann. inc.) : $\overset{\pounds}{120}\cdot$ (ann. inc.)

III.

Or thus, mentally.

Annual Income = As many times £3· as there are seventy-fives in 3000· (or twenty-fives in 1000·)
= 40· times £3·
= £120·

Fifth Example worked out. If I buy into the £3½ per cents., at £87½ nett, what rate of Interest shall I receive on the money which I invest?

Example completed.

I.

Interest on £87½ invested = £3½

Interest on £1· invested = $\frac{\pounds 3\frac{1}{2}}{87\frac{1}{2}} = \frac{\pounds 7}{175}$

Interest per cent. on invest. = $\frac{\pounds 7\cdot}{175} \times 100\cdot = \pounds 4\cdot$ per cent.

II.

Or thus, by a Simple Proportion.

Abbreviated Question.

If $\overset{\pounds}{87\frac{1}{2}}$ (Principal) produce $\overset{\pounds}{3\frac{1}{2}}$ (ann. Int.)
100· .... ?

Statement and Result.

$$\text{As } \overset{£}{87\tfrac{1}{2}} \text{ (Prin. inv.)} : \overset{£}{100\cdot} \text{ (Prin. inv.)} :: \overset{£}{3\tfrac{1}{2}} \text{ (an. Int.)} : \overset{£}{4\cdot} \text{ (an. Int.)}$$

PROOF.

$$\left.\begin{array}{l}\text{An. Int. of £}87\tfrac{1}{2}\text{ at £}4\cdot\\ \quad\text{per cent., per annum.}\end{array}\right\} = \overset{£}{87\cdot5} \times \cdot04 = \overset{£}{3\cdot75} \text{ or } \overset{£}{3\tfrac{3}{4}}.$$

Sixth Example worked out. Supposing the current price of any species of Stock to depend solely on the Rate of Interest obtainable at the same time elsewhere, what would the £3¾ per cents be worth, if the general Interest of money were £5· per cent. per annum?

The question amounts to this: "What Principal will produce £3¾ annual Interest, at £5· per cent. per annum?"

Example completed.

I.

Prin. to gain £5· Annual Interest = £100·
„ £1· „ „ = £100· ÷ 5· = £20·
„ £3¾ „ „ = £ 20· × 3¾ = £75·

II.

Or thus, by a Simple Proportion.

Abbreviated Question.

$$\left.\begin{array}{l}\text{If } \overset{£}{5\cdot} \text{ (an. Int.)}\\ \quad 3\tfrac{3}{4} \text{ (an. Int.)}\end{array}\right\} \begin{array}{l}\text{.................. accrue from } \overset{£}{100\cdot} \text{ (Prin.)}\\ \text{..................} \qquad\qquad\quad ? \qquad „\end{array}$$

Statement and Result.

$$\text{As } \overset{£}{5\cdot} \text{ (an. Int.)} : \overset{£}{3\tfrac{3}{4}} \text{ (an. Int.)} :: \overset{£}{100\cdot} \text{ (Prin.)} : \overset{£}{75\cdot} \text{ (Prin.)}$$

PROOF.

$$\left.\begin{array}{l}\text{Annual Interest on £}75\cdot\\ \quad\text{at £}5\cdot\text{ per cent., per an.}\end{array}\right\} = \overset{£}{75\cdot} \times \cdot05 = \overset{£}{3\cdot75} = \overset{£}{3\tfrac{3}{4}}$$

Seventh Example worked out. Which would be the more profitable investment, the £3½ per cents., at £87½, or the £4· per cents., at £92½?

The question is: "Will a certain Principal produce the greater Interest at £3½ for every £87½, or at £4· for every £92½?"

Example completed.

By the first investment:

$$\text{Interest on every £}87\tfrac{1}{2} = \overset{£}{3\tfrac{1}{2}}$$

$$\text{Interest on every £}1\cdot = \frac{£3\frac{1}{2}}{78\frac{1}{2}} = \frac{£7}{175}$$

By the second investment:

$$\text{Interest on every £}92\tfrac{1}{2} = \overset{£}{4\cdot}$$

$$\text{Interest on every £}1\cdot = \frac{£4}{92\frac{1}{2}} = \frac{£8}{185}$$

We have now to compare the two Fractions $\frac{7}{175}$ and $\frac{8}{185}$

$$\frac{7}{175}=\frac{7\times185}{175\times185}=\frac{1295}{175\times185}; \text{ and } \frac{8}{185}=\frac{8\times175}{185\times175}=\frac{1400}{185\times175}$$

Since, then, the latter of the two Fractions is greater than the former, the second investment is the more profitable.

Eighth Example worked out. What nominal amount in the £4· per cents., at £95· can be purchased by the proceeds of sale of £10000· of the £3¾ per cents., at £90·?

Example completed.

Every £100· sold produces £90·
Every £100· bought costs £95·

$$\text{Proceeds of Sale} = \frac{£10000\cdot\times90\cdot}{100\cdot}$$

$$\text{No. of times £100$\cdot$ bought} = \frac{£10000\cdot\times90\cdot}{100\cdot\times95\cdot}$$

$$\text{Nominal amount bought} = \frac{£10000\cdot\times90\cdot\times100\cdot}{100\cdot\times95\cdot}$$

$$=\frac{£10000\cdot\times90\cdot}{95\cdot}=\overset{£}{9473}\cdot\overset{s.}{13}\cdot\overset{d.}{8\tfrac{4}{19}}$$

The same Example as an Inverse Proportion. If a certain sum of money will buy a nominal £10000· at £90· for every £100·, how much will the same amount buy, at £95· for every £100·?

If, at 90· p. c. . . . £10,000· are bought,
at 95· p. c. . . . ?

Statement and Result.

$$\text{As } \overset{\pounds}{95\cdot}\ (\text{per } 100\cdot) : \overset{\pounds}{90\cdot}\ (\text{per } 100\cdot) :: \overset{\pounds}{10{,}000\cdot} : \frac{10000\cdot \times 90\cdot}{95\cdot}$$

The foregoing Examples comprise the principal varieties of question arising in the purchase and sale of Stocks; and, from inspection of these, short rules may be deduced applicable to similar cases.

I. *To find what Capital will purchase any nominal Stock.*

Multiply Stock by Price, and Divide by 100· Ex. I.

II. *To find what nominal Stock any Capital will purchase.*

Multiply Capital by 100·, and Divide by Price. Ex. II.

III. *To find the price of that Stock of which a given Capital will purchase a stated amount.*

Multiply Capital by 100·, and Divide by Stock. Ex. III.

IV. *To find gross annual income from any Capital invested, at a stated price, in Stock producing a certain annual Interest.*

Multiply Capital by Interest, and Divide by Price. Ex. IV.

V. *To find Rate of Interest obtained on Capital invested in Stock producing a certain annual Interest.*

Multiply Interest per cent. of Stock by 100· and Divide by Price. Ex. V.

VI. *To find the natural value of any Stock; that is to say, its value compared with other investments, producing a stated Interest.*

Multiply Interest of Stock by 100· and Divide by stated Interest of other investments. Ex. VI.

VII. *To find which of two or more kinds of Stock is the more profitable Investment.*

Divide Interest of each kind of Stock by its Price. Compare the results (as Fractions). Ex. VII.

VIII. *To find what amount of one kind of Stock can be purchased by the proceeds of sale of another Stock.*

Multiply Stock sold by its Price and Divide by Price of Stock bought. Ex. VIII.

EXERCISE CXXVI.

A.

What amount of Cash will purchase

1· £1500· in 3· p. cents, at 92· ?
2· £1600· „ 3· p. cents, at 89· ?
3· £2800· „ 3½ p. cents, at 75· ?
4· £3100· „ 3½ p. cents, at 85· ?
5· £1280· „ 4· p. cents, at 86· ?
6· £1900· „ 4· p. cents, at 98· ?
7· £4220· „ 4½ p. cents, at 92· ?
8· £2100· „ 4½ p. cents, at 84· ?
9· £1050· in 3· p. cents, at 96· ?
10· £2900· „ 4· p. cents, at 90· ?
11· £8250· „ 3½ p. cents, at 92½?
12· £4200· „ 4½ p. cents, at 87¼?
13· £2025· „ 4· p. cents, at 90¾?
14· £1500· „ 3½ p. cents, at 75½?
15· £4125· „ 3· p. cents, at 82⅛?
16· £6000· „ 4½ p. cents, at 90⅜?

B.

What nominal amount of Stock can be bought

1· For £8302· 8·s. at 86· ?
2· For £53418· 7·fl. 5·c., at £92· 5·fl.?
3· For £8474· 1·s. 3·d., at £81⅞?
4· For £1355· 6·fl. 2·c. 5·m., at £90· 7·s. 6·d.?
5· For £1837· 14·s. 6·d., at £91·6?
6· For £3397·5, at £75· 5·fl.?
7· For £3882· 4·fl., at £92· ?
8· For £3664· 5·fl., at £87· 2·fl. 5·c.?
9· For £1767· at £93· ?
10· For £11088· at £96· ?
11· For £870· at £90· ?
12· For £4419·1875, at £79·625?

C.

1· What is the price of that Stock, of which £3100· cost £2635·?

2· What annual income will arise from the investment of £7631· 5·s. in the £3½ per cents., at £92½?

3· What rate of Interest will accrue on Capital invested in the £4· per cents., at £90·?

4· When the general rate of Interest on loans is £5· per cent., per annum, what is the natural price of the £3· per cents., Government securities?

5· If the natural price of the three-and-a-half per cents. be £116⅔, what is the current Interest of money?

6· How much Capital must be invested in the £4½ per cents., at £84·, to produce an annual income of £94· 10·s.?

7· At what price of the three-and-a-halfs will £1132· 10·s. invested in them produce £52· 10·s. per annum?

8· What is the rate of Interest on that stock by investing in which, at £87¼, £3664· 10·s. will produce £189· per annum?

9· Stock purchased at £75· made an annual return of £4· 13·s. 4·d. on the Capital invested. What rate of Interest did that Stock bear?

10· The general rate of Interest being £6¼, the natural price of a certain Stock is £72· What Interest is borne by that Stock?

11· What rate of Interest is obtained on Capital by investing it in the £3· per cents. at £96·?

12· Whether of the two is the more profitable investment, the three-and-a-halfs, at £75·, or the four per cents., at £90·, and by how much per cent. on the Capital invested?

13· Would the £3½ per cents, at £92¼, or the £3· per cents, at £89·, be the more profitable investment; and by how much per cent. on Capital?

14· Sold out £2900· of the £4· per cents. at £92·, and invested the proceeds in the £4½ per cents, at £96· What nominal amount of the latter stock was purchased?

15· What alteration in annual income would result from this transfer?

16· How much Capital will purchase a sufficient amount of the £3½ per cents at £85· to produce an annual return of £108· 10·s.?

17· What rate per cent. would that be on the Capital invested?

FELLOWSHIP, OR PARTNERSHIP.

Fellowship or Partnership includes all questions requiring the distribution of a profit, a loss, or an additional outlay among partners, in direct proportion to the respective amounts of Capital invested by each of them in the undertaking whence the said profit, loss, or necessity for further expenditure, arises. It is divided into two parts:

I. **Single** or **Simple Fellowship**, or **Fellowship without time**; where all the partners are concerned for the same length of time.

II. **Double**, or **Compound Fellowship**, or **Fellowship with time**; where the several partners are associated, and their respective capitals employed, for different lengths of time.

It is evident that all Compound Fellowship questions may be converted into Simple Fellowship ones, by the parties closing their accounts and starting anew whenever a partner enters or quits the concern. And this would be the more natural mode of proceeding, and hence the more likely one to be adopted in actual business.

Questions in Simple Fellowship merely require the distribution of a stated sum into proportional parts, a process already fully explained on pages 158· and 159·

FIRST EXAMPLE WORKED OUT. SIMPLE FELLOWSHIP. Four persons associate to work a quarry; A contributing towards the preliminary expenses £150·, B £200·, C £80· and D £70· These sums being disbursed, it is agreed to expend £2000· for machinery. How much should each partner pay?

Example completed.

Original Capital = £500·, of which the portion contributed

£	£ £
By A = 150· or $\frac{3}{10}$	∴ A must pay $\frac{3}{10}$ of the 2000· or 600·
By B = 200· or $\frac{2}{5}$	B „ „ $\frac{2}{5}$ of „ 2000· or 800·
By C = 80· or $\frac{4}{25}$	C „ „ $\frac{4}{25}$ of „ 2000· or 320·
By D = 70· or $\frac{7}{50}$	D „ „ $\frac{7}{50}$ of „ 2000· or 280·
500·	2000·

We might also say, if we preferred to work by "The Rule of Three,"

$$\text{As}\left\{\begin{matrix}\text{Whole}\\ \text{Capital}\end{matrix}\right\} : \left\{\begin{matrix}\text{Each Part-}\\ \text{ner's Cap.}\end{matrix}\right\} :: \left\{\begin{matrix}\text{Whole Profit}\\ \text{Loss, or Out.}\end{matrix}\right\} : \left\{\begin{matrix}\text{That Part.'s}\\ \text{P. or L. or O.}\end{matrix}\right\}$$

In this example, then, the statements of Proportion would be

	£	£	£		£
1st.	As 500·	: 150·	:: 2000·	: A's new contribn.	or 600·
2nd.	As 500·	: 200·	:: 2000·	: B's „ „	or 800·
3rd.	As 500·	: 80·	:: 2000·	: C's „ „	or 320·
4th.	As 500·	: 70·	:: 2000·	: D's „ „	or 280·

In reality, however, the property would be considered as divided into *fifty shares*, of which A would hold 15·, B 20·, C 8·, and D 7·, and then any Profit, Loss, or Additional Outlay would be also divided into fifty shares, of which there would be allotted to A 15·, to B 20·, to C 8·, and to D 7·

EXERCISE CXXVII. A.

EXAMPLES IN SIMPLE FELLOWSHIP.

1· Two men bought a house for £450·, of which the one paid £200· and the other the remainder. Repairs being required to the amount of £72·, how much should each of them pay?

2· A and B join in renting grazing ground for £63· A fattens on it 180· sheep, and B 72· What should each pay of the rent?

3· Divide £11· 9·s. 8¼d. amongst three children, in proportion to their respective ages, which are 5·, 7·, and 9· years.

4· Four persons provide horses for a coach: A for 9· miles per day, B for 11· miles, C for 10· miles, and D for 6· Now A's fair share of the profits for one year was £84· 11·s. 9¼d. What were the shares of B, C, and D?

5· A bankrupt's nett estate is worth £987· 1·s. 6·d. He owes to A £347· 8·s.; to B £315·; to C £280· 14·s. 3·d.; to D £213· 16·s. 6·d.; to E £110· 3·s. 9·d.; to F £47· 15·s.; and to G £1· 13·s. 6·d. What should each creditor receive?

6· Distribute £185· 5·s. 7½d. into four parts, which shall be to each other in the ratios of ½, ⅓, ¼, and ⅕.

See Second Example worked out on Page 158·

7· Divide £1· so that A shall get half as much as B; C twice as much as A and B; and D as much as A and C.

8· A testator, ill-versed in fractional Arithmetic, directed that his eldest son should receive $\frac{1}{2}$ of his property; his second, $\frac{1}{3}$; and his youngest, $\frac{1}{4}$ of it. How much should each receive out of £1690· 6·s. 6·d.?

9· Apportion £103· 1·s. 6$\frac{3}{4}$d. between A, B, and C, so that for every £5· received by A, B shall get £4·; and for every £8· received by B, C shall have £9·

10· Woody fibre is composed of 9· parts, by weight, of Carbon, 8· of Oxygen gas, and 1· of Hydrogen gas. What weight of each of these elementary substances is there in 1· cwt. 3· qr. 2· lb. of woody fibre?

11· The proceeds of a certain charity, amounting to £18· 12·s. 8·d., are to be distributed among four indigent families, consisting, respectively, of 3·, 5·, 7·, and 11· members. What should each family receive?

12· The gross value of a ship and cargo was £15000·, of which only part was insured; viz.: In one office, £2250·; in a second office, £3500·; in a third, £1750·; and in a fourth, £4500· Meeting with disaster, the ship and cargo incurred such damage that the owner's portion of the loss amounted to £375· 14·s. 10·d. How much had each of the underwriters to pay?

13· In the joint stock of two partners, A invests £1· 15·s. for every £1· invested by B. What was the whole gain, if A's fair share was £93· 4·s. 9$\frac{1}{4}$?

14· Three persons fund £2700· to prosecute a certain undertaking, A putting in £1500·, B £900·, and C the remainder. The latter, however, contributing also his skill and experience as manager of the affair, is allowed for these 10· per cent. of the gross profits, which were £800· What should each receive?

COMPOUND FELLOWSHIP.

In solving any question under this head, the amounts employed for *different* times must be reduced to the amounts which would be their equivalents if all were employed for the *same* length of time. The question will thus be converted into an example of Simple Fellowship.

SECOND EXAMPLE WORKED OUT. A having commenced a business with £200·, is joined after 6· months by B with £700·, a month later by C with £500·, and at the end of the next month by D, also with £500· At the close of the first year, the nett profits are found to be £150· 6·s. 3·d. What portion belongs to each partner?

	£	
Here A has in the business	200·	for 12· months
B ,, ,, ,,	700·	for 6· months
C ,, ,, ,,	500·	for 5· months
D ,, ,, ,,	500·	for 4· months.

Now, since the value of money is measured by the Interest accruing on it, and since £200· in 12· months will gain only the same Interest as 12· times £200· will in 1· month, the investments of the several partners are respectively equivalent to

£				£	
200·	×	12·	=	2400·	for 1· month
700·	×	6·	=	4200·	,, ,,
500·	×	5·	=	2500·	,, ,,
500·	×	4·	=	2000·	,, ,,

Joint investments = £11100· for 1· month.

Then, by Simple Fellowship, or Distribution into Proportional parts:

			£ s. d.		£ s. d.
A's share of Gain	=	$\frac{24}{111}$ of	150· 6· 3·	=	33· 17· 1·
B's ,, ,,	=	$\frac{42}{111}$ of	150· 6· 3·	=	56· 17· 6·
C's ,, ,,	=	$\frac{25}{111}$ of	150· 6· 3·	=	27· 1· 8·
D's ,, ,,	=	$\frac{20}{111}$ of	150· 6· 3·	=	32· 10· 0·
		Whole gain		=	£150· 6· 3·

EXERCISE CXXVII. B.

Examples in Compound Fellowship.

1· The grass of a field was hired jointly by 5· farmers. A put into it 14· oxen for 8· weeks; B, 20· for 5· weeks; C, 10· for 6· weeks; and D, 17· for 4· weeks. What portion of the rent should each pay?*

2· Supposing C's fair share of the rent to be £11· 15·s., what would be due from each of the others?*

3· In their joint undertaking, A invested £500· for 9· months; B, £600· for 7· months; C, £400· for 3 months; and D, £300 for 5· months. Their ultimate nett gain was £965· 5·s. 6·d. Apportion it equitably.*

4· A and B start a business with capitals in the ratio of 5· to 8·, but at the end of 3· months, the former adds $\frac{1}{4}$ to his previous investment. They share, as the first half-year's profits, £199· 8·s. 8$\frac{1}{4}$d. How much falls to the lot of each?*

5· In a joint stock business, A, B, C, and D, (the only partners,) originally invest, respectively, £300·, £600·, £800·, and £350·. At the end of 4· months, A increases his capital by £100·, and 3· months later by the same sum. B withdraws £100· at the end of the third month, but 2· months afterwards puts in £200·. At the commencement of the second half year, C adds £100 to his previously-invested capital, but, at the end of that month, withdraws £150·. D's original stock remains unaltered to the close of the seventh month, but he then augments it by £190· What part of the first year's gain or loss accrues to each of the four co-partners?*

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* The examples thus marked are fair specimens of Compound Fellowship, as usually exemplified in books of Arithmetic. They all, however, presuppose,

6· A commenced a business with £200·, and was joined by B, with £700·, four months before the first division of profits, and by C, with £1300·, two months after B. Now, the profits were at the regular rate of £7½ per cent. per annum, and amounted to £52· 10·s. How long was A in the business before B joined him?

In the resolution of this question, the successive steps are:

Capital to gain £52· 10·s. in 1· year =
Capital to gain ditto in 1· month =
Of which B's and C's are equivalent to =
∴ *For 1· month, A's Capital would be* =

Then, by Division,

Whole No. of months A trades in Partnership till first Division of Profits =
∴ *No. of months A trades alone* =

EXCHANGE.

This is merely the reduction of a given amount in coinage of one country to its equivalent in that of another.

The Par of Exchange is the actual comparative value of any two nations' coinages, with reference only to the intrinsic worth of the metals composing them respectively.

The Course of Exchange is the variable amount of the one coinage which, at any given time, is held equivalent, in commercial transactions, to a fixed sum in the other.

The Course of Exchange between any two countries varies with the state of trade between them.

To obviate, as far as possible, the risk and expense of transmitting bullion from one country to another, merchants employ Bills of Exchange, which effect the desired object thus:

	£
Suppose A in London owes C in Riga	5000·
Whilst D in Riga owes B in London	3500·

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and depend upon, a variety of conditions which are most unlikely to exist in real business. For instance, they all assume:

I. That the business in question is such that it can be contracted or expanded at pleasure, so as to permit the entry or withdrawal of partners, without affecting its stability or profitable character.
II. That the rate of profits is equable.
III. That no premium is due to, or demanded by, the earlier from the later partners, for establishing the business, and bringing it up to a paying position; but that late entering partners are admitted on the same terms as the original promoters of the concern. Neither of which assumptions is consistent with the real nature of general business.

Then B in London may draw on D a Bill of Exchange, payable in Riga, for £3500·, and sell it to A. The latter will remit it to C in Riga, who will present it to D for payment. By this contrivance, D of Riga can pay B in London £3500·, and A of London pay C in Riga the like amount, the double transaction being completed without actual transmission of coin either way.

To discharge, by similar means, the remaining £1500· which he owes to C, A must find, in London or elsewhere, some other person to whom that amount is due from Riga. If London has received from Riga a greater amount of goods than it has sent thither,

The Balance of Trâde is against London.

In this case, some London debtors must either purchase Bills of Exchange drawn elsewhere upon Riga, for which Bills they must pay a premium, or they must incur the expense of transmitting and insuring bullion. And £100· in London being thus no longer sufficient to pay £100· in Riga, English money is below par in Riga.

The Course of Exchange is against London.

First Example worked out. Exchange,

I. £2400· 15·s. for French money, at 25· francs 40· centimes per £1·, and,

II. 34360· francs for sterling, at 25· francs per £1·

Example completed.

I.

$$\text{£}2400\cdot\ 15\cdot\text{s.} = (2400{\cdot}75 \times 25{\cdot}4)\ \text{francs.}$$
$$= 60979\cdot\ \text{francs}\ 5\cdot\ \text{centimes.}$$

II.

$$34360\cdot\ \text{francs} = \text{£}(34360\cdot \div 25\cdot)$$
$$= \text{£}\ 1374\cdot\ 8\cdot\text{s.}$$

Second Example worked out. What is the course of Exchange between London and Hamburg, when £1650· are equivalent to 22687· marks 8· schillings?

Example completed.

$$\text{Course of Exchange} = \text{No. of marks for £}1\cdot$$
$$= 22687\tfrac{1}{2} \div 1650\cdot = 13\tfrac{3}{4}\ \text{marks.}$$
$$= 13\cdot\ \text{marks}\ 12\cdot\ \text{schillings.}$$

Arbitration of Exchanges.

It might happen that when the Course of Exchange with Amsterdam is against London, that of London is against (say) Lisbon, whilst that of Lisbon is against Amsterdam. In such a case, London, by paying Amsterdam *through* Lisbon, might not only escape the loss incident to direct exchange with Amsterdam, but might even make a profit out of the transaction.

By "paying through Lisbon" is meant, drawing or buying Bills in London on Lisbon, cashing in Lisbon, and, with the proceeds, purchasing Lisbon Bills on Amsterdam. By this process, London, in the case proposed, would obtain two discounts instead of paying a premium.[*]

The resulting Rate of Exchange, by this circuitous route, is called the **Par of Arbitration** between London and Amsterdam through Lisbon.

To arbitrate exchanges is to ascertain whether the direct or any one of a given number of circuitous routes is the more advantageous.

In **Simple Arbitration of Exchanges** only three places are concerned. In **Compound Arbitration of Exchanges** more than three places are concerned.

THIRD EXAMPLE WORKED OUT. SIMPLE ARBITRATION OF EXCHANGES. Bills on Paris are bought in London at £1· for 25·4 francs, and sold in Hamburg at 100· marks for 187· francs. What is the resulting "Par of Arbitration" between Hamburg and London?

Here, London pays Hamburg through Paris.

Example completed.

£1· = 25·4 francs. 1· mark = 1·87 franc.

$$\therefore \text{ No. of marks for £1}\cdot = \frac{25\cdot4}{1\cdot87} = 13\tfrac{109}{187} \text{ marks.}$$

$$= 13\cdot \text{ marks } 9\tfrac{61}{187} \text{ schillings.}$$

FOURTH EXAMPLE WORKED OUT. COMPOUND ARBITRATION OF EXCHANGES. A Bill, London upon Hamburg, is bought at £1· for 13· marks 10¼ schillings, and sold at Amsterdam at 35¾ florins for 40· marks. With the proceeds are then purchased French Bills, at 57¼ florins for 120· francs. Find the Par of Arbitration between London and Paris by the given route.

Example completed.

$$\because\ 1\cdot \text{ franc} = \frac{57\frac{1}{4}}{120} \text{ florins; and } 1\cdot \text{ florin} = \frac{40}{35\frac{3}{4}} \text{ marks;}$$

$$\therefore\ 1\cdot \text{ franc} = \left(\frac{57\frac{1}{4}}{120} \times \frac{40}{35\frac{3}{4}}\right) \text{ florins.} \qquad \text{PRINCIPLE IX.}$$

$$\text{But, } 13\tfrac{41}{64} \text{ marks} = £1\cdot$$

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[*] The word "discount" is very generally employed, as here, to signify any *diminution* from Par, as "premium" is any excess above Par.

$$\text{And, No. of francs in £1}\cdot = \frac{\text{No. of marks in £1}\cdot}{\text{No. of marks in 1}\cdot\text{ franc.}}$$

$$= 13\tfrac{41}{64} \div \left(\frac{57\frac{1}{4}}{120} \times \frac{40}{35\frac{3}{4}}\right)$$

$$= \frac{13\frac{41}{64} \times 120\cdot \times 35\frac{3}{4}{}^{*}}{1\cdot \times 57\frac{1}{4} \times 40\cdot} \quad \text{PRINCIPLE XVII.}$$

$$\doteq 25\cdot \text{ francs } 55\tfrac{1405}{3664} \text{ centimes.}$$

* Observe, that all the *buying prices* come into the denominator of this fraction, and, of course, will do so in every similar example. Whence this simple Rule:

To find the Par of Arbitration between the first and last of any number of places successively concerned in a circuitous exchange.

Divide the continued product of all the *Selling Prices* by the continued product of all the *Buying Prices.*

To Arbitrate Exchanges.

I. Find the Par of Arbitration by each of the given routes.

II. Compare these with each other, and with the direct Course of Exchange.

The route, whether direct or circuitous, which yields the highest Par of Arbitration between the first and last places in question, will, of course, be the most advantageous.

FIFTH EXAMPLE WORKED OUT. £1881· are to be remitted from London to Paris, either:

I. Direct, at £1· for 25· francs; or,

II. To Hamburg, at £1· for 13· marks 12· schillings; thence to Amsterdam, at 40· marks for 36· florins; thence to Paris, at 57· florins for 120· francs; or,

III. London to Berlin, at £1· for 6· dollars 15· groschen; Berlin to Brussels, 7· dollars for 40· schillings; Brussels to Hamburg, 39· schillings 6· grotes for 14· marks; Hamburg to Paris, 13· marks 12· schillings for 26· francs.

Which route is the most advantageous, and by how much

I. In French money?
II. In Sterling money?
III. Per cent. on amount remitted?

Example completed.

No. of francs for £1· by direct exchange = 25· fr.

$$\text{by 1st circuit} = \frac{13\tfrac{3}{4} \times 36\cdot \times 120\cdot}{1\cdot \times 40\cdot \times 57\cdot} = 26\tfrac{1}{19}\,\text{fr.}$$

$$\text{by 2nd circuit} = \frac{6\tfrac{1}{2} \times 40\cdot \times 14\cdot \times 26\cdot}{1\cdot \times 7\cdot \times 39\tfrac{1}{2} \times 13\tfrac{3}{4}} = 24\tfrac{778}{869}\,\text{fr.}$$

∴ The first circuitous route is the most advantageous of the three, exceeding the direct one by $1\frac{1}{19}$ franc per £1·; and producing a profit

		fr.		
I. In French money	=	$1\frac{1}{19} \times 1881\cdot$	=	francs. 1980·
II. In Sterling money	=	fr. (1980· ÷ 25·)	=	£ 79· s. 4· d. 0·
III. Per cent. on remittance	=	$\dfrac{£79\frac{1}{5} \times 100\cdot}{1881\cdot}$	=	£ 4· s. 4· d. $2\frac{330}{627}$

Sixth Example worked out. What amount sterling, by this route, would be equivalent to the £1881· by the direct route?

That is to say, How many £, at $26\frac{1}{19}$ francs, are equal to £1881· at 25· francs?

Example completed. By Inverse Proportion.

As $26\frac{1}{19}$: 25· :: £1881· : £1805·

PROOF.

fr.				francs.
25·	×	1881·	=	47025·
$26\frac{1}{19}$	×	1805·	=	47025·

EXERCISE CXXVIII. A.

Examples in Exchange.

I. France.

100· Centimes = 1· Franc.

Exchange for Francs:

	£	s.		fr.	cen.
1·	340·	10·	at	24·	50·*
2·	965·	15·	at	25·	30·*
3·	784·	5·	at	25·	75·*

Exchange for £.

	francs.		fr.	cen.
4·	9854·	at	25·	75·*
5·	3682·	at	24·	25·*
6·	7680·	at	25·	60·*

II. Belgium.

100· Centimes = 40· Grotes = 1· Florin or Franc.
12· Grotes = 1· Schilling.

Exchange for Florins.

	£	s.		sch.	gr.
7·	500·	10·	at	40·	6·*
8·	950·	15·	at	39·	4·*

Exchange for £.

	flo.	cen.		sch.	gr.
9·	9980·	80·	at	39·	10·*
10·	6436·		at	40·	9·*

III. HOLLAND.

100· Cents = 1· Florin. 5· Cents = 1· Stiver.
50· Stivers = 1· Rix-Dollar.

Exchange for Florins.

	£	s.		flo.	c.
11·	675·	10·	at	12·	75·*
12·	496·	4·	at	11·	90·*

Exchange for £.

	r. dol.	sti.		flo.	c.
13·	348·	30·	at	12·	45·*
14·	763·	30·	at	11·	75·*

IV. HAMBURG.

12· Pfennings = 1· Schilling. 16· Schillings = 1· Mark.

Exchange for Marks.

	£		m.	sc.
15·	846·	at	13·	4·*
16·	973·	at	12·	14·*

Exchange for £.

	m.	sc.		m.	sc.
17·	6455·	4·	at	13·	8·*
18·	9386·	10·	at	13·	12·*

V. RUSSIA.

100· Copecs = 1· Silver Ruble.

Exchange for Rubles.

	£		d.	
19·	750·	at	37¼	per ruble.
20·	964·	at	38¾	„

Exchange for £.

	ru.	co.		d.	
21·	7484·	00·	at	37¼	per ru.
22·	9355·	25·	at	36⅞	„

VI. PRUSSIA.

30· Silver Groschen = 1· Dollar.

Exchange for Dollars.

	£		d.	gr.
23·	600·	at	6·	20·
24·	945·	at	6·	24·

Exchange for £.

	d.	gr.		d.	gr.
25·	3640·	20·	at	6·	25·*
26·	7364·	15·	at	6·	28½*

VII. AUSTRIA.

60· Kreutzer = 1· Florin. 90· Kreutzer = 1· Rix-Dollar.

Exchange for Florins.

	£		fl.	kr.
27·	963·	at	10·	5·
28·	786·	at	9·	55·

Exchange for £.

	flo.	kr.		flo.	kr.
29·	6438·	50·	at	10·	6·*
30·	7863·	45·	at	10·	2½*

31· Exchange 195· Rix-Dollars, 50· Kreutzer, for £, at 9· flo. 48· kr.*

32· Exchange £1347· 10·s. for Rix-Dollars, at 10· flo. 2 kr.*

EXERCISE CXXVIII.

B.

1· If 25· Francs be given for 11·7 Dutch Florins, 39· Florins for 45· Hamburg Marks, and 2¼ Marks for 1· Russian Ruble, how many Rubles are equivalent to 1000· Francs?

2· How much English Standard gold is a Franc equivalent to, if it weigh $\frac{10}{31}$ of a gramme, and 31·1 grammes = 1· oz. Troy; the value of English gold being to that of French as 3151· : 3100·?

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* Per £1· Sterling.

3· How many Francs = £1· "at Par"?

4· What is the arbitrated rate of exchange ("Par of Arbitration") between London and Madrid, when Bills on Paris, bought in London at 25·4 Francs per £1· are sold in Madrid at 3·75 Francs for 1· Dollar of 20· Reals?

5· How much is London then above or below Par at Madrid, the Par of Exchange being 3·s. per Dollar?

6· Of the three routes, whether is the most advantageous for remitting from London to St. Petersburg:

- I. The Direct, at 37·d. per Silver Ruble; or
- II. London to Paris, £1· for 25¼ Francs; Paris to Lisbon, 6·5 Francs per Milree; Lisbon to Vienna, 4½ Milrees for 12· Florins; Vienna to St. Petersburg 10·75 Florins for 6·5 Rubles; or
- III. London to Hamburg, £1· for 13· Marks 12· Schillings; to Berlin, 10· Marks for 5½ Dollars; to Madrid, 6¼ Dollars for 7 Milrees; to St. Petersburg, 14· Milrees for 12½ Rubles?

EQUATION OF PAYMENTS.

It may be occasionally necessary in business to solve such a question as the following.

"Supposing A to owe me £245· due 11· months hence, and £600· at 4· months hence, when should both sums be paid at once, so that neither of us should lose or gain?"

The time required to be determined is called the **Equated Time**. And, in this instance, the equated time is 6· months hence.

So that, the £245· will be paid 5· months before its time.
And, the £600· will run 2· months over its time. Therefore,

I ought to allow him 5· months' *Discount* on £245·
He ought to allow me 2· months' *Interest* on £600·

If these allowances be equal, then, 6· months hence, is, as stated above, a true Equated Time for the payment of the two debts. Now, at £5· per centum per annum,

5· months' *Discount* on £245· = £5· which I allow him.
2· months' *Interest* on £600· = £5· which he allows me.

Both debts, then, being paid at the end of 6· months, the advantages are exactly balanced *up to that, the Equated Time.*

Otherwise:

Let the natural duration of the longest debt (11· months) be called **The Term.**

Then, seeing that whoever of the two persons holds the moneys may derive Interest from them, for whatever portion of the Term they remain in his hands, it will be seen that payment of both debts at the end of $6\frac{5}{169}$ months would also effect an equitable settlement. For, in that event,

The £245· would be paid $4\frac{164}{169}$ months *before* due.
The £600· would be paid $2\frac{5}{169}$ months *after* due.

Then,

I ought to allow him *Interest* on £245· for $4\frac{164}{169}$ mos.
He ought to allow me *Interest* on £600· for $2\frac{5}{169}$ mos.

But, at £5· per centum, per annum,

£ s. d.
Interest on £245· for $4\frac{164}{169}$ mos. = $5 \cdot 1 \cdot 5\frac{3}{4}\,\frac{1}{169}$, allowed by me.
Interest on £600· for $2\frac{5}{169}$ mos. = $5 \cdot 1 \cdot 5\frac{3}{4}\,\frac{1}{169}$, allowed to me.

Therefore, by payment of both debts at the end of $6\frac{5}{169}$ months, the advantages will be exactly balanced *up to the end of the Term* (11· months.)

To illustrate further: Suppose each of the debts to be paid when due. Then, by the end of the Term,

£ £ s.
I should gain 7· months' Interest on . . . 600· = 17· 10·
By Eq. of P., I gain $4\frac{164}{169}$ months' Interest on 845· = 17· 10·

So that I neither gain nor lose by the end of the term.

Again, If both debts were paid when due, then by the end of Term,

£ £ s. d.
He would gain 11· months' Interest on 245· = 11· 4· 7·
Together with 4· months' Interest on 600· = 10· 0· 0·
By Eq. he gains $6\frac{5}{169}$ months' Interest on 845· = 21· 4· 7·

So that, by paying both debts at the Equated Time ($6\frac{5}{169}$ months) neither party gains or loses up to the end of the Term (11· months).

There are, then, *two Equated Times* for the equitable payment of these debts.

The first (6· *months*) *is the true Equated Time, if the balance of advantages be referred to the day of equated payment.*

The second ($6\frac{5}{169}$ *months*) *is the true Equated Time, if the balance of advantages be referred to the end of the Term.*

Now, although the first result (6· months) happens, in this particular example, to be a simpler one than the second ($6\frac{5}{169}$ months), this is not always the case. Moreover, a result of the first kind, even for two debts only, could not be found without the aid of Algebra; whereas a result of the second kind, for the Equated Time to pay three, four, or any number of debts can be determined by the same easy Arithmetical process which is employed to solve questions in Compound Fellowship.

FIRST EXAMPLE WORKED OUT. If a person owe £500· due in 3· months, £400· in 5· months, £300· in 6· months, and £200· in 8· months; when should the whole debt be paid, so that neither debtor nor creditor should lose?

Here, we replace each debt by the sum which would produce the same Interest *in one month*. And this sum is found by multiplying the debt by the number of months it had to run before becoming due. For, in the production of Interest,

$$\begin{array}{llll}
\overset{\pounds}{500\cdot} \text{ for } \overset{\text{m.}}{3\cdot} = \overset{\pounds}{500\cdot} \times 3\cdot = & \overset{\pounds}{1500\cdot} & \text{for 1 month.} \\
400\cdot \text{ for } 5\cdot = 400\cdot \times 5\cdot = & 2000\cdot & ,, \; ,, \quad ,, \\
300\cdot \text{ for } 6\cdot = 300\cdot \times 6\cdot = & 1800\cdot & ,, \; ,, \quad ,, \\
\underline{200\cdot} \text{ for } 8\cdot = 200\cdot \times 8\cdot = & \underline{1600\cdot} & ,, \; ,, \quad ,, \\
1400\cdot \qquad\qquad\qquad 1400\cdot) & \underline{6900\cdot} & ,, \; ,, \quad ,,
\end{array}$$

The four debts are = £1400· due $4\frac{13}{14}$ months hence.

$\therefore$ $4\frac{13}{14}$ mos. is the Equated Time for paying all at once.

PROOF.

By Equation of payments, the Creditor will surrender, at (say) £5· per cent., per annum,

$$\text{Interest on } \overset{\pounds}{500\cdot} \text{ for } 1\tfrac{13}{14} \text{ months} = \overset{\pounds}{4\cdot}\ \overset{\text{s.}}{0\cdot}\ \overset{\text{d.}}{4\tfrac{2}{7}}$$

But the Debtor will also surrender, at the same rate,

$$\begin{array}{lllrrr}
 & & & \pounds & s. & d. \\
\text{Interest on } \overset{\pounds}{400\cdot} \text{ for } \frac{1}{14} \text{ month} & & = & 0\cdot & 2\cdot & 4\frac{4}{7} \\
\quad ,, \quad \text{on } 300\cdot \text{ for } 1\frac{1}{14} \text{ month} & & = & 1\cdot & 6\cdot & 9\frac{3}{7} \\
\quad ,, \quad \text{on } 200\cdot \text{ for } 3\frac{1}{14} \text{ months} & & = & \underline{2\cdot} & \underline{11\cdot} & \underline{2\frac{2}{7}} \\
\left.\begin{array}{l}\text{Making, in all, the same allowance as} \\ \quad \text{his Creditor . . . .}\end{array}\right\} & & = & 4\cdot & 0\cdot & 4\frac{2}{7}
\end{array}$$

TO FIND EQUATED TIME FOR THE PAYMENT OF ANY NUMBER OF DEBTS.

First. Multiply each debt by the No. of times (years, or months, or days,) it has to run.

Second. Add the products together.

Third. Divide by the sum of the debts.

SECOND EXAMPLE WORKED OUT. One thousand pounds becoming due 8· months hence, and £200· of it being paid at the end of 3· months, how long should the remainder be allowed to stand over?

Here, we have, "Given the Equated Time (8· mo.) of both debts, and the time of payment of one of them (3· mo.) to find the time of payment of the other debt."

Debtor is entitled to

Int. on £1000· for 8· mo., or on £8000· for 1· mo.

But he gets

Int. on 200· for 3· mo., or on 600· for 1· mo.

There remains then, due to him

Int. on . . . . 7400· for 1· mo.

Or on £800· for (7400· ÷ 800· or,) 9¼ months.

∴ The £800· may stand over until the end of 9¼ months.

PROOF.

The £200· are paid 5· months *before* due.
The £800· are paid 1¼ months *after* due.

	£	s.	d.
∴ Debtor sacrifices Int. on £200· for 5· mo. =	4·	3·	4·
Creditor sacrifices Int. on £800· for 1¼ mo. =	4·	3·	4·

THIRD EXAMPLE WORKED OUT. A certain debt would become due in 2· years and 9· months. Now, supposing ¼, ⅓, and ⅛ of it to be paid at the ends of the first, second, and third years, respectively, when should the remainder be paid?

The four instalments being ¼, ⅓, ⅛, and $\frac{7}{24}$; that is to say, $\frac{6}{24}$, $\frac{8}{24}$, $\frac{3}{24}$, and $\frac{7}{24}$, call the debt £24·

Then, the instalments are £6·, £8·, £3·, and £7·

$$\begin{array}{lllll} & \pounds 24\cdot \times 2\tfrac{3}{4} & = & \pounds 66\cdot & \text{for } 1\cdot \text{ year.} \\ \text{But} & \left.\begin{array}{l} 6\cdot \times 1\cdot = 6\cdot \\ 8\cdot \times 2\cdot = 16\cdot \\ 3\cdot \times 3\cdot = 9\cdot \end{array}\right\} & = & 31\cdot & ,, \quad ,, \\ & & 7\cdot) & \overline{35\cdot} & ,, \quad ,, \end{array}$$

The £7· must run for 5· years.

PROOF.

	£	s.	d.
Debtor surrenders Int. of £6· for 1¾ yrs. =	0·	10·	6·
Int. of £8· for 9· mo. =	0·	6·	0·
	0·	16·	6·

	£	s.	d.
Creditor surrenders Int. of £3· for 3· mo· =	0·	0·	9·
Int. of £7· for 2¼ yrs. =	0·	15·	9·
	0·	16·	6·

INCLUSIVE, OR COVERING, INSURANCE.

If the Premium (see page 352·) be £5· per cent., and a person ensure his property at its exact value, he would pay to the Company £5· on every £100· worth of it; and, in the event of its destruction, he would receive from them the exact value of the property insured, thus losing £5· per cent., the Premium paid.

But if, representing every £95· worth of property as being worth £100·, he were to pay on it the Premium on £100·, then, for every £95· worth destroyed he would expect to receive from the Company, £100·, covering both the value of the property and also the Premium paid.

A. To cover £95· and Premium, insure a nominal £100·

B. To cover £1· and Premium, insure ” $\frac{\pounds 100\cdot}{95\cdot}$

C. To cover, say, £3800· and Premium, insure $\frac{\pounds 100\cdot \times 3800\cdot}{95\cdot} = 4000\cdot$

PROOF.

	£
Real value to be covered . . . . =	3800·
Premium Paid on fictitious value (£4000·) . . . . =	200·
Amount insured, to cover real value and Premium =	4000·

Now, in line C., the £100· is *constant*. And the Divisor, £95· is = £100·—(Premium on £100·) Therefore,

To ascertain what nominal value must be insured in order to cover a given real value and the Premium paid.[*]

First. Multiply real value by 100·

Second. Divide Product by the Difference between £100· and the Premium on £100·

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[*] This scheme, which is usually included in arithmetical books, involves a false statement by the Insured to the Insurers; and it contains within itself the elements of its own defeat, the Insurance Company being only bound to pay the actual worth of the property destroyed, and not the fictitious value which the Insured may choose to assign to it. If, however, the Insurance Company agree to accept the arrangement by which the Premium as well as the property is covered, then the Insured would *gain by the destruction of the insured property*, for, in that case, his premium would be restored to him; whereas, if the property remain safe, the premium would be lost.

EXERCISE CXXIX.

EQUATION OF PAYMENTS AND INCLUSIVE INSURANCE.

Prove every Example.

1· What is the equated time, for payment in one sum, of three debts: viz. £700· due 3· months hence, £400· in 5· months, and £180· in 6· months?

2· A debt of £1000· is due thus: £300· at the end of each of the first three months, and £100· at the end of the fourth month. When would a single payment of the whole be equitable?

3· If £364· 5·s. be due 8· months hence, £548· 10·s. at 4· months, £91· 15·s. at 5· months, and £145· 10·s. at 7· months, what is the equated time for paying the whole?

4· On the first day of April bills are drawn which will fall due as follows: £400· on May 1rst., £350· on May, 11th., and £750· on June 20th.; when should the whole be paid, if in one sum?

5· When is a debt of £700· due, if it can be equitably discharged by monthly instalments of £100· each?

6· When should £1800· be paid down, to be equivalent to six equal monthly instalments?

7· How long has a debt to run which might be equitably paid in nine equal monthly instalments?

8· When is the whole of a debt due, if it might be fairly paid at the rate of $\frac{1}{10}$ of it per month?

9· At the end of one month, £600· are to be paid, and then £150· at the end of each of the next 4· months. When (instead of this arrangement) should the whole £1200· be paid, so that neither Debtor nor Creditor should lose?

10· £1000· being due in 2· months, would the Debtor or Creditor gain, and how much, if the money were paid in 4· monthly instalments, the first three being £300· each?

11· Which of them would gain, and how much, supposing the £100· to be

I. The first instalment?
II. The second instalment?
III. The third instalment?

12· A debt of £240· will be due at the end of 4· months and 13· days. Now, if £60· be paid at 2· months' end, and £48· two months later, it is required to find when other £40 must be paid, so that the remainder of the debt may stand over to the end of the sixth month.

13· A certain debt becoming due at the end of 12· months, when must the first half be paid to allow the remainder to run to the end of 17· months?

14· Of £600· due in 8· months, £300· are paid in 2· months, and £200· in 3· months. How long must the remainder run?

What nominal value must be insured

15· At £5· per cent., to cover £380· and Premium?
16· At £2· per cent., to cover £690· 18·s. and Premium?
17· At £6⅓ per cent., to cover £1379· and Premium?

18· What amount is covered, with Premium, by insuring £700· at 2⅓ per cent.?

PROFIT AND LOSS.

Questions under this head require no special rule, each being solved by applying to its conditions the principles and processes already made familiar.

EXERCISE CXXX.

Explain every line. *Prove every Example.*

1· If by selling a horse for £12· I gain £20· per cent., what did it cost me?

Gain = $\frac{1}{5}$ *of Cost Price,* = $\frac{1}{6}$ *of Selling Price.*

2· A loss of £20· per cent. is incurred by selling a certain article for £12· What was the Cost Price?

3· What is gained per cwt., and per cent. on Cost Price, by selling for £17· 18·s. 4½d., 8· cwt. 3· qrs. 7· lb., which cost £14· 2·s.?

I.

Gain on $8\frac{13}{16}$ *cwt.* = £3· 16·*s.* 4½*d.*
Gain on $\frac{1}{16}$ *cwt.* =
Gain on 1· *cwt.* =

II.

Gain on £$14\frac{1}{10}$ = £3· 16·*s.* 4½*d.*
Gain on £ $\frac{1}{10}$ =
Gain on £ 1· =
Gain per cent. =

4· If a trader gain 5·s. 7½d. by selling an article for 33·s. 9·d., what does he gain per cent. on his outlay?

5· A grocer bought 2· cwt. 13· lb. of sugar at 3·d. per lb., and 4· cwt. 15· lb. at 3½d. He sold 1· cwt. 3· qr. 4· lb. of the latter at a loss of 1·d. per lb. All the rest he sold at a uniform rate, and thus cleared 14·s. 0½d. What was this uniform price per lb.?

6· An article which was sold for 31·s. 7½d. returned a profit of 5·s. 7½d. How much per cent. was gained
I. On Cost Price?
II. Out of Selling Price?

7· How much per cent. *out of Selling Price* is equal to 25· per cent. *on Cost Price?*

8· Bought goods to the amount of £100· nett, and resold them at a profit of £25· per cent. *out of Selling Price.* By how much was this gain more than £25· per cent. *on Cost Price?*

9· Which is the greater gain, 5·s. *on* every £1· of Cost Price, or 4·s. *out of* every £1· of Selling Price?

10· Bought sugar at £37· 6·s. 8·d. per ton, and retailed it at 4½d. per lb. What was the gain per cent. on Cost Price?

11· A and B lay out the same amount, in the same kind of goods, at the same rate. The first sells at a profit of 15· per cent. *on Cost Price*, and the second at 15· per cent. *out of selling price.* What was the difference of their gains?
I. On every £1· of Cost Price?
II. Out of every £1· of Selling Price?

12· If, by selling an article for £40·, I lose £20· per cent., what must I have sold it for to gain £20· per cent. both on C. P.?

13· By selling a bullock for 15· guineas, a grazier lost 20· per cent. on C. P. What would it have fetched if he had gained 20· per cent. on C. P.?

Loss = ¼ *of First Selling Price* = ¼ *of* £15· 15·s.

14· The Loss in selling an article for £21· 18·s. 9·d., is 10· per cent. on Cost Price. What must it have been sold for in order to gain 10· per cent. out of Selling Price?

15· A man purchases at £25· each, railway shares, on which £18· per share have been paid. He pays up the calls to £20· and sells at £28½. What does he gain per cent. on his outlay?

EVOLUTION.

As Involution (page 53·) is the raising of a number to any required power, so Evolution is the exactly opposite process, its object being to ascertain of what number any given one is a certain power.

For Example: To solve the question, "Of what number is 361· the square?" we must find (or "extract") the square root of 361·

Every number is called

The "square root" of its square, or 2nd power;
The "cube root" of its cube, or 3rd power;
The "fourth root" of its fourth power; and so on.

Thus, $\because\ 2^2 = 4\cdot;\ 2^3 = 8\cdot;\ 2^4 = 16\cdot;\ 2^5 = 32\cdot$; &c.
$\therefore\ 2\cdot =$ sq. root of $4\cdot =$ cube root of $8\cdot =$ 4th root of $16\cdot$ &c.

Arithmetical Symbol. Instead of the word "root," we employ the **Radical Sign**, which is thus formed, $\surd$. It represents the letters r^t, the first and last of the word "root."

Roots, like powers, are distinguished by their index or exponent. The square root is indicated thus, $\surd$ or $\sqrt[2]{}$. For the words "cube root of," we write $\sqrt[3]{}$; for "fourth root of," $\sqrt[4]{}$; and so on.

SQUARE AND CUBE NUMBERS.

A number which is the square of another, and which, consequently, has an exact square root, is called a **square number**; and one which has an exact cube root, is called a **cube number.** The first ten squares and cubes should be remembered.

TABLE.

Roots.	Squares.	Cubes.	Roots.	Squares.	Cubes.
1·	1·	1·	6·	36·	216·
2·	4·	8·	7·	49·	343·
3·	9·	27·	8·	64·	512·
4·	16·	64·	9·	81·	729·
5·	25·	125·	10·	100·	1000·

EXTRACTION OF THE SQUARE ROOT.

Pointing for Square Root. Since $1^2 = 1$·; $10^2 = 100$·; $100^2 = 10000$·; $1000^2 = 1000000$·; and so on; it is plain that

Of any single digit the square is under 100·
Of any No. containing 2· places, the sq. is between 100· and 1000·
Of any No. containing 3· places, the sq. is between 1000· and 100000·

And, conversely,

If the sq. has not more than *two* places, the root has *one*.
If the sq. has *three* or *four* places, the root has *two*.
If the sq. has *five* or *six* places, the root has *three*.

So that

A square number never has more, and can only have one less, than twice as many places as its Square Root. And, for the units' place, and for every two places besides the units, in the Square Number, there will be one place in its Square Root.

Therefore, if we place a point over the units' place of the number whose Square Root is to be found, and also a point over every alternate place to the right and left of units, there will be one place of figures in root for every such point.

The pointed figure, and next one to the left, (if any) form a **Period** for Square Root.

There are three such periods in $\dot{2}2\dot{2}0\dot{1}$·

When there is an odd number of decimal figures, it is necessary to add a cipher to complete the right-hand period, as in $2\dot{2}\cdot 2\dot{0}1\dot{0}$

In such a case, there is no exact Square Root; for, if a Square Number end in 0, it will end in 00, since every number ending in 0 must have its Square in 00, by the Principles of Multiplication.

We may, of course, add any number of periods (of 00 each) to the right-hand of decimals.

Formation of a Square Number. If we take any Number and multiply it by itself, the product is a Square Number, of which the former one is the Square Root. Now, it will be found that the square may also be obtained from the root by adding together

I. *The square of the tens.*
II. (*Twice tens* + *units*) × *units.*

Or, more concisely, putting t for "*tens*," u for "*units*," and a full point for the sign of multiplication, the number to be squared will be represented by $t + u$, and the fact stated above, thus:

$$(t+u)^2 = t^2 + (2.t+u).u$$

EXAMPLES.

$64\cdot = 6$ *tens* + 4 *units*	$643\cdot = 64$ *tens* + 3 *units*
$t^2 = 60^2 \quad = 3600\cdot$	$640^2 \quad = \quad 409600\cdot$
$(2.t+u)u = 124\cdot \times 4\cdot = \underline{\ 496\cdot}$	$1283\cdot \times 3\cdot = \quad \underline{3849\cdot}$
$64^2 = 64\cdot \times 64\cdot \quad = \underline{\underline{4096\cdot}}$	$643^2 \quad = \quad \underline{\underline{413449\cdot}}$

The process for extraction of the Square Root is derived from the foregoing analysis of a Square Number; and consists, essentially, of two parts:

I. Having ascertained the tens of root, we subtract their square.

II. Dividing remainder by *twice the tens* of root we find its units' figure.

Then, to prove that this is the true units' figure of root, we add it to twice the tens, and multiply the sum by it.
If the Product = Remainder, the Square Root is found.

FIRST EXAMPLE WORKED OUT. A SQUARE NUMBER OF TWO PERIODS. INTEGERS. Find the Square Root of 7225·

The Square Root of $7\dot{2}2\dot{5}\cdot$ can only have two places of figures; and the square of the tens of root must be the greatest square number under 72 hundreds.

∴ Square of tens = 64 hundreds. ∴ Tens' figure of root is 8
Put 8 (*tens*) in root.
Subtract square of 8 (*tens*) = 64 (*hundreds*).
Twice 8 (*tens*) = 16 (*tens*).

This is the **Trial Divisor,** contained 5· times in 825·

$$\begin{array}{rl}
 & 7\dot{2}2\dot{5}\cdot(85\cdot = \text{sq. root.} \\
 & \underline{64} \qquad = t^2 \\
2\,.\,t+u = 16,5\cdot) & 825\cdot \\
(2\,.\,t+u)\times u = & \underline{825\cdot}
\end{array}$$

∴ 5 is units' figure of root. Put 5 in root beside the 8
Put 5 also on the right of the 16 (*tens*) of *Trial Divisor.*

This makes the **Complete Divisor** 165·

Multiply *Complete Divisor* (165·) by 5· **Subtract Product** (825·)

The two quantities subtracted make up the square of 85· But as nothing remains, the quantities subtracted = also 7225·

∴ Square of 85· = 7225· ∴ 85· = $\sqrt{7225}$· which was to be found.

SECOND EXAMPLE WORKED OUT. SQUARE NUMBER HAVING THREE PLACES IN ROOT. Extract $\sqrt{236196}$·

The Square Root of $2\dot{3}6\dot{1}9\dot{6}$· will contain hundreds', tens', and units' places. But we may, when convenient, read hundreds as tens; and this we shall do to simplify our explanation in the present instance.

I. We shall find, precisely as in former example, the Square Root of the greatest square number contained in the two left-hand periods 2361 (*hundreds*).

This will give us two figures of the root, which are, separately taken, its hundreds' and tens' figures. But,

II. Calling these the tens of the root, (as they really are, when taken together,) we shall employ them to form a Second Trial Divisor, by whose aid we shall discover the units' figure of the root.

Example completed.

$2\dot{3}6\dot{1}9\dot{6}$·(48,6· = Square Root.
16
8,8) 761
704* (*with former subtrahend*)† $= t^2$
$2.\,t+u=$ ‡96,6·) 5796·
5796· $= (2\,.\,t+u) \times u$

* Thus far, the work is similar to that of previous example.

† Down to this point we have subtracted square of 48 (*tens*). To the remainder we take down next period (96·) making Dividend 5796·

‡ Having now in the Root 48 (*tens*), our *Second Trial Divisor* must be twice 48 (*tens*) = (twice 40 + twice 8) *tens*.

But we have already, First Complete Divisor = (twice 40 + once 8) *tens*. To this, therefore, we have only to add 8 (*tens*), and Second Trial Divisor is formed = 96 (*tens*).

96 (*tens*) are contained 6· times in 579 (*tens*). Therefore 6· appears to be the units' figure of root. To test it, add 6· to Second Trial Divisor, making Second Complete Divisor 966·, which multiplied by 6· = 5796· This being subtracted leaves no remainder.

The three Subtrahends are, therefore, equal to the given number, 236196· They also together make up the square of 486· For

$$\left.\begin{array}{lcr}\text{1st Subt}^{\text{nd}} & = & 160000\cdot \\ \text{2nd}\ \ ,, & = & 70400\cdot\end{array}\right\} = (48\ \textit{tens})^2 = t^2$$

$$\text{3rd}\ \ ,, \quad = \quad \underline{5796\cdot} \quad = (\text{twice } 48\ \textit{tens} + 6)\times 6 = (2\,.\,t+u)\times u$$

$$486\cdot \times 486\cdot = \underline{236196\cdot} \quad = t^2+(2\,.\,t+u)\times u \quad \text{See Page } 391\cdot$$

$$\therefore \sqrt{236196\cdot} = 486\cdot$$

When, therefore, the given number contains three or more periods, work the left-hand two as if they were all. Then consider the two root-figures thus found as tens. To right of remainder take down next period, and proceed as if this were the only period left. This will give a third root-figure. Consider all three root-figures, taken together, as tens, and work with next period, if any. Proceed thus, until all the periods are worked out. Note that

I. Each new Trial Divisor is formed from last previous Complete Divisor by adding to it last-found figure of root.

II. Each Trial Divisor is equal to *twice the root so far found.*

1st Trial Divisor = twice left-hand root-figure.
2nd Trial Divisor = twice two left-hand root-figures.

III. The sum of any Subtrahend and all previous ones is equal to square of root so far found.

Sum of first 3· Subtrahends = square of first 3· root-figures.
Sum of first 7· Subtrahends = square of first 7· root-figures.

The next example worked out will shew how to manage ciphers in root.

To Extract the Square Root of any Number.

First. Point the number into periods, by placing a dot over the units' place, and one also over every alternate place to right and left of units.

Second. Find square root of greatest square number less than first (left-hand) period. Put this square number under first period, and its square root (in the place of a quotient) as first root-figure.

Third. Subtract square number from first period. To right of remainder take down second period. This forms First Dividend.

Fourth. Form First Trial Divisor, by doubling first root-figure.

Fifth. Ascertain how often Trial Divisor is contained in Dividend (exclusive of right-hand figure). Quotient is new root-figure. Put it to the right of former root-figure.

Sixth. Join new root-figure to right of Trial Divisor. Multiply Complete Divisor by new root-figure. Put product under Dividend. Subtract. Take down next period, thus making New Dividend.

Seventh. Form *New Trial Divisor* by adding last-found root-figure to *Former Complete Divisor*.

Eighth. Repeat Fifth, Sixth, and Seventh steps. Continue such repetition with every successive period.

Add decimal periods (of 00 each) when necessary.

THIRD EXAMPLE WORKED OUT. APPROXIMATE SQUARE ROOT OF A NUMBER THAT IS NOT SQUARE. Find square root of 3· to ten places of decimals.

Example completed.

$\dot{3}\cdot0\dot{0}$ (1·7320508075 &c.
1·

27) 200
189

343) 1100
1029

3462) 7100
6924

346405) 1760000
1732025

34641008) 279750000
277128064

3464101607) 26219360000
24248711249

34641016145) 197064875100
173205080725

23859794375

SQUARE ROOT OF A FRACTION.

To find the square root of a Fraction, find those of Numerator and of Denominator. Or, reduce the fraction to the decimal form, and extract square root.

$$\text{Thus: } \sqrt{\frac{169}{1024}} = \frac{\sqrt{169}}{\sqrt{1024}} = \frac{13}{32}. \qquad \text{For } \frac{13}{32} \times \frac{13}{32} = \frac{169}{1024}.$$

EXERCISE CXXXI.

Prove every example which has an exact square root. Trace the process.

1· Extract the square roots of 2916·, 22·09, and 1536·64
2· Find $\sqrt{41\cdot9904}$, and $\sqrt[2]{9\cdot891025}$
3· Of what number is 378018·388224 the square?
4· What number multiplied by itself will produce 3721976064·?
5· The product of two equal factors being 870·25, what is each of them?
6· Extract square roots of ·0000121104, and of ·004970673009
7· Extract square root of 7· to six places of decimals.
8· Extract, to seven places of decimals, $\sqrt{8\cdot}$ and $\sqrt{11\cdot}$
9· Find $\sqrt{\frac{60025}{339889}}$, and $\sqrt{\frac{96721}{2460016}}$
10· Shew that $\sqrt{\cdot\dot{1}} = \cdot\dot{3}$
11· Find the square root of $\cdot\dot{0}1234567\dot{9}$
12· Also of $113\cdot\dot{7}$, and of $\cdot02\dot{7}$

Extract the square roots

13· Of $9579\frac{33}{64}$.
14· Of 36650916·
15· Of 19·044496
16· Of ·00156816
17· Of 93760489·
18· Of $1\frac{24}{25}$, and of $38\frac{11}{25}$
19· Of 9971220736·
20· Of 139·, and of 14· to five places each.
21· Of $881\frac{55}{361}$ and of $\cdot\dot{0}025$
22· Of 216270112515625·
23· Of 49505999·6025
24· Of ·099856, and of $\cdot3402\dot{7}$
25· Of $\cdot\dot{0}20408163265306122448979591836734693877555\dot{1}$

USES OF THE SQUARE ROOT.

FIRST EXAMPLE WORKED OUT. SQUARES AND PARALLELOGRAMS. A square field has an area of 18·a. 3·r. 25·p. statute measure, how long is its side?

Since *Area of a Square = square of its side,*
∴ *Side of Square = square root of Area.*

Example completed.

Area = 18·90625 acres = 189·0625 supl. chains.
∴ Side = $\sqrt{189\cdot0625}$ su. ch. = 13·75 lineal chains.

PARALLELOGRAM REDUCED TO A SQUARE.

SECOND EXAMPLE WORKED OUT. It is required to find the side of a Square having the same Area as a Parallelogram of which the base measures 144· chains, and the perpendicular height 22· chains 9· links.

Example completed.

Area of Square = (144· × 22·09) su. ch. = 3180·96 su. ch.
∴ Side of Square = $\sqrt{3180{\cdot}96}$ superficial chains.
= 56·4 lineal chains.

THE MEAN PROPORTIONAL.

When the second term of a Proportion is the same as the third, it is a mean proportional between the first and fourth.

Let m represent a mean proportional between 6· and 24· Then,

$$\text{As } 6\cdot : m :: m : 24\cdot$$

Therefore, by PRINCIPLE XXIV.,

$$m \times m = 6\cdot \times 24\cdot$$
$$m^2 = 144\cdot$$

And, these two quantities being equal, their square roots are also equals

$$\therefore m = \sqrt{144\cdot} = 12\cdot$$
$$6\cdot : 12\cdot :: 12\cdot : 24\cdot$$

The Mean Proportional between two quantities is the Square Root of their Product.

The second example worked out shews, then, that if a square be equal to a parallelogram, the side of the square is a mean proportional between the length and perpendicular height of the parallelogram.

THE CIRCLE.

THIRD EXAMPLE WORKED OUT. What is the diameter of a circle containing 70· ac. 2· rd. 29·76 po. statute measure?

"A circle is a plane figure, contained by one line, which is called the circumference, and is such that all straight lines drawn from a certain point within the figure to the circumference are equal to one another. And this point is called the centre of the circle. The diameter of a circle is a straight line passing through the centre, and terminated both ways by the circumference." *Euclid.*

The Area of a circle cannot be exactly ascertained. It has, however, been proved to be nearly = ·7854 of the square, each side of which touches the circumference of the circle externally.

The side of such a square is = diameter of the circle about which it is circumscribed.

$\therefore$ Area of Square = diameter^2 (of circle).
$\therefore$ Area of Circle = ·7854 of square of diameter.

And, conversely,

$\because$ Diameter^2 = Area of circle $\div$ ·7854
$\therefore$ Diameter = $\sqrt{}$(Area $\div$ ·7854)

Example completed.

Area of circle = 70 a. 2 r. 29·76 p. = 706·86 sup. ch.
$\therefore$ Sq. of diam. = (706·86 $\div$ ·7854) sup. ch. = 9· sup. ch.
$\therefore$ Diameter = $\sqrt{}$9· sup. ch. = 3· lineal chains.

To find the circumference of a circle (nearly).

Multiply Diameter by 3·1416

To find the solidity of a cylinder (nearly).

Multiply Area of circular end by units in length.

Solidity of Cylinder = Area of End × Length.
" " = Diam.^{2} × ·7854 × Length.
$\therefore$ Diameter^2 = Solidity $\div$ ·7854 $\div$ Length.
$\therefore$ Diameter = $\sqrt{}$(S $\div$ L $\div$ ·7854).
Also, Length = Solidity $\div$ Area of End.
= Solidity $\div$ Diam.^{2} $\div$ ·7854

THE RIGHT-ANGLED TRIANGLE.

"In any right-angled triangle, the square described upon the side opposite to the right angle is equal to the sum of the two squares described upon the sides containing the right angle." *Euclid.* Book I. Prop. 47·

The side opposite to the right angle is called the ***hypotenuse;*** the others are the ***base*** and ***perpendicular.***

$\because$ Hyp.^{2} = Base^2 + Perp.^{2} $\therefore$ Hyp. = $\sqrt{}$(Base^2 + Perp.^{2})

Also,

$\because$ Base^2 = Hyp.^{2} − Perp.^{2} $\therefore$ Base = $\sqrt{}$(Hyp.^{2} − Perp.^{2})

And,

$\because$ Perp.^{2} = Hyp.^{2} − Base.^{2} $\therefore$ Perp. = $\sqrt{}$(Hyp.^{2} − Base.^{2})

Hence, any two sides of a right-angled triangle being given, the third side may be found.

EXERCISE CXXXII.

Prove every Example.

1· Find the side of a square field containing 15·ac. 2·r. 20·po. statute measure.

2· A square court-yard has an area of 77· square yards 64· square inches. How long is its side?

3· The length and breadth of a rectangle are respectively 34· chains, 81· links, and 25· chains. Find the side of the square of equal area.

4· A triangular field 84·1 ch. long, and 28·8 ch. in perpendicular height, is to be exchanged for a square piece of land of equal value per acre. Find the side of the square.

5· Find the diameter of a circle whose approximate area is 10·ac. 1·r. 21·9064 po. statute.

6· How long a ladder will reach a window 22·ft. 6·in. high, from the opposite side of a street 30·ft. broad?

7· The foot of a tower 63·ft. 8·in. high, is washed by a river, from the opposite bank of which to the summit of the tower a line must be 79·ft. 7·in. to reach. Required the width of the river.

8· Two men start from the same point at the same time. One travels 21· miles due west, and the other 28· miles due south. How far are they then asunder?*

9· The end of an oblong table measures 4·ft. 6·in., and the diagonal 7·ft. 6·in. What is its area?

10· How may 4205· trees be arranged in a rectangle five times as long as broad?

11· A certain number of men, each earning as many farthings as there were men in the party, gained altogether £90· 13·s. 0¼d. How many were there, and what did each earn?

12· The base of a right-angled triangle being 62·92 chains, and its area 148· ac. 1· r. 33·5584 po. statute, what is the length of its hypotenuse?

13· Required solidity of a cylinder 10·ft. long and 3·ft. in diameter.

14· Find the weight of a cylinder of granite 6½ ft. long and 2· ft. in diameter; sp. gr. 2·688

15· How much water per hour might pass through a circular pipe 10· inches in diameter in the clear, at the uniform rate of 8· feet per second?

16· How long is a cylinder which, having a diameter of 3· feet, contains 63· cu. ft. 1066·8672 cu. in.?

17· Find diameter of a cylinder 5·ft. 8·in. long, and containing 33· cu. ft. 1136·4408 cu. in.

EXTRACTION OF THE CUBE ROOT.

Pointing for Cube Root. Since $1^3 = 1\cdot$; $10^3 = 1000\cdot$; $100^3 = 1000000\cdot$, &c.

∴ Of any single digit the cube is under 1000·

Of any No. containing 2· places the cube is between 1000· and 1000000·

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* Taking no note of the earth's curvature.

And, conversely,

If the cube has not more than *three* places, the root has *one*.
If the cube has *four*, *five*, or *six* places, the root has *two*.
If the cube has *seven*, *eight*, or *nine* places, the root has *three*.

Hence,

A Cube Number never has more, and can only have two less, than three times as many places as its Cube Root has. And, for the units' place, and for every three places besides, in the Cube Number, there will be one place in the Cube Root.

Consequently, we point for cube root by placing a dot over units' place, and a dot also over every third place to the right and left of units, thus forming *periods of three places each*, except the left-hand period, which may consist of *one* or of *two*. Hence, 37148632·1584, when pointed for cube root, thus:

$$3\dot{7}14\dot{8}63\dot{2}\cdot15\dot{8}40\dot{0}$$

contains four periods of three places each, and one of two places only, ciphers being added to complete the right-hand period in Decimals. *For every period there will be one place of figures in the Cube Root.*

Formation of a Cube Number. The cube of any number may be made up by the addition of two quantities, namely:

I. The cube of the tens.
II. $\{(\text{Tens}^2 \times 3\cdot) + (\text{Tens} \times 3\cdot \times \text{Units}) + (\text{Units} \times \text{Units})\}$

EXAMPLE.

$$94\cdot = 9 \text{ tens} + 4 \text{ units.}$$

$$\left.\begin{array}{l}\text{Cube}\\ \text{of } 94\cdot\end{array}\right\} \begin{array}{ll} = 90^3 & = 729000\cdot \\ + \{(90^2 \times 3\cdot) + (90\cdot \times 3\cdot \times 4\cdot) + 4^2\} \times 4\cdot & = \underline{101584\cdot} \end{array}$$

$$\text{For, } 94\cdot \times 94\cdot \times 94\cdot = 830584\cdot$$

EXTRACTION OF THE CUBE ROOT.

The cube of a number, as constructed above, may be more concisely expressed thus:

$$(t + u)^3 = t^3 + (3.\ t^2 + 3.\ t.\ u + u^2) \times u$$

where t represents the *tens* of given number, u its *units*, and the full point stands for the sign of multiplication.

In extracting the cube root, then, we first find tens of root, and subtract their cube. Remainder is Dividend.

Complete Divisor is $(3.\ t^2 + 3\ .\ t.\ u + u^2)$ which will be contained u times in Dividend.

But, Trial Divisor is $3.\,t^2$; that is 3 times sq. of tens.

In the preparation of these Divisors we employ two columns.

In the right-hand column we put $3.\,t^2$ = *Trl. Divsr.*
In the left-hand column we first get $3.\,t + u$, which we then multiply by u, and carry out (in right-hand column) the product, which is $= 3.\,t.\,u + u^2$
Then, by addition of right-hand column, we get $3.\,t^2 + 3.\,t.\,u + u^2 =$ *Comp. Div.*
which we multiply by *units* of root, making

$$(3.\,t^2 + 3.\,t.\,u + u^2) \times u.$$

FIRST EXAMPLE WORKED OUT. CUBE NUMBER OF TWO PERIODS. Extract the cube root of 830584·

Example completed.

tens		$83\dot{0}58\dot{4}$·(94· cu. rt.
*18	*hunds.*	729 *thous.* $= t^3$
9 × 9 (*tens*) × 3· =	*243 Tr. Div.	101584·
†27,4·×4· =	1096·†	
	‡25396· × 4· =	101584· $= (3.\,t^2 + 3.\,t.\,u + u^2) \times u$

EXPLANATIONS OF LINES.

Left Column.	**Right Column.**
* 18(0·) = 2. *tens.*	*243(00·) = $3.\,t^2$
9(0·) = *tens.*	† 1096· = $3.\,t.\,u + u^2$
† 274· = 3. *tens* + *units.*	‡ 25396· = $3.\,t^2 + 3.\,t.\,u + u^2$

EXPLANATION OF THE PROCESS.

Having pointed the number, we find our first period to be 830 (thous.) Greatest cube No. under this is 729 (thous.), of which the cube root is 9 (tens). We put 9 in root, and cube of 9 (that is 729) under the first period; subtract, and take down second period, making dividend 101584·

We then put twice 9 (tens) in left-hand column, under which we write also 9 (*tens*), which we × 9 (tens) and by 3·, carrying out product 243 (*hunds.*) in right-hand column for Trial Divisor. This being contained 4 times in the 1015 hunds. of Dividend, we put 4 for units' figure of root. We then add together the two lines in left-hand column, and affix to the right of the sum the new root-figure (4); multiply the number thus formed by 4, and carry out product under Trial Divisor, but two places to the right. (*Why?*) We then add the product, thus placed, with the Trial Divisor, making Complete Divisor 25396·, which we multiply by 4·, and carry out product under Dividend.

EXTENSION OF THE PROCESS.

To obtain the cube root of a number of more than two periods, we extend the method as we did in square root. That is to say, having worked first two periods, as if these were all, we then consider the two root-figures thus found as *tens*, and work out the next period. Then, considering the *three* root-figures as tens, we work out the next period, and so on; always considering the root-figures so far found as tens, whilst working for the next root-figure.

SECOND EXAMPLE WORKED OUT. A CUBE NUMBER OF MORE THAN TWO PERIODS. Extract the cube root of 847396215847·

Example completed.

		847396215847·(9463·*
18		729
9 × 9 × 3	= 243	) 118396
*274 × 4	= 1096	
	*25396 × 4	= 101584*
	16	
8	26508†	) 16812215
2826 × 6	= 16956	
	2667756 × 6 =	16006536
	36	
12	2684748	) 805679847·
28383 × 3	= 85149·	
	268559949· × 3 =	805679847·

* Thus far, the work and the two root-figures found, are the same as in the first example.

THE NEW WORK.

† IN RIGHT-HAND COLUMN. As the Trial Divisor is always *three times the square of tens* of root, and as we now (whilst seeking a third root-figure) consider the 94 as tens, Second Trial Divisor must be three times the square of 94 (*tens*). And this is easily made up by merely adding 16 (which is the square of last-found root-figure) with former Complete Divisor, and with the number standing over that Divisor.

The square of any number $= \textit{tens}^2 + \textit{twice tens} \times \textit{units} + \textit{units}^2$.
$\therefore$ square of 94 $= 90^2 + 2 \times 90 \times 4 + 4^2 = A$.
And 3 times square of 94 $= 3 \times 90^2 + 6 \times 90 \times 4 + 3 \times 4^2$

But, from last example, we learn
that 1096 $= 3 \times 90 \times 4 + 4^2$
and that 25396 $= 3 \times 90^2 + 3 \times 90 \times 4 + 4^2$
$\therefore$ By adding 16 $= 4^2$
we get 26508 $= 3 \times 90^2 + 6 \times 90 \times 4 + 3 \times 4^2$
$= 3$ times $(90^2 + 2 \times 90 \times 5 + 4^2) = 3$ times A.
$= 3$ times 94^2; which was to be found.

Each new Trial Divisor is, therefore, found by adding square of last-found root-figure to former Complete Divisor, together with the number standing over it.

‡ In left-hand column we must get 3 times tens + units (see former example), that is, 3 times 94 *tens* + 6; or, which is the same thing, 94 *tens* × 3, with 6 placed on the right of the product (Principle viii.).

Now, 3 times 94 = 3 times 90 + 3 times 4
But the number last found in this column, namely, 274 is = 3 times 90 + once 4
To which add 8 = twice 4
And the sum 282 = 3 times 90 + 3 times 4
= 3 times 94
And 2826 = 3 times 94 tens + 6

Each new number in the left-hand column can therefore be derived from the last previous one by adding to it twice the last former figure of root, and annexing new root-figure to the sum..

The Sum of any Subtrahend and all previous ones is equal to the Cube of Root so far found.

Having seen, in First Example, that this is true of the first *two* Subtrahends, we can shew why it must be true of the first *three;* thus: Let $t = 94$ *tens*, and $u = 6$

First Subtrahend = 729
Second Subtrahend = 101584

Sum of 1st two Subtrahends = 830584 $= t^3$
$3.t^2 = 26508$
$2826 = 3.t + u$
$\times 6 = 3.t.u + u^2 = 16956$

$3.t^2 + 3.t.u + u^2 = 2667756$
Third Subtrahend $= 16006536 = (3.t^2 + 3.t.u + u^2).u$
$t^3 + (3.t^2 + 3.t.u + u^2).u = 846590536$
$= (t+u)^3$ (See p. 399·)
$= (94 \text{ tens} + 6)^3$
Sum of 1st three Subtrahends $= 946^3$*

---

* Prove this also by Involution.

In the same manner it may be shewn that the sum of the first four Subtrahends is equal to the cube of the first four root-figures; but, in that demonstration, *t* will represent 946 (*tens*), and *u* will stand for the fourth root-figure (3).

Now, the four Subtrahends are together equal to the given number 847396215847·, for after subtracting them from it, nothing remains.

But they are also = cube of 9463·

∴ 847396215847· = cube of 9463·

∴ 9463· = cube root of 847396215847·

To extract the Cube Root of any Number.

First. Point the number into periods by placing a dot over the units' place, and also one over every third place to the right and left of units.

Second. Put under first period the greatest cube number less than it, and put cube root of that number for first root-figure. Subtract; and to right of remainder bring down next period, making *First Dividend.*

Third. Put root-figure in left-hand column. Multiply by itself, and by 3, carrying out product in right-hand column. This is *First Trial Divisor.*

Fourth. Ascertain how often Trial Divisor is contained in Dividend. Quotient is new root-figure.

Fifth. Over the former root-figure standing in left-hand column, put twice that figure. Add the two lines.

Sixth. To the sum annex new root-figure. Multiply by new root-figure. Carry out product in right-hand column, under Trial Divisor, but two places to the right. Add with Trial Divisor. The sum is *First Complete Divisor.* Multiply this by new root-figure. Carry out product (which is Second Subtrahend) under First Dividend. Subtract. Take down next period, making *Second Dividend.*

Seventh. Under Complete Divisor put square of last-found root-figure. Add with Complete Divisor, and with the number above it (three lines). This makes new Trial Divisor, with which find new root-figure (as in Fourth Step).

Eighth. Add twice last-found root-figure to the number in left-hand column. Repeat Sixth, Seventh, and Eighth steps as often as required to complete the example.

Note i. When a cipher occurs in the root, annex one cipher to number in left-hand column, and two ciphers to Trial Divisor. Take down next period, &c.

Note ii. To find the cube root of a Vulgar Fraction, take those of its Numerator and Denominator.

EXERCISE CXXXIII.

Prove exact roots. Trace the process.

Extract the Cube Root

1· Of 31554496·
2· Of 25672375·
3· Of 14706125·
4· Of ·004096
5· Of 138·991832
6· Of 907882814987·
7· Of $\cdot\dot{0}3\dot{7}$, and of $11\frac{25}{64}$
8· Of 83110180·544
9· Of ·015625 & $12\cdot\dot{7}0\dot{3}$
10· Of 350447359·153527
11· Find $\sqrt[3]{99425242966\cdot036381696}$
12· Extract $\sqrt[3]{31418376777267496615936\cdot}$

The Fourth, Sixth, and other Roots.

Any root whose exponent is a power of 2· or 3·, or the product of any of their powers, may be extracted by means of the methods explained above. Thus:

4th root = square root of square root.
6th root = square root of cube root.
= cube root of square root.
8th root = sq. root of sq. root of sq. root.
9th root = cube root of cube root, and so on.

EXERCISE CXXXIII. (*continued.*)

13· Extract the fourth, or biquadrate, root of 9971220736·
14· Find the sixth root of 216270112515625·
15· What is the ninth root of 31418376777267496615936·
16· Extract, to seven places each, $\sqrt[3]{3\cdot}$ and $\sqrt[3]{48\cdot}$
17· How long is the edge of a cube, the solidity of which is 112· cu. ft. 10·′ 11·″ 4·‴?
18· A cubical tank will contain 23· tons 18· cwt. 1· qr. 21· lb. 15· oz. of water, sp. gr. 1· How much ground does it stand on, the material being 3· inches thick?
19· A certain square prism is 4· ft. 6· in. long, by 4· ft. wide and 1· ft. 6· in. thick. Required the edge of an equal cube.
20· What is the whole superficial area of a cube whose solidity is 496· cu. ft. 1·′ 11·″ 11·‴?
21· How long is the internal edge of a cubical vessel which will contain 16· hhds. 31· gals. 3· qts. 3· gills?

APPENDIX.

Note to Page 175·

OLD AND NEW STYLES OF RECKONING TIME.

THE JULIAN AND GREGORIAN CALENDARS. The necessity for two years of different lengths, the Common Year of 365· days, and the Bissextile or Leap Year of 366·, arises from the fact that the earth does not make an exact number of diurnal revolutions on her axis during her annual circuit round the sun. Hence the Tropical Year is not an exact multiple of the Solar Day, but contains 365· such days, and a fraction (365·24224 days).

Down to the time of JULIUS CÆSAR, no practical notice of this fraction was taken in the computation of years, and inextricable confusion of dates was the result. For, taking the fraction at exactly $\frac{1}{4}$ of a day, the neglect of it would make every fourth year's date expire one day too soon; one day, that is, before the earth had completed her fourth annual journey round the sun. And, as 365·×4· = 1460·, in that number of dated years one whole such year would be gained. For 1460· such years would only represent 1459· complete revolutions of the earth about the sun; and, hence, the year which would be called 1460· after any given epoch, would be, in reality, only 1459· years after it. The seasons, too, would, by this time, have made the entire round of the year. The longest day, for example, would have fallen, in succession, on every date in the year.

These considerations shew the serious nature of the error which Julius Cæsar sought to remedy, by the addition of an intercalary day (Feb. 29th) to every fourth year; a contrivance for retarding dates that would have been perfectly effectual if their rate of acceleration had been exactly one-fourth of a day per annum. They had not, however, been advancing so rapidly as this; for the Tropical Year is less than 365·25 days, by ·00776 of a day, amounting to 6·984 days in 900· Julian Years. The Julian scheme, therefore, over-corrected the error which it was intended to counteract, and retarded the advance of dates, more than was necessary, by nearly 7· days in 900· years. This new error had produced such inconvenience by the time of POPE GREGORY XIII., that, in 1582·, he promulgated, for its correction, the rule which bears his name. It is that which is explained on page 175·, and it prescribes that, in every 400· years, three which, under the Julian scheme, would have been leap years of 366· days each, shall be common years of 365· days each. This plan accelerates the Julian speed of dates by 6·75 days in 900· years, and, thus, so very nearly counteracts the error, that 10,000· Gregorian Years differ by only 2·6 days from the like number of Tropical Years. And this error might be reduced to a single day in 100,000· years, by (extending the Gregorian principle one step further) declaring that every year divisible by 4000· shall be a common year of 365· days, as suggested by Sir John Herschel.

PROOF.

		Days.
To 365· days × 100,000·	=	36;500,000·
Add 1· day for every 4th year	=	25,000·
∴ 100,000· JULIAN YEARS contain		36;525,000·
Subtract 3· days for every 400· years = 750· d. And 1· day for every 4000· years = 25· d.	=	775·
∴ 100,000· GREGORIAN YEARS contain		36;524,225·
But 100,000· TROPICAL YEARS contain		36;524,224·
Error in 100,000· Gregorian Years = only		1·

When the Gregorian Calendar was promulgated, in 1582·, the accumulated error of the Julian computation amounted to 10· days, which number it was, consequently, necessary to strike out, and this was done by calling the next day to October 4th, October 15th, instead of October 5th.

The Gregorian Calendar was not adopted in England until 1752·, when 11· nominal days were cancelled, by placing the 14th of September next to the 2nd of that month. And these two days are, respectively, the first of **New Style,** and the last of **Old Style,** in English Chronology.

Russia is the only European country in which O.S. is still retained. There is now a difference of twelve days between the two Styles, so that a Russian's New Year's Day is our January 13th.

Note to Pages 87· and 207·

PURE SUBTRACTION.

When borrowing is necessary, Subtraction is always effected by the aid of Addition. For, we say either,

I. *Borrowed Quantity + Minuend-figure — Subtrahend figure.*
or, II. *Borrowed Quantity — Subtrahend-figure + Minuend-figure.*
But, we might dispense with the Addition, by saying,
III. *Borrowed Quantity — (Subtrahend-figure — Minuend-figure).*

EXAMPLE OF PURE SUBTRACTION.

From 3056002143·
Take 98153687·
2957848456·

Here, though we cannot take 7· from 3· we can take 3· of the 7· from the 3·; leaving only 4· to be taken from the borrowed *ten.* The Work, without employing Addition, is as follows:

3 from 7 *four;*	4 from *ten*	6
3 from 8 *five;*	5 from *ten*	5
0 from 6 *six;*	6 from *ten*	4
1 from 3 *two;*	2 from *ten*	8
	5 from *nine*	4
	1 from *nine*	8
5 from 8 *three;*	3 from *ten*	7
4 from 9 *five;*	5 from *ten*	5
	0 from *nine*	9
	0 from *two*	2

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